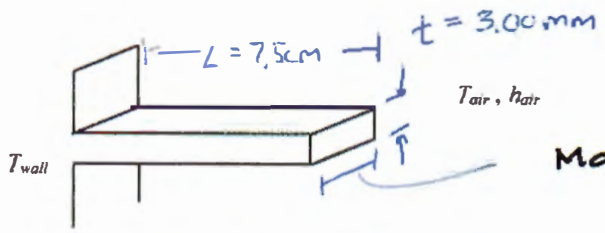
Example

A straight aluminum fin (k = 200 W/m-K) is 3.00 mm thick and 7.5 cm long. It protrudes from a wall whose temperature is maintained at 300°C. The ambient air temperature is T_{air} = 50°C with h_{air} = 10 W/m²-K. Calculate the heat loss from the fin per unit depth assuming

(a) an infinitely long fin, and
(b) an insulated tip with a corrected fin length.



Make $w = 1\ m$

(a) $\dot{Q} = \sqrt{hPkA_c} \cdot (T_b - T_\infty)$

$T_b = T_{wall}$

$T_\infty = T_{air}$

$P = 2 \cdot t + 2 \cdot w$

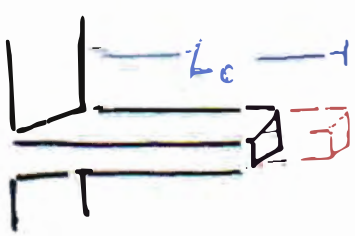
$= (2 \cdot 0.003\ m) + (2 \cdot 1\ m) = 2.006\ m$

$A_c = t \cdot w = (0.003\ m)(1\ m) = 0.003\ m^2$

$$\dot{Q} = \sqrt{10 \frac{W}{m^2 \cdot K} \cdot 2.006\ m \cdot 200 \frac{W}{m \cdot K} \cdot 0.003 m^2} \cdot (300 - 50)\ ^\circ C$$

$= 867\ W$ ANS

(b)



$L_c = L + A_c/P$

$= 7.5\ cm + \frac{0.003\ m^2}{2.006\ m} \left\langle \frac{100\ cm}{m} \right\rangle$

$= 7.65\ cm$

$\dot{Q} = \sqrt{hPkA_c} \cdot \tanh(mL_c) \cdot (T_b - T_\infty)$

$$m = \sqrt{\frac{hP}{kA}} = \sqrt{\frac{10\ \frac{W}{m^2 K} \cdot 2.006\,m}{200\ \frac{W}{m \cdot K} \cdot 0.003\,m^2}} = 5.782\ 1/m$$

$\dot{Q} = \ldots = 360\ W$ ← ANS

Discussion:

These are very different. I would not trust infinitely long.

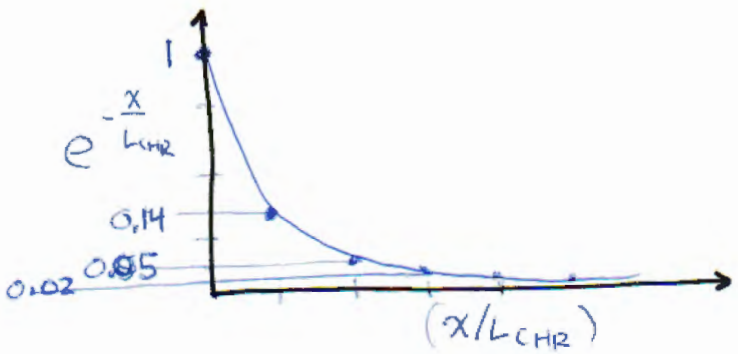
How can we tell when infinitely long is valid?

Remember

$$\left.\frac{T(x) - T_\infty}{T_b - T_\infty}\right|_{\infty\ LY\ LONG} = e^{-mx} = e^{-\frac{x}{(1/m)}} = e^{-\frac{x}{L_{CHR}}}$$

A characteristic length of this fin

$$L_{CHR} = \frac{1}{m} = \frac{1}{5.782\ \frac{1}{m}} = 0.1730\ m = 17.3\ cm \gg L$$



For a fin that is

$L = 4L_{CHR}$ or $5L_{CHR}$

long, infinitely long approximation is not so bad!

It's bad for this fin, though.