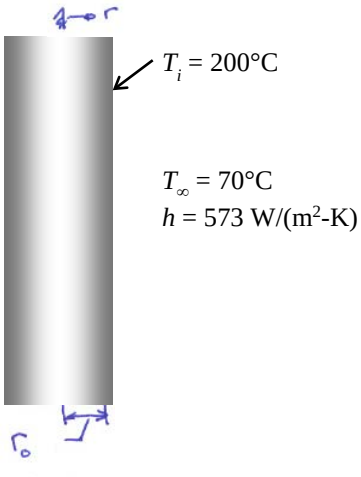
Example

A one meter long aluminum cylinder 15.0 cm in diameter and initially at 200°C is suddenly exposed to a convection environment at 70°C and h = 573 W/(m^{2}-K).

(a) Calculate the temperature at a radius of 1.73 cm 1 min after the cylinder is exposed to the environment.
(b) Calculate the heat lost 1 min after the cylinder is exposed to the environment. Express your answer in J.



PROPERTY DATA (TABLE

$k = 237$ W/m·K

$\rho = 2702$ kg/m^3

$c = 903$ J/kg·K

$\alpha = k/\rho c = 97.1 \times 10^{-6}$ m^2/s

HINT FIND θ_o FIRST.

$$Bi = \frac{hr_o}{k} = \frac{(573)(0.075)}{237} = 0.1813$$

$$Fo = \frac{\alpha t}{r_o^2} = \frac{(97.1\times10^{-6})(60\text{ s})}{(0.075)^2} = 1.036 > 0.2 \text{ OK}$$

TABLE

$\lambda_1 = 0.5842$

$A_1 = 1.0439$

$$\theta = A_1 \exp(-\lambda_1^2 Fo)\, J_0(\lambda_1 r/r_o)$$

$$= (1.0439)\exp(-(0.5842)^2(1.036))\, J_0(\lambda_1 r/r_o)$$

$$\theta = (0.7421)\, J_0\left(\frac{\lambda_1 r}{r_o}\right)$$

$$\frac{\lambda_1 r}{r_o} = 0.5842\left(\frac{1.73}{7.5}\right) = 0.1348$$

TABLE $J_0(0.1348) = 0.9955$

$$\theta = (0.7421)(0.9955) = 0.7387$$

$$\theta = \frac{T - T_\infty}{T_i - T_\infty} \qquad T(r,t) = (\theta)(T_i - T_\infty) + T_\infty$$

$$= (0.7387)(200 - 70)^\circ C + 70^\circ C$$

$$= \boxed{166^\circ C}$$

(b) $$\frac{Q}{Q_{MAX}} = 1 - 2\theta_o \frac{J_1(\lambda_1)}{\lambda_1}$$

TABLE

$$J_1(0.5842) = 0.2798$$

$$\frac{Q}{Q_{MAX}} = 1 - (2)(0.7421)\left(\frac{0.2798}{0.5842}\right) = 0.2891$$

$$Q_{MAX} = mc(T_i - T_\infty) = \rho \forall c (T_i - T_\infty)$$

$$= \rho\left(\frac{\pi D^2}{4} \cdot 1\right) c (T_i - T_\infty)$$

$$= (2702)\left[\frac{\pi (0.15)^2}{4}\right](903)(200 - 70) = \underline{5.61\ MJ}$$

$$Q = (\quad) Q_{MAX} = (0.2891)(5.61) = \boxed{1.62\ MJ}$$