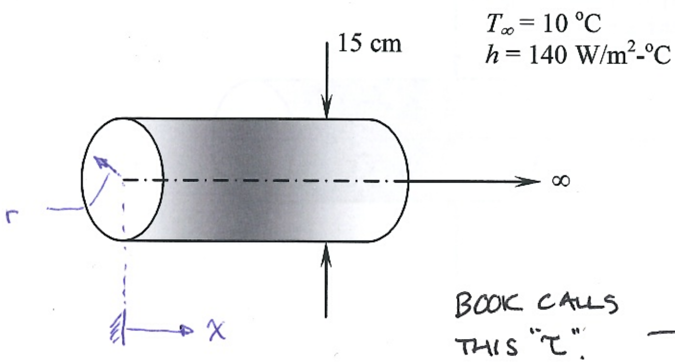
Example

A semi-infinite aluminum cylinder (k = 237 W/m-°C, α = 9.71 x 10^{-5} m²/s) of diameter D = 15 cm is initially at a uniform temperature of T_i = 150°C. The cylinder is now placed in water at 10°C, where the convection heat transfer coefficient is h = 140 W/m²-°C. Determine the temperature at the center of the cylinder 10 cm from the end surface 8 min after the start of the cooling.

$$\theta(r,x,t) = \theta_{\substack{\text{SEMI}\\ \infty \\ \text{MEDIUM}}}(x,t) \times \theta_{\substack{\infty\\ \text{CYLINDER}}}(r,t)$$



∞ CYLINDER:

$$Bi = \frac{h r_o}{k} = \frac{(140)(0.075)}{237} = 0.0443$$

$$Fo = \frac{\alpha t}{r_o^2} \approx \frac{(9.71 \times 10^{-5})(8 \times 60)}{0.075^2} = 8.286$$

BOOK CALLS THIS "τ".

> 0.2 → 1 TERM APPROX OK.

FROM Bi $\lambda_1 = 0.3251$ $A_1 = 1.0133$

AT CENTER ($r \to 0$)

$$\theta_{0,\infty\,\text{CYLINDER}} = \frac{T - T_\infty}{T_i - T_\infty} = A_1 \exp(-\lambda_1^2\, Fo) = \ldots\ 0.422$$

SEMI-∞ MEDIUM:

$$\theta_{\text{SEMI}\,\infty} = 1 - \frac{T - T_i}{T_\infty - T_i}$$

CAREFUL! THIS LAST TERM IS WHAT I HAVE A SOLUTION FOR!

$$= 1 - \left[\text{erfc}\left(\frac{x}{2\sqrt{\alpha t}}\right) - \exp\left(\frac{hx}{k} + \frac{h^2 \alpha t}{k^2}\right) \text{erfc}\left(\frac{x}{2\sqrt{\alpha t}} + \frac{h\sqrt{\alpha t}}{k}\right)\right]$$

$$= 1 - \left[\text{erfc}(0.232) - \exp(0.0753)\,\text{erfc}(0.360)\right]$$

$$= 1 - \left[0.745 - (1.078)(0.6107)\right] = 0.913$$

SO:

$$\frac{T(r=0, x=0.10\text{m}, t=8\text{ min}) - T_\infty}{T_i - T_\infty} = (0.913)(0.422) = 0.385$$

$$\frac{T - 10°C}{150°C - 10°C} = 0.385$$

$$\boxed{T = 63.95°C}$$