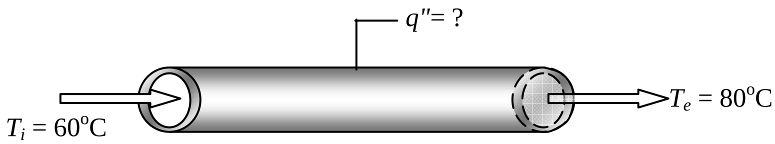
Name________________________ CM________

ROSE-HULMAN Institute of Technology
Department of Mechanical Engineering

ME 462 Thermal Design

ALE – Calculating convective heat transfer coefficients

1. Water flows through a section of 2.54-cm inner diameter tube 3.0 m long. The water enters the section at 60°C with a velocity of 2 cm/s and leaves at 80°C. Assuming that the flow is *fully developed* by the time it enters the region of interest and that the wall is subject to constant wall heat flux
   a. Find the convective heat transfer coefficient h.
   b. find the heat flux on the inner surface in W/m^2.
   c. Find the inner *wall* temperatures at the inlet and the exit.



Hints:

- For internal flow an energy balance on the fluid is another tool for finding $\dot{Q}$ (and thus q").
- How does the surface to fluid to temperature difference vary for fully developed flow with q" = constant? Is there any difference between local and average h, then?
- Are you going to use T_{film} for properties here, or some other T?

Answers:
a. 114 (W/m^2·^{o}C)
b.3468 W/m^2
c. $T_{s,in} = 90.4^oC$, $T_{s,out} = 110.4^oC$

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2. Repeat Problem 1 with a fluid velocity of 2 m/s instead of 2 cm/s.

Answers:
a. 10,300 (W/m^2·^{o}C)
b. 346,715 W/m^2
c. $T_{s,in} = 93.7^oC$, $T_{s,out} = 113.7^oC$

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3. The core of a high powered electro-magnet is to be cooled using glycerin (c = 2447 J/kg-C^o, ρ = 1258 kg/m^3, k = 0.2860 W/m-C^o, μ = 0.6582 kg/m-s) flowing through a 20 mm inner diameter coil wrapped around it. The core produces 1000 W, and keeps the surface temperature of the tubing essentially constant at 47°C. The glycerin enters the coil at 25°C and leaves at 35°C.
   a. Find the required flowrate of glycerin to achieve this cooling.
   b. Find the convective heat transfer coefficient (h) from the tube wall to the glycerin and the required length of tubing.

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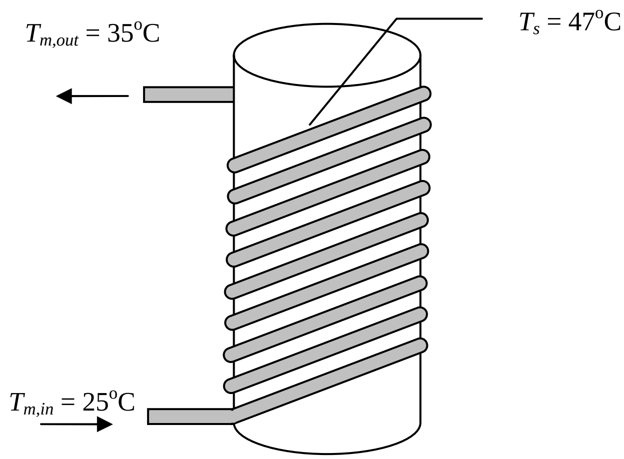

Hints:

- Again, remember that for internal flow an energy balance on the fluid is another tool useful tool.
- Careful before you assume fully developed flow!
- For part b: If you are looking for the required length of tubing, do you want h_{local} or $h_{average}$?

Answers:

a. 0.0409 kg/s
b. 2532 W/m^2-C^o
c. 12.87 m

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4. An *annular* flow passage 5 m long is formed by placing a 2"-nominal schedule 40 cast iron pipe (OD = 6.034 cm) inside a 4"-nominal schedule 40 cast iron pipe (ID = 10.23 cm). If the flow passage conveys 8 liters/s of methanol (ρ = 788.4 kg/m^3, μ = 0.586x10^{-3} kg/m-s, c = 2115 J/kg-C^o, k = 0.286 W/m-C^o), find the average heat transfer coefficient over the 5 m length.

Hints: Not a circular flow passage. What do you use instead of D?

Answers:

2532 W/m^2-C^o. If use *equivalent diameter* with only inner surface heated, 2114 W/m^2-C^o

a) FIND PROPERTIES AT

$$T_{BULK} = \frac{T_{IN} + T_{EXIT}}{2} = 70°C$$

ρ = 977.5 kg/s
μ = 0.404×10^{-3} kg/m-s
k = 0.663 W/m-C°
C = 4190 J/kg-C°
Pr = 2.55

$$Re = \frac{\rho V D}{\mu} = \frac{977.5 \frac{kg}{m^3} \times .02 \frac{m}{s} \times 0.0254\,m}{0.404 \times 10^{-3} \frac{kg}{m\text{-}s}} = \underline{1229}$$

∴ FLOW IS LAMINAR. CONST q'' & FULLY-DEVELOPED MEANS CORRECT NUSSELT CORRELATION IS

$$Nu = \frac{hD}{k} = 4.36$$

$$h = \frac{k\,Nu}{D} = \frac{0.663 \frac{W}{m\text{-}C°} \cdot 4.36}{0.0254\,m} = \boxed{114\ W/m^2\text{-}C°}$$

b) $q'' = \dot{Q}/A$

ENERGY BALANCE ON FLUID

$$\dot{Q} = \dot{m} C (T_{EXIT} - T_{IN})$$

$$\dot{m} = \rho A V = \left(977.5 \frac{kg}{m^3}\right) \frac{\pi (0.0254)^2 m^2}{4} \cdot (0.02) \frac{m}{s} = \underline{9.906 \times 10^{-3} \frac{kg}{s}}$$

$$\dot{Q} = \left(9.906 \times 10^{-3} \frac{kg}{s}\right)\left(4190 \frac{J}{kg\text{-}C°}\right)(80 - 60)\,C°$$

$$= \underline{830\ W}$$

$$q'' = (830\,W) / (\pi \cdot 0.0254 \cdot 3)\,m^2 = \boxed{3468\ W/m^2}$$

c) FOR F-D FLOW W/ q'' = CONST, $T_{WALL} - T_{FLUID}$ = CONST.

a) $\mathrm{Re} = \frac{\rho V D}{\mu} = \frac{(997\ \mathrm{kg/m^3})(2\ \mathrm{m/s})(0.0254)\ \mathrm{m}}{0.404 \times 10^{-3}\ \mathrm{kg/m\text{-}s}} = \underline{122{,}900}$

FLOW IS <u>TURBULENT</u>, F-D W/ $q'' = $ CONST.

FOR TURBULENT FLOW, THE BOUNDARY CONDITION DOESN'T MATTER MUCH.

<u>ONE</u> (CERTAINLY NOT THE ONLY) CORRELATION IS THE

Dittus Boelter eqn:

$\mathrm{Nu} = (0.023)\,\mathrm{Re}^{0.8}\,\mathrm{Pr}^{n}$ $\quad n = 0.3$ FOR COOLING; $= 0.4$ " HEATING

$= (0.023)(122{,}900)^{0.8}(2.55)^{0.4}$

$= 394$

$h = \frac{\mathrm{Nu} \cdot k}{D} = \frac{(394)(0.663\ \mathrm{W/m\text{-}C^\circ})}{0.0254\ \mathrm{m}} = \boxed{10{,}300\ \mathrm{W/m^2\text{-}C^\circ}}$

b) $\dot{Q} = \dot{m} c\,(T_{EXIT} - T_{IN})$

$\dot{m} = \rho A V = \left(997\ \frac{\mathrm{kg}}{\mathrm{m^3}}\right) \frac{\pi (0.0254)^2}{4}\ \mathrm{m^2} \cdot 2\ \frac{\mathrm{m}}{\mathrm{s}} = \underline{0.9906\ \frac{\mathrm{kg}}{\mathrm{s}}}$

$\dot{Q} = (0.9906\ \frac{\mathrm{kg}}{\mathrm{s}})(4190\ \frac{\mathrm{J}}{\mathrm{kg\text{-}^\circ C}})(80 - 60)^\circ\mathrm{C} = 83{,}000\ \mathrm{W}$

$q'' = \dot{Q}/A = \dot{Q}/(\pi D L) = \ldots = \boxed{346{,}700\ \mathrm{W/m^2}}$

c) $T_{WALL,\,IN} = q''/h + T_{IN} = \ldots = \boxed{93.7^\circ\mathrm{C}}$

$T_{WALL,\,OUT} = q''/h + T_{OUT} = \ldots \boxed{113.7^\circ\mathrm{C}}$

a) $\dot{Q} = \dot{m}c(T_{m,out} - T_{m,in})$

$$\dot{m} = \frac{\dot{Q}}{c(T_{m,out} - T_{m,in})} = \frac{1000\ W}{2447\ \frac{J}{kg\text{-}C^\circ}\ (35-25)C^\circ} =$$

$$= \boxed{0.0409\ kg/s}$$

b) $Re = \dfrac{\rho V D}{\mu}$ $\qquad \dot{m} = \rho A V$

$$V = \frac{\dot{m}}{\rho A} = \frac{\dot{m} \cdot 4}{\rho \pi D^2}$$

$$Re = \frac{\cancel{\rho}\left(\frac{4\dot{m}}{\cancel{\rho}\pi D^{\cancel{2}}}\right)\cancel{D}}{\mu} = \frac{4\dot{m}}{\pi \mu D}$$

A HANDY REL'N TO REMEMBER FOR <u>CIRCULAR</u> PASSAGES

$$= \frac{(4)(0.0409\ \frac{kg}{s})}{(\pi)(0.6582\ \frac{kg}{m\cdot s})(0.020\ m)} = \underline{3.96}$$

<u>LAMINAR FOR SURE</u>

CHECK FOR F-D FLOW

$$\frac{L_t}{D} = 0.05\ Re\ Pr$$

WHAT IS Pr?

$$Pr = \frac{\nu}{\alpha} = \frac{\mu/\rho}{k/\rho c} = \frac{\mu c}{k}$$

$$Pr = \frac{(0.6582)(2447)}{(0.2860)} \frac{\frac{kg}{m\cdot s}\cdot\frac{J}{kg\cdot C}}{\frac{W}{m\cdot C}}$$

$$= \underline{5631}\ !!!$$

$$L_t = 0.05\ Re\ Pr\ D$$

$$= (0.05)(3.96)(5631)\cdot(0.020\ m) = \underline{22.3\ m}$$

BETTER NOT ASSUME F-D FLOW!!

USE AVG. Nu CORRELATION FOR DEVELOPING REGION:

$$Nu \overset{?}{=} 3.66 + \frac{(0.065)(D/L)\,Re\,Pr}{1 + (0.04)\left[(D/L)\,Re\,Pr\right]^{2/3}} \qquad (1)$$

NOTE THAT THIS DEPENDS ON L, WH/ WE DON'T KNOW YET!

HENCE, PART c) & b) MUST BE SOLVED SIMULTANEOUSLY & ITERATIVELY:

$$\dot{Q} = hA\,(LMTD)$$

$$\dot{Q} = \overset{?}{h}\,(\pi D L)(LMTD) \qquad (2)$$

$$Nu = \frac{hD}{k} \qquad (3)$$

EQNS (1) → (3) CAN BE SOLVED ITERATIVELY. I WOULD GUESS Nu = 3.66 FIRST (F-D VALUE), FIND h FROM (3) & THEN L FROM (2). THEN I WOULD USE (1) TO FIND A BETTER Nu & REPEAT. OR USE EES (ATTACHED)

RESULT:

$$\boxed{\begin{aligned} Nu &= 5.24 \\ h &= 749 \text{ W/m}^2\,^\circ C \\ L &= 12.9 \text{ m} \end{aligned}}$$

$k = 0.286$ [W/m-C]

$Re = 3.96$

$Pr = 5631$

$T_{in} = 25$ [C]

$T_{out} = 35$ [C]

$T_{wall} = 47$ [C]

$D = 0.02$ [m]

$$LMTD = \frac{T_{wall} - T_{in} - \left[T_{wall} - T_{out}\right]}{\ln\left[\dfrac{T_{wall} - T_{in}}{T_{wall} - T_{out}}\right]}$$

$$Nus = h \cdot \frac{D}{k}$$

$$Nus = 3.66 + \frac{0.065 \cdot \dfrac{D}{L} \cdot Re \cdot Pr}{1 + 0.04 \cdot \left[\dfrac{D}{L} \cdot Re \cdot Pr\right]^{\left[2/3\right]}}$$

$$\dot{Q} = h \cdot \pi \cdot D \cdot L \cdot LMTD$$

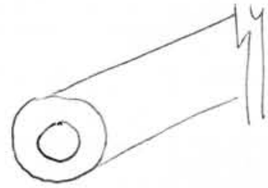
$\dot{Q} = 1000$ [W]

$k_{meth} = \mathbf{k}\left[\text{'Methanol'}, T = 25, P = 101\right]$

SOLUTION

Unit Settings: [kJ]/[C]/[kPa]/[kg]/[degrees]

$D = 0.02$ [m]	$\boxed{h = 74.93 \text{ [W/m}^2\text{-C]}}$	$k = 0.286$ [W/m-C]	$k_{meth} = 0.1983$
$\boxed{L = 12.87 \text{ [m]}}$	$LMTD = 16.5$ [C]	$\boxed{Nus = 5.24}$	$Pr = 5631$
$\dot{Q} = 1000$ [W]	$Re = 3.96$	$T_{in} = 25$ [C]	$T_{out} = 35$ [C]
$T_{wall} = 47$ [C]			



$$D_h = \frac{4A}{P} = \frac{\cancel{4}\left[\frac{\cancel{\pi}\cdot ID_{out}^2}{\cancel{4}} - \frac{\cancel{\pi}\, OD_{in}^2}{\cancel{4}}\right]}{\cancel{\pi}\, ID_{out} + \cancel{\pi}\, OD_{in}}$$

$$= \frac{ID_{out}^2 - OD_{in}^2}{ID_{out} + OD_{in}} = \frac{\cancel{(ID_{out} + OD_{in})}(ID_{out} - OD_{in})}{\cancel{(ID_{out} + OD_{in})}}$$

$$= ID_{out} - OD_{in}$$

$D_h = 10.23 - 6.034 = \underline{4.196 \text{ cm}}$

$\mathrm{Re} = \frac{\rho V D_h}{\mu}$

$\dot{V} = VA$

$$V = \dot{V}/A = \frac{8.5 \text{ l/s}}{\frac{\pi(10.23^2 - 6.034^2)}{4} \text{cm}^2}$$

$$= 0.1585 \frac{\text{l}}{\text{cm}^2\text{s}} \times \left(\frac{1000 \text{ cm}^3}{\text{l}}\right)$$

$$= 158.5 \text{ cm/s} = \underline{1.585 \text{ m/s}}$$

$$\mathrm{Re} = \frac{\left(788.4 \frac{\text{kg}}{\text{m}^3}\right)\left(1.585 \frac{\text{m}}{\text{s}}\right)\left(4.196\times10^{-2} \text{ m}\right)}{\left(0.586\times10^{-3} \frac{\text{kg}}{\text{m-s}}\right)} = \underline{89{,}477}$$

TURBULENT

$\mathrm{Nu} = 0.023\, \mathrm{Re}^{0.8} \mathrm{Pr}^{0.4}$

$\mathrm{Pr} = \frac{\mu c}{k} = \ldots = 4.33$

$= (0.023)(87{,}477)^{0.8}(4.33)^{0.4}$

$= 371.6 = \frac{h \cdot D_h}{k}$

$$h = \frac{\mathrm{Nu}\cdot k}{D_h} = \frac{(371.6)(0.286 \text{ W/m}^\circ\text{C})}{4.196\times10^{-2} \text{ m}}$$

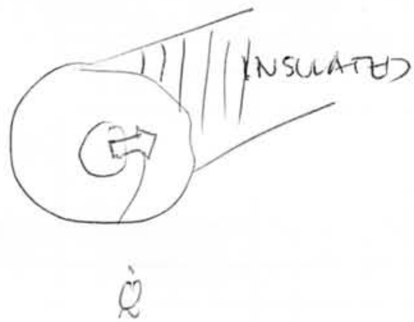
$$= \boxed{2532 \text{ W/m}^2\text{-C}^\circ}$$

SOME FOLKS USE AN <u>EQUIVALENT</u> DIAMETER FOR Nu GIVEN BY

$$D_e \equiv \frac{4A}{P_{HEATED}}$$

WHERE P_{HEATED} IS ONLY THAT PORTION of THE PERIMETER WHERE $\dot{Q}$ IS CROSSING.

FOR EXAMPLE, LET'S SAY THAT THE OUTER PERIMETER IS INSULATED, BUT $\dot{Q}$ CROSSES THE INNER DIAMETER:



$$D_e = \frac{4A}{P} = \frac{\cancel{4}\left[\frac{\pi \, ID_{out}^2}{\cancel{4}} - \frac{\pi \, OD_{in}^2}{\cancel{4}}\right]}{\pi \, OD_{in}}$$

$$= \underline{11.31 \text{ cm}}$$

THIS GIVES:

$$Re = \frac{\rho V D_e}{\mu} = \frac{(788.4)(1.585)(11.31\times10^{-2})}{(0.586\times10^{-3})} = 241{,}177$$

$$Nu = 0.023\, Re^{0.8} Pr^{0.4} = \ldots = 836 \implies h = \frac{Nu \cdot k}{D_e}$$

$$h = \frac{(836)(0.286)}{(11.31\times10^{-2})} = \boxed{2114 \ W/m^2\text{-}C^\circ}$$