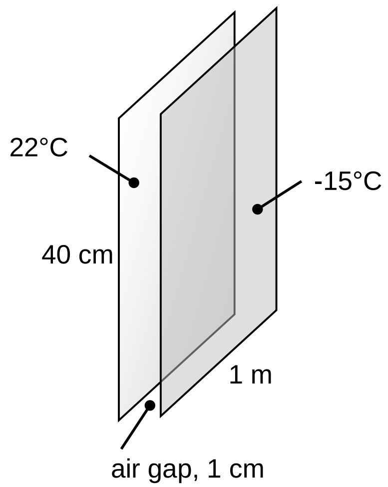
Example

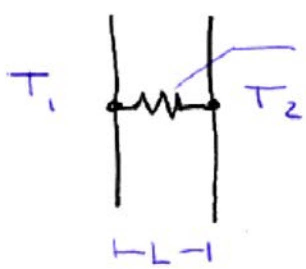
A double pane window is 40 cm high and 1 m wide. The air gap between the two pieces of glass is 1 cm. The inside and outside temperatures of the window are 22°C and -15°C, respectively. Neglecting the thermal resistance of the glass,

(a) calculate the rate of heat transfer through the glass ignoring the effects of natural convection; i.e., if heat transfer is by conduction only.
(b) Calculate the rate of heat transfer through the window considering natural convection.
(c) Repeat part b) if the gap thickness is increased to 2 cm. Discuss the results.



Properties @ $T = \frac{(22-15)°C}{2} = 3.5°C$: $k = 0.0246\ W/m\cdot K$, $\nu = 1.4\times10^{-5}\ m^2/s$, $Pr = 0.717$, $\beta = 0.0036\ K^{-1}$

(a)



$$R_{TH} = \frac{L}{kA} = \frac{0.01\ m}{\left(0.0246 \frac{W}{m\cdot K}\right)(0.40\ m)(1\ m)}$$

$$= 1.016\ °C/W = 1.016\ \frac{°}{W}$$

$$\dot{Q} = \frac{T_1 - T_2}{R_{TH}} = \frac{(22-(-15))°C}{1.016\ °C/W} = \boxed{36.4\ W}$$

(b) Procedure is to find $k_{EFF} = Nu \cdot k$ & use k_{EFF} in above calculation.

$$Ra = \frac{g\beta(T_1-T_2)L^3}{\nu^2} Pr = \frac{9.81 \frac{m}{s^2} \cdot 0.0036 K^{-1} \cdot (22-(-15)) K\ (0.01)^3 m^3}{(1.4\times10^{-5})^2\ m^4/s^2} \cdot 0.717$$

$$= 4742$$

Using $Nu = 0.22\left[\left(\frac{Pr}{0.2+Pr}\right) Ra\right]^{0.28}\left(\frac{H}{L}\right)^{-1/4} = \ldots = 0.873$ Reject. Why?

Using $Nu = 0.42\, Ra^{1/4}\, Pr^{0.012}\left(\frac{H}{L}\right)^{-0.3}$

$$= \ldots = 1.148$$

$$\dot{Q} = \frac{T_1 - T_2}{\frac{L}{k_{eff} A}} = \frac{T_1 - T_2}{\frac{L}{(k\,Nu) A}} = \frac{(22 - (-15))\,°C}{\frac{0.01\ m}{\left(0.0246\ \frac{W}{m \cdot K}\right)(1.148)(0.40)(1)\,m^2}}$$

$$= \boxed{41.8\ W}$$

Note the Çengel does not have any one correlation that meets all our requirements in terms of Rayleigh number range, Pr number, H/L, etc. This happens!

(c) Procedure is the same. Highlights:

$Ra = 37{,}937$

$Nu = 0.42 \cdot Ra^{1/4}\, Pr^{0.012} \left(\frac{H}{L}\right)^{-0.3} = \ldots = 2.37$

$k_{eff} = 0.0585\ W/m \cdot K$

$$\boxed{\dot{Q} = 43.3\ W}$$

Note that trends are difficult to anticipate. You might think, for example, that doubling the air layer should *decrease* $\dot{Q}$, but it increases it instead, most likely due to increased circulation.