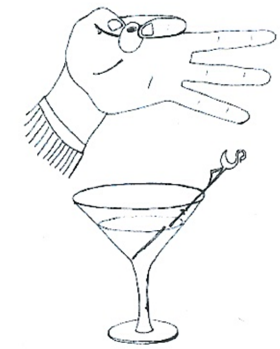
Example



Let's take one last look at the frozen olive problem. We drop a frozen olive initially at a temperature of T_i = 0°C into a martini at a temperature T_∞ = 5°C. We then stir the martini with a flamingo swizzle stick resulting in a convection coefficient of h = 10 W/(m² C°). The olive is modeled as a sphere with 1-cm diameter with ρ = 850 kg/m³, k = 0.350 W/(m² C°) and c_p = 1780 J/(kg C°)

(a) Find the Biot number for the olive in the martini. Is the lumped capacitance model OK?
(b) Find the time constant for the olive in the martini.
(c) How long does it take the olive to warm up to 4°C?
(d) What it the *rate* of heat transfer into the olive when T = 4°C? What is the total amount of heat transferred (Q with no dot!) to the olive during this time?

(a) $$Bi = \frac{h L_{CHE}}{k} \qquad L_{CHE} = \frac{V}{A} = \frac{(4/3)\pi r^3}{4\pi r^2} = \frac{r}{3} = \frac{D}{6} = \frac{0.01\,\text{m}}{6} = 0.00167\,\text{m}$$

$$Bi = \frac{10\,\frac{W}{m^2 K}\cdot 1.67\times10^{-3}\,\text{m}}{0.350\,\frac{W}{m\cdot K}} = 0.048 < 0.1$$

Lumped capacitance OK.

(b) $$\tau_c = \frac{\rho V c_p}{hA} = \frac{\rho c}{h}\cdot\frac{D}{6} = \frac{850\,\frac{kg}{m^3}\cdot 1780\,J/kg\cdot K}{10\,\frac{W}{m^2\cdot K}}\cdot 0.00167\,\text{m} \quad \boxed{252\text{ sec}}$$

(c) $$\frac{T-T_\infty}{T_i-T_\infty} = \exp(-t/\tau_c) \qquad \frac{4°C-5°C}{0°C-5°C} = e^{-t/252\text{ sec}}$$

$$\boxed{t = 406\text{ sec}}$$

(d) $$\dot{Q} = hA_{SURF}(T_\infty - T) \quad \text{(B/C it is \underline{into} olive)}$$

$$= 10\,\frac{W}{m^2 K}\cdot 4\pi\left[\frac{0.01}{2}\right]^2 m^2\,(5°C-4°C) = \boxed{0.00314\text{ W}}$$

Does $Q = \dot{Q}\Delta t$? No!! $\dot{Q} \neq$ CONST

$$Q = \int_{t=0}^{t}\dot{Q}\,dt = \int_{t=0}^{t} hA(T_\infty - T)\,dt$$

where $T = (T_i - T_\infty)e^{-t/\tau_c} + T_\infty$ = NO FUN...

Anyone see an easier way?

Thermal energy balance, finite time:

$$U_2 - U_1 = Q_{IN,12} + \cancel{W}^{\,0}$$

$$mc(T_2 - T_1) = Q_{IN,12}$$

$$= \rho \forall c (T_2 - T_1)$$

$$= \left(850\ \frac{\cancel{kg}}{\cancel{m^3}}\right)\left(\frac{4}{3}\pi \left(\frac{0.01\ \cancel{m}}{2}\right)^{\cancel{3}}\right)\left(1780\ \frac{J}{\cancel{kg\cdot K}}\right)(4-0)\cancel{°C}$$

$$= \boxed{3.17\ J}$$