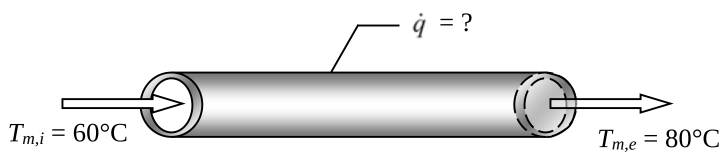
Example

Water flows through a section of 2.54-cm diameter tube 3.0 m long. The water enters the section at 60°C with a velocity of 2 cm/s. Assuming that the flow is **fully developed** (buzza buzza buzz) by the time it enters the region of interest and that the wall is subject to constant wall heat flux,

(a) calculate the wall heat flux (in W/m²) needed to heat the water to 80°C.
(b) Calculate the wall temperatures at the inlet and the exit.
(c) Repeat part a) and b) if the velocity of the water is increased to 2 m/s.



Fluid properties

$$@\ T_B = \frac{T_i + T_e}{2} = 70°C$$

ρ = 977.5 kg/m³

μ = 0.404 x10⁻³ kg/m·s

k = 0.663 W/m·K

c_p = 4190 J/kg·°C

Pr = 2.55

(a) Use cons. of energy:

$$0 = \dot{Q} + 0 + \dot{m}h_i - \dot{m}h_e$$

$$\dot{Q} = \dot{m}(h_e - h_i) = \dot{m}\left[c_p(T_e - T_i) + v(\cancel{P_e - P_i}^{\,0})\right]$$

$$\dot{m} = \rho A V = \rho \frac{\pi D^2}{4} \cdot V$$

$$= 977.5\ \frac{kg}{m^3} \cdot \frac{\pi \cdot 0.0254^2\ m^2}{4} \cdot 0.02\ \frac{m}{s}$$

$$= \underline{9.906 \times 10^{-3}\ kg/s}$$

$$= (9.906 \times 10^{-3}\ \frac{kg}{s})(4190\ \frac{J}{kg \cdot C})(80°C - 60°C)$$

$$= 830\ W$$

$$\dot{q} = \frac{\dot{Q}}{A_{SURFACE}} = \frac{\dot{Q}}{\pi D L} = \frac{860\ W}{(\pi)(0.0254\ m)(3.0\ m)} = \boxed{3468\ W/m^2}$$

NOT $A_{CROSS\text{-}SECTION}$!

(b) Since we want <u>wall</u> temperatures, we need:

$$\dot{q} = h(T_{s,e} - T_e) \quad \& \quad \dot{q} = h(T_{s,i} - T_i)$$

If we can find h, we've got it.

$$Re = \frac{\rho V D}{\mu} = \frac{977.5 \frac{kg}{m^3} \cdot 0.02 \frac{m}{s} \cdot 0.0254\ m}{0.464 \times 10^{-3}\ kg/m\text{-}s} = \underline{1229}$$

→ LAMINAR FLOW

- Laminar
- Fully-developed
- Round duct
- $\dot{q}$ = CONST BC

} … $Nu = 4.36$

$$Nu = \frac{hD}{k} \longrightarrow h = \frac{Nu \cdot k}{D} = \frac{(4.36)(0.663 \frac{W}{m\,°C})}{0.0254\ m} = 113.9 \frac{W}{m^2\,°C}$$

At inlet:

$$\dot{q} = h(T_{s,i} - T_i) \qquad T_{s,i} = T_i + \frac{\dot{q}}{h} = 60°C + \frac{3468\ W/m^2}{113.9\ W/m^2\,°C}$$

$$\boxed{T_{s,i} = 90.4°C}$$

At exit:

$$\dot{q} = h(T_{s,e} - T_e) \qquad T_{s,e} = T_e + \frac{\dot{q}}{h} = 80°C + \frac{3468\ W/m^2}{113.9\ W/m^2\text{-}°C}$$

$$\boxed{T_{s,e} = 100.4°C}$$

Remember, w/ $\dot{q}$ = CONST BC & fully-developed flow, $T_s - T_m$ is the same @ all locations.

(c) The process is the same, the numbers change. Here are the highlights:

$$\dot{m} = \ldots = 0.9906 \text{ kg/s}$$

$$\dot{Q} = \ldots = 83{,}000 \text{ W}$$

$$\dot{q} = \ldots = 343{,}715 \text{ W/m}^2$$

$$Re = \ldots = 122{,}900 \longrightarrow \text{TURBULENT FLOW}$$

- Turbulent
- Fully-developed
- Round duct
- $\dot{q}$ = CONST B.C.

$\left.\right\}$ Many choices.

One easy one:

$$Nu = 0.023\, Re^{0.8}\, Pr^{0.4}$$

$$= (0.023)(122{,}900)^{0.8}(2.55)^{0.4}$$

$$= 394$$

$$h = \frac{Nu \cdot k}{D} = \frac{(394)(0.663 \text{ W/m}\cdot\text{K})}{0.0254 \text{ m}} = \underline{10{,}295 \text{ W/m}^2\,{}^\circ\text{C}}$$

Note how much higher!

Inlet: $T_{s,i} = T_i + \frac{\dot{q}}{h} = \ldots = 93.7^\circ\text{C}$

$$T_{s,e} = T_{s,e} + \frac{\dot{q}}{h} = \ldots = 113.7^\circ\text{C}$$

For turbulent flow, a $T_s - T_m$ of a little over 10°C gives 100x more $\dot{q}$ than for laminar flow.