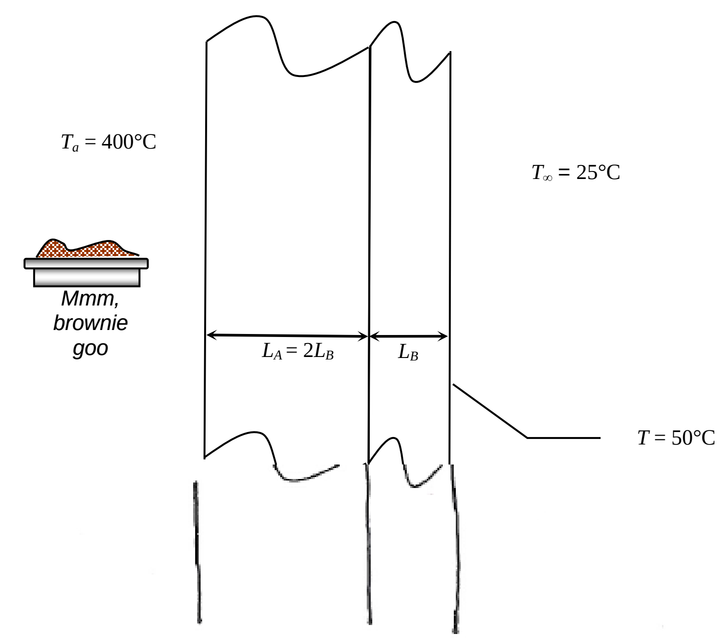
Example

Dr. Thom bakes lots of brownies. In the process, he drips large amounts of brownie goo in his oven. He therefore is looking for a self-cleaning oven. One such oven design involves the use of a composite window separating the oven cavity from the room. The composite consists of two high temperature plastics (A and B) with thermal conductivities $k_A = 0.15$ W/(m·°C) and $k_B = 0.08$ W/(m·K) and thicknesses $L_A = 2L_B$. During the self-cleaning process, the oven air temperature is $T_a = 400$°C, while the room air temperature is $T_\infty = 25$°C. Convective heat transfer coefficients in and out of the oven are approximately 25 W/(m^2·°C).

(a) Find the minimum window thickness $L = L_A + L_B$ needed to ensure a temperature of 50°C on the outer window surface. (Hint: Use the resistance analogy and draw a thermal circuit. Assume that the cross sectional area of the window in 1 m^2 to make life easier.)

(b) Repeat part (a) if there is also a *radiation heat transfer coefficient* inside the oven of $h_r = 25$ W/(m^2·°C).

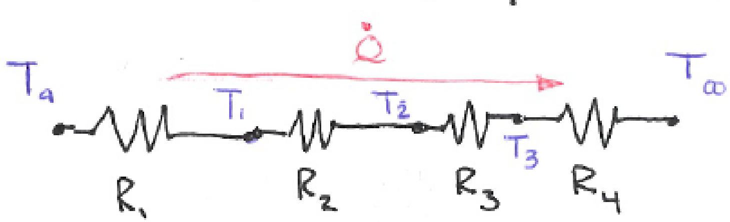


$$R_1 = \frac{1}{h_a A} = \frac{1}{\left(25\frac{W}{m^2 °C}\right)(1 m^2)} = 0.04 \text{ °C/W}$$

$$R_2 = \frac{L_A}{k_A A} = \frac{2L_B}{k_A A} \quad (1)$$

$$R_3 = \frac{L_B}{k_B A} \quad (2)$$

$$R_4 = \frac{1}{h_\infty A}$$



Applying V = IR (OR $\Delta T = \dot{Q} R$) across all resistors:

$$\dot{Q} = \frac{T_a - T_\infty}{R_1 + R_2 + R_3 + R_4} \quad (3)$$

Need another eqn: Apply "Ohm's Law" across resister 4 only:

$$\dot{Q} = \frac{T_3 - T_\infty}{R_4} \quad (4)$$

Now we can solve. Rest is algebra.

Combining (3) & (4)

$$\frac{T_3 - T_\infty}{R_4} = \frac{T_a - T_\infty}{R_1 + R_2 + R_3 + R_4}$$

Substituting expressions for Rs

$$\frac{T_B - T_\infty}{\frac{1}{h_\infty A}} = \frac{T_a - T_\infty}{\frac{1}{h_a A} + \frac{2L_B}{k_A A} + \frac{L_B}{k_B A} + \frac{1}{h_\infty A}}$$

$$h_\infty (T_B - T_\infty) = (T_a - T_\infty)\left[\frac{h_a h_\infty k_A k_B}{k_A k_B h_\infty + 2L_B h_a k_B h_\infty + L_B h_a k_A h_\infty + h_a k_A k_B}\right]$$

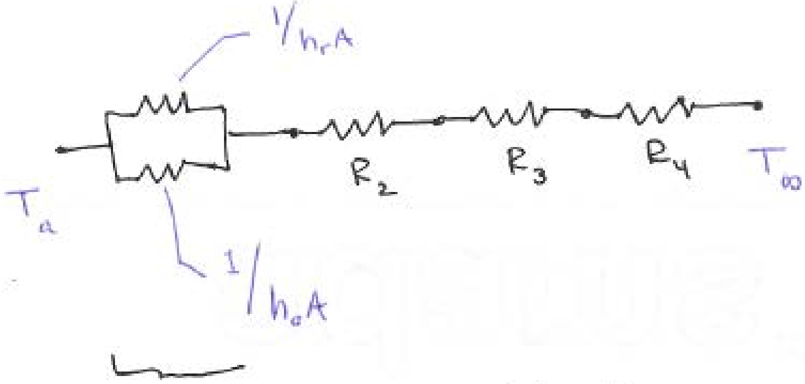
$$L_B = \frac{\frac{T_a - T_\infty}{T_B - T_\infty} k_A k_B h_a - k_A k_B h_\infty - k_A k_B h_a}{2 h_a k_B h_\infty + h_a k_A h_\infty}$$

$$= \ldots = 0.0201 \text{ m} = 2.01 \text{ cm}$$

$$\therefore L_A = 2 \times 2.01 \text{ cm} = 4.02 \text{ cm}$$

$$\boxed{L_A + L_B = 6.03 \text{ cm}}$$

(b) New circuit is



These resisters in parallel add as you expect.

Result...

$L_B = 2.09$ cm

$L_A = 4.18$ cm