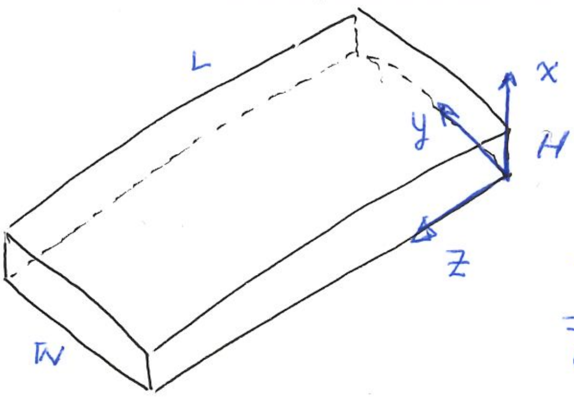
Example

Jeff Spicoli is trying out a new surfboard designed for use on the northern California coast. Since the NoCal waters are noticeably colder than those at Sunset Cliffs, the new board makes use of electrical resistance heating. The surfboard has rectangular cross section and has a width W that is much greater than its thickness H. The bottom of the surfboard is initially in contact with the ocean at its lower surface, and the temperature throughout the board is approximately equal to that of the ocean T_0. Suddenly Spicoli turns on the heater and catches a tasty wave such that an electric current is passed through the entire board and an air-stream of temperature T_∞ is passed over the top surface at a constant rate. The bottom surface continues to be maintained at T_0.

Assuming the board has a constant thermal conductivity k, obtain the differential equation and the boundary and initial conditions that could be used to determine the temperature as a function of time and position in the board.



General 3-D conduction equation

CONST k

$$\frac{\partial}{\partial t}(\rho c T) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\cancel{\frac{\partial T}{\partial y}}\right)_{\to 0} + \frac{\partial}{\partial z}\left(k\cancel{\frac{\partial T}{\partial z}}\right)_{\to 0} + \dot{e}_{gen}$$

$$\rho c \frac{\partial T}{\partial t} = k\frac{\partial^2 T}{\partial x^2} + \dot{e}_{gen}$$

$$\boxed{\underbrace{\frac{1}{\alpha}\frac{\partial T}{\partial t}}_{\text{Need 1 IC}} = \underbrace{\frac{\partial^2 T}{\partial x^2}}_{\text{Need 2 BCs}} + \underbrace{\frac{\dot{e}_{gen}}{k}}_{}}$$

Because of $\dot{W}_{ELEC.}$

I.C. @ $t=0$

$$\underbrace{T(x, t=0) = T_0}$$

BCs

@ $x=0$ $\quad \underbrace{T(x=0,t) = T_0}$

@ $x=H$ $\quad \dot{q}_{COND} = \dot{q}_{CONV}$

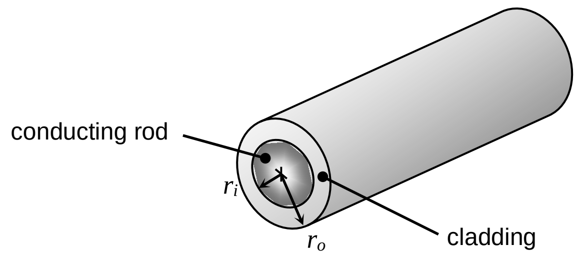
$\dot{q}_{CONV}$

$\dot{q}_{COND}$

$$\underbrace{-k\frac{\partial T}{\partial x}\bigg|_{x=L,t}} = \underbrace{h\left(T(x=L,t) - T_\infty\right)}$$

Example

Electric current is passed through a long conducting rod of radius r_i and thermal conductivity k_r, resulting in a uniform volumetric heat generation of $\dot{e}_{gen}$. The rod is wrapped in an *electrically* non-conducting cladding with outer radius r_o and thermal conductivity k_c. The entire rod/cladding combination is immersed in a flowing fluid with known heat transfer coefficient h and temperature T.



(a) Reduce the conduction equation for steady-state conditions and state the appropriate boundary conditions for the conducting rod.

(b) Reduce the conduction equation for steady-state conditions and state the appropriate boundary conditions for the cladding.

a) USE CYL COOR:

$$\frac{1}{r}\frac{\partial}{\partial r}\left(k_r r \frac{\partial T}{\partial r}\right) + \underbrace{\cancel{\frac{1}{r^2}\frac{\partial}{\partial \phi}\left(k \frac{\partial T}{\partial \phi}\right)}}_{0 \text{ WHY? WHY NOT!?}} + \underbrace{\cancel{\frac{\partial}{\partial z}\left(k\frac{\partial T}{\partial z}\right)}}_{0 \text{ (LONG)}} + \dot{e}_{gen} = \rho c \underbrace{\cancel{\frac{\partial T}{\partial t}}}_{\text{S-S}}$$

$$\boxed{k_r \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) + \dot{e}_{gen} = 0}$$

- BC @ $r = r_i$

$$\dot{q}_r A = \dot{q}_c A$$

WHY THE DISTINCTION?

$$\boxed{-k_r \frac{\partial T_r}{\partial r}\Big|_{r_i} = -k_c \frac{\partial T_c}{\partial r}\Big|_{r_i}}$$

- BC @ $r=0$ $\quad \boxed{\frac{\partial T_r}{\partial r} = 0 \Big|_{r=0}}$ WHY?

b) $$\boxed{k_c \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right) = 0}$$

ALREADY USED! NOW WHAT?

- BC @ $r = r_i$ $$-k_r \frac{\partial T_r}{\partial r}\Big)_{r_i} = -k_c \frac{\partial T_c}{\partial r}\Big)_{r_i}$$

so: BC $r = r_i$ $$\boxed{T_r(r=r_i) = T_c(r=r_i)}$$

- BC @ $r = r_o$ $$\boxed{-k_c \frac{\partial T_c}{\partial r}\Big)_{r=r_o} = h\left(T_c(r=r_o) - T_\infty\right)}$$

$$\dot{Q}_{cond} \Rightarrow \Big| \Rightarrow \dot{Q}_{conv}$$