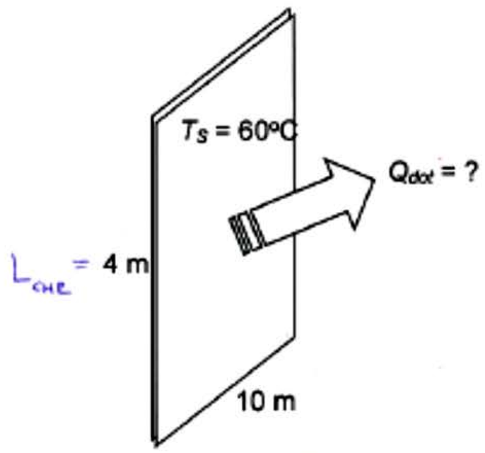
Example

A large vertical plate 4.0 m high is maintained at 60°C and exposed to atmospheric air at 10°C. Calculate the heat transfer rate from the plate if it is 10 m wide.

$$\dot{Q} = \underbrace{h}_{?} A (T_s - T_{air}) \quad (1)$$



Properties:

$$T_f = \frac{T_s + T_\infty}{2} = \frac{60+10}{2}\ °C = 35°C$$

$\rho = 1.145$

$k = 0.02625$ W/m·K

$Pr = 0.7268$

$\nu = 1655 \times 10^{-5}$ m²/s

$$\beta = \frac{1}{T_f} = \frac{1}{(35+273)K} = 0.00325\ K^{-1}$$

$$Gr = \frac{\beta g (T_s - T_{air}) L_{char}^3}{\nu^2} = \frac{(0.00325 \frac{1}{\cancel{K}})(9.81 \frac{m}{s^2})(60-10)\cancel{K}(4^3 \cancel{m^3})}{(1.655 \times 10^{-5})^2\ \cancel{m^4/s^2}}$$

$$= 372.5 \times 10^9$$ ⟸ These numbers are often really big!

$$Gr \times Pr = Ra = (372.5 \times 10^9)(0.7268) = 270.7 \times 10^9$$

With simpler correlation:

$$Nu = 0.1\, Ra_L^{1/3} = (0.1)*(270.7 \times 10^9)^{1/3} = 647$$

$$Nu = \frac{h L_{char}}{k} \qquad h = \frac{k Nu}{L_{char}} = \frac{(0.02625\ W/m \cdot K)(647)}{4\ m} = 4.25\ \frac{W}{m^2 \cdot °C}$$

(1) becomes

$$\dot{Q} = (4.25\ \frac{W}{\cancel{m^2}\ \cancel{°C}})(4\ \cancel{m} \times 10\ \cancel{m})(60-10)\cancel{°C} = \boxed{8.49\ kW}$$

Using more complicated (& more accurate) correlation:

$$Nu = \left\{ 0.825 + \frac{0.387\, Ra^{1/6}}{\left[1 + (0.492/Pr)^{9/16}\right]^{8/27}} \right\} = 701$$

... $h = 4.60\ W/m^2\text{-}K$

... $\dot{Q} = \ldots = \boxed{9.20\ kW}$

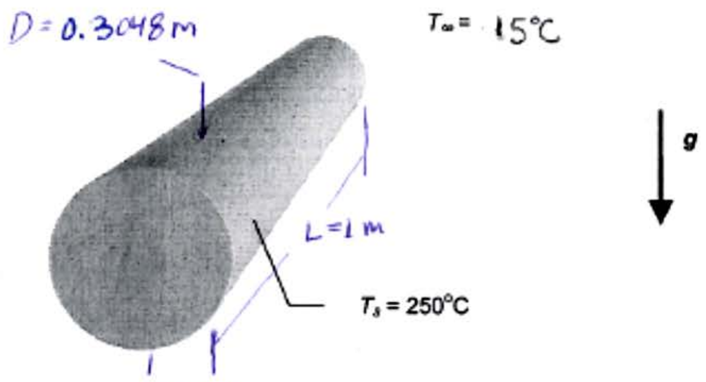
About 8% higher.

Example

The surface of a horizontal pipe 1 ft (0.3048 m) in diameter is maintained at a temperature of 250°C in a room where the ambient air is at 15°C. Calculate the free-convection heat loss per meter of length.

$$T_f = \frac{T_s + T_\infty}{2}$$

$$= 405.5\,K$$



PROPERTIES:

$k = 0.03348$ W/m·K

$\nu = 2.67 \times 10^{-5}$ m²/s

$Pr = 0.703$

$\beta = ? = 1/T_f = 0.002466\,K^{-1}$

CAREFUL TO USE ABSOLUTE

$$Ra = \frac{g\beta(T_s - T_\infty)D^3}{\nu^2} Pr$$

$$= \frac{(9.81\,\frac{\cancel{m}}{\cancel{s^2}})(0.002466\,\frac{1}{\cancel{K}})(250-15)\cancel{K}(0.3048)^3\cancel{m^3}}{(2.67\times10^{-5})^2\,\frac{\cancel{m^4}}{\cancel{s^2}}}\,0.703$$

$$= \underline{1.581 \times 10^8}$$

FOR THIS GEOMETRY & Ra RANGE:

$$Nu = \left[0.6 + \frac{0.387\,Ra^{1/6}}{[1 + (0.559/Pr)^{9/16}]^{8/27}}\right]^2 = \frac{hD}{K}$$

$$= \underline{65.1}$$

$$h = \frac{Nu \cdot k}{D} = \frac{(65.1)(0.03348\,W/m\cdot K)}{0.3048\,m} = 7.15\,W/m^2\cdot K$$

$$\dot{Q} = hA(T_s - T_\infty) = h(\pi D \cdot L)(T_s - T_\infty)$$

$$= \left(7.15\,\frac{W}{\cancel{m^2}\cdot\cancel{K}}\right)(\pi \cdot 0.3048\,\cancel{m} \cdot 1\,\cancel{m})(250 - 15)\cancel{K}$$

$$= 1{,}610\,W = \boxed{1.61\,kW}$$