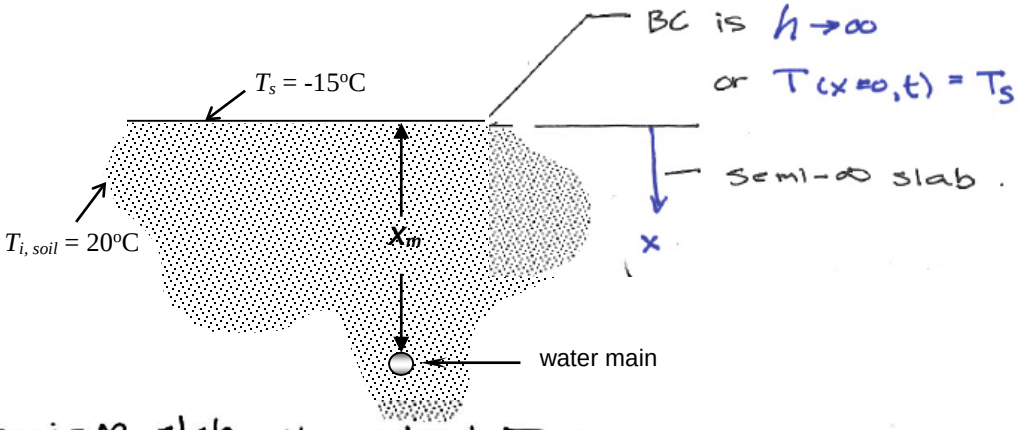
Example

In laying water mains, utilities are concerned about the possibility of freezing during cold periods. What minimum burial depth would you recommend for a water main under the following conditions: Soil, initially at a uniform temperature of 20°C, is subjected to a constant surface temperature of –15°C for 60 days. Assume the properties of soil to be ρ = 2050 kg/m^3, k = 0.52 W/m-°C, c = 1840 J/kg-°C and $\alpha = (k/\rho c)$ = 0.138 x 10^{-6} m^2/s.



Modeling as semi-∞ slab w/ constant T B.C.:

$$\frac{T(x,t) - T_i}{T_s - T_i} = \text{erfc}\left[\frac{x}{2\sqrt{\alpha t}}\right] \longrightarrow T(x = X_m, t = 60 \text{ days}) = 0°C$$

$$\therefore \quad \frac{0 - 20°C}{-15°C - 20°C} = 0.5714 = \text{erfc}\left[\frac{X_m}{2\sqrt{\alpha \cdot 60 \text{ days}}}\right]$$

From table,

$$\frac{X_m}{2\sqrt{\alpha t}} \approx 0.40$$

(That is, $\text{erfc}(0.40) \approx 0.5714$)

$$\therefore X_m = (0.40) \times 2 \times \left(0.138 \times 10^{-6} \frac{m^2}{s} \times 60 \text{ days} \left\langle \frac{24 \text{ hr}}{\text{day}} \right\rangle \left\langle \frac{3600 \text{ s}}{\text{hr}} \right\rangle\right)^{1/2}$$

$$= \boxed{0.677 \text{ m}}$$