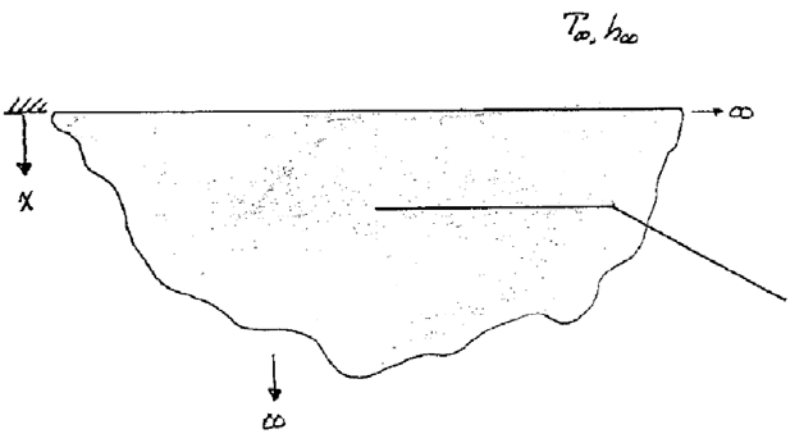
NOTES: Conduction in a semi-∞ medium

CONDUCTION IN A SEMI-∞ MEDIUM (TRANSIENT, THAT IS.)

A SEMI-∞ MEDIUM, INITIALLY AT T_i THROUGHOUT IS SUDDENLY EXPOSED TO A CONVECTIVE MEDIUM WITH h & T_∞.

FIND: $T = T(x, t)$



WRITE THE *Conduction Equation* FOR ANY POINT IN HERE:

REDUCE IT (PER ASSUMPTIONS)

[EQN 1]

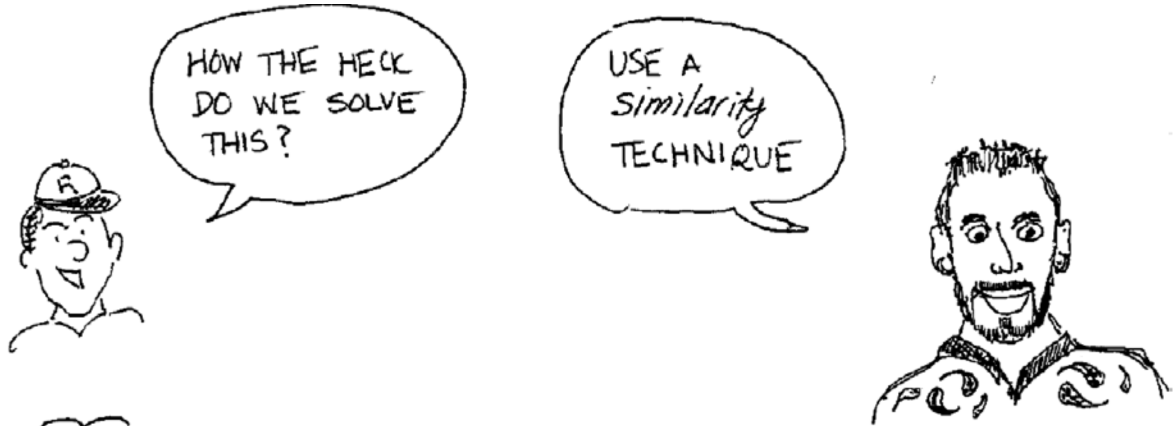
INITIAL & BOUNDARY CONDITIONS

I.C.

B.C. # 1

B.C. # 2

NOTES: Conduction in a semi-∞ medium



DEFINE:

$$\eta \equiv \frac{x}{(4\alpha t)^{1/2}} \quad \Longleftarrow \quad \boxed{\text{SIMILARITY VARIABLE}}$$

TRANSFORM DERIVATIVES:

$$\frac{\partial T}{\partial t} = \frac{dT}{d\eta} \times \frac{\partial \eta}{\partial t} = \left[\qquad\qquad\qquad \right] \frac{dT}{d\eta}$$

$$\frac{\partial^2 T}{\partial x^2} =$$

SUBSTITUTE INTO [1]

NOW IT'S AN *O.D.E.*, NOT A *P.D.E.*!!

[EQN 2]

TRANSFORM I.C. & B.C.s

- I.C. & B.C. #1 COLLAPSE INTO 1 B.C. (*If you can't make this happen, you can't use a similarity technique...*)

$$\left.\begin{array}{l} T(x, t=0) = T_i \\ T(x \to \infty, t) = T_i \end{array}\right\} \quad T(\qquad) =$$

B.C. #2

$$+k\left.\frac{\partial T}{\partial x}\right|_{x=0} = h\left(T_{x=0} - T_{\infty}\right)$$

NOW LET'S INTEGRATE [2]

$$\frac{1}{dT/d\eta}\, d\left(\frac{dT}{d\eta}\right) = -2\eta$$

$dT =$

$T =$

$T =$

C_1 & C_2 COME FROM ____________

BLAH, BLAH, BLAH...

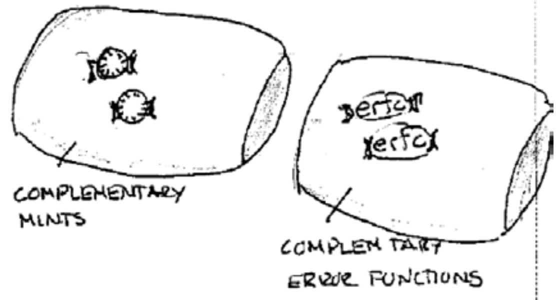
RESULTS:

$$\boxed{\frac{T(x,t) - T_i}{T_{\infty} - T_i} = \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) * \operatorname{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)}$$

where

$$\mathrm{erfc}(u) \equiv 1 - \frac{2}{\sqrt{\pi}} \int_0^u \exp(-u^2)\, du$$

COMPLEMENTARY ERROR FUNCTION



NOTE

$$\frac{T(x,t) - T_i}{T_\infty - T_i} = 1 - \theta$$

PER OUR PREVIOUS NOTATION.

IF B.C. #2 IS $T(x=0,t) = T_s$ INSTEAD:

$$\boxed{\frac{T(x,t) - T_i}{T_\infty - T_i} = \mathrm{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right)}$$

WHAT DO YOU THINK $T(x,t)$ LOOKS LIKE?