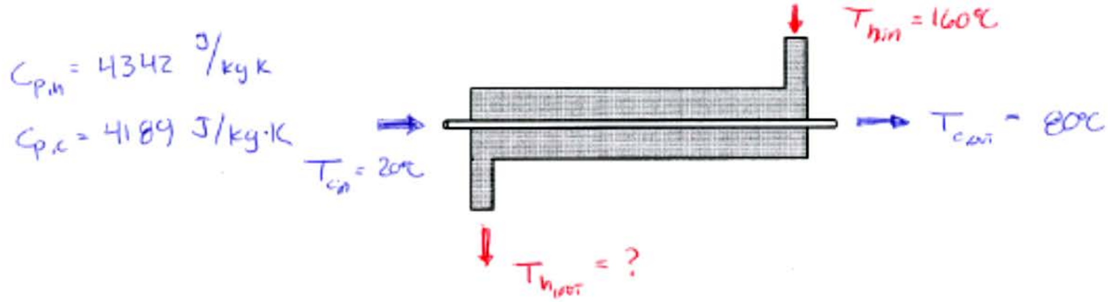
Example

A counter-flow double-pipe heat exchanger is to heat water from 20°C to 80°C at a flow rate of 1.2 kg/s. The warmer fluid is geothermal water available at 160°C and a flow rate of 2 kg/s. The inner tube is thin-walled with a diameter of 1.5 cm. If the **overall heat transfer coefficient** is 640 W/m²-C°, find the required heat exchanger length.



CoE on hot fluid

$$\frac{d}{dt}(E) = \dot{Q}_{IN} + \dot{W}_0 + \dot{m}_h(h_{h,in}) - \dot{m}(h_{h,out})$$

$$\dot{Q}_{out} = -\dot{Q}_{IN} = \dot{m}_h(h_{h,in} - h_{h,out}) = \dot{m}_h c_{p,H}(T_{h,in} - T_{h,out})$$

CoE cold fluid

$$\frac{d}{dt}(E) = \dot{Q}_{IN} + \dot{W}_0 + \dot{m}_c(h_{c,in} + \ldots) - \dot{m}_c(h_{c,out} + \ldots)$$

$$\dot{Q}_{IN} = \dot{m}_c(h_{c,out} - h_{c,in}) = \dot{m}_c c_{p,c}(T_{c,out} - T_{c,in})$$

$$= \left(1.2\ \frac{kg}{s}\right)\left(4189\ \frac{J}{kg \cdot K}\right) \cdot (80 - 20)\ °C \left\langle \frac{W \cdot s}{J} \right\rangle$$

$$= 302{,}000\ W$$

Back to hot fluid. Solve for $T_{h,out}$

$$T_{h,out} = T_{h,in} - \frac{\dot{Q}_{out}}{\dot{m}_h c_{p,h}} = \ldots = 125°C$$

Now can use LMTD relation:

$$\dot{Q} = UA\,\Delta T_{LM} \qquad A = \frac{\dot{Q}}{U\,\Delta T_{LM}}$$

$$\Delta T_{LM} = \frac{(T_{h\,out} - T_{c\,in}) - (T_{h\,in} - T_{c\,out})}{\ln\left(\frac{T_{h\,out} - T_{c\,in}}{T_{h\,in} - T_{c\,out}}\right)} = \frac{(125-20)-(160-80)}{\ln\left(\frac{125-20}{160-80}\right)}$$

$$= 92°C$$

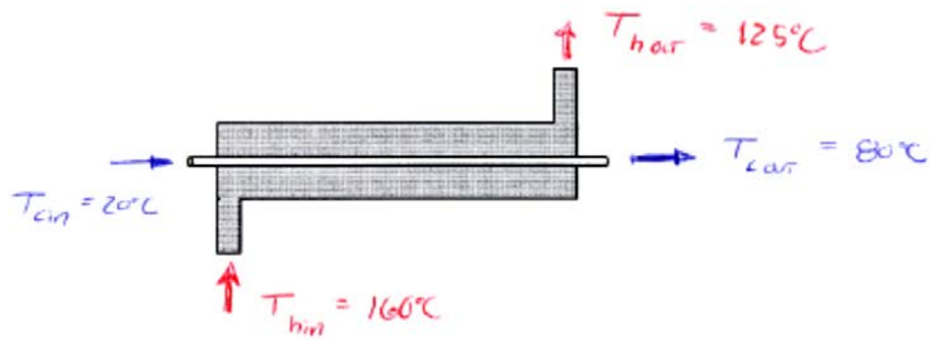
$$A = \frac{302{,}000\ W}{\left(640\ \frac{W}{m^2\,°C}\right)(92\,°C)} = 5.13\ m^2$$

$$A = \pi D L$$

$$L = \frac{A}{\pi D} = \frac{5.13\ m^2}{(\pi)(0.015\ m)} = \boxed{109\ m}$$

Example

Reconsider the last example, but this time make the heat exchanger a *parallel flow* design. As before, the heat exchanger is a double-pipe design, and is used to heat water from 20°C to 80°C at a flow rate of 1.2 kg/s. The warmer fluid is geothermal water available at 160°C and a flow rate of 2 kg/s. The inner tube is thin-walled with a diameter of 1.5 cm. If the overall heat transfer coefficient is 640 W/m^2-C°, find the required heat exchanger length.



Do not need to redo the conservation of energy. The rate of heat transfer remains unchanged & so do the temperatures. <u>LMTD</u> changes.

$$\Delta T_{LM} = \frac{(T_{hin} - T_{cin}) - (T_{hout} - T_{c,out})}{\ln\left(\frac{T_{hin} - T_{cin}}{T_{hout} - T_{cout}}\right)} = \frac{(160-20)-(125-80)}{\ln\left(\frac{160-20}{125-80}\right)}$$

$$= 83.7°C$$ Note it is <u>smaller</u> than $\Delta T_{LM,CF}$

$$A = \frac{\dot{Q}}{U\,\Delta T_{LM}} = \frac{302{,}000\ W}{\left(640\ \frac{W}{m^2\ °C}\right)(83.7°C)} = 5.64\ m^2$$

$$L = \frac{A}{\pi D} = \frac{5.64\ m^2}{(\pi)(0.015\ m)} = \boxed{120\ m}$$