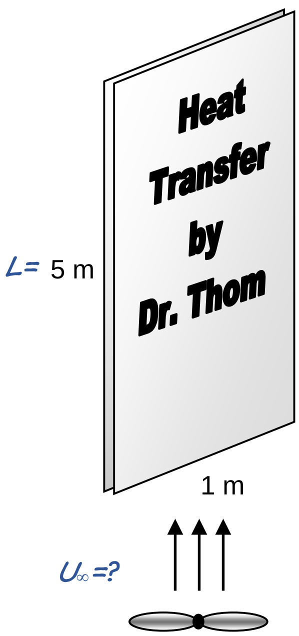
Example

In a fit of temporary insanity, a frustrated Rose student painted a piece of ply wood to resemble a giant novelty-sized heat transfer book, took it to the front lawn, and set it on fire. Luckily, the fire was put out quickly and no one was hurt. Sometime after the fire was put out, it was observed that the "book" temperature was 85°C and the surrounding air temperature was 29°C. A small fan was placed beneath the "book" to aid in its cooling.

(a) Determine the minimum air velocity for which natural convection is negligible.
(b) Find the rate of heat transfer from the "book" if the air velocity is 5 m/s.



(a) Properties @ $T_{film} = \frac{T_s + T_\infty}{2} = \frac{85+29}{2}\,^\circ C = 57^\circ C = 330\ K$

$k = 0.0283\ W/m\cdot K$

$\nu = 1.86 \times 10^{-5}\ m^2/s$

$Pr = 0.708$

$\beta = 0.00303\ K^{-1}$

For minimum $\bar{U}_\infty$, set

$$\frac{Gr}{Re^2} = 0.1 \qquad Gr = \frac{g\beta(T_s - T_\infty)L^3}{\nu^2}$$

$$= \frac{9.81\ \frac{m}{s^2} \cdot 0.00303\ K^{-1}\,(85-29)K\,(5)^3 m^3}{(1.86\times10^{-5})^2\ m^4/s^2}$$

$$= 6.015 \times 10^{11}$$

$$\therefore Re = (0.1/Gr)^{1/2}$$

$$= \frac{\bar{U}_\infty L}{\nu}$$

$$\bar{U}_\infty = \left(\frac{0.1}{Gr}\right)^{1/2} \frac{\nu}{L}$$

$$= \left(\frac{0.1}{6.015\times10^{11}}\right)^{1/2} \cdot \frac{1.86\times10^{-5}\ m^2/s}{5\ m} = \boxed{9.12\ m/s}$$

(b) $\bar{U}_\infty < \bar{U}_{\infty,min}$ $\longrightarrow$ Must find $\overline{Nu}$ for combined forced & natural convection.

$$Nu = \left[Nu_{forced}^{n} \pm Nu_{nat'l}^{n} \right]^{1/n}$$

n = 3 for vertical plates

(+) because natural convection "helps."

Forced convection

$$Re_L = \frac{U_\infty L}{\nu} = \frac{(5\ m/s)(5\ m)}{(1.86\times10^{-5}\ m^2/s)} = 1{,}344{,}086 \longrightarrow \text{Turbulent.}$$

However,

$$Re_{cr} = 5\times10^5 = \frac{U_\infty x_{cr}}{\nu} = \frac{(5\ m/s)\ x_{cr}}{1.86\times10^{-5}\ m^2/s}$$

$\longrightarrow x_{cr} = 1.86\ m$ Almost 40% of plate is laminar. Need combined laminar/turbulent average Nu correlation:

$$Nu_f = (0.037\ Re^{4/5} - 871)\ Pr^{1/3} = \ldots = 1872$$

Natural convection

$$Ra = Gr \cdot Pr = (0.708)(6.015\times10^{11}) = 4.259\times10^{11}$$

Vertical plate: $Nu_n = (0.1)(Ra)^{1/3} = 752$

$$Nu = \left[1872^3 + 752^3 \right]^{1/3} = 1912 = \frac{hL}{k}$$

$$h = \frac{Nu\cdot k}{L} = \frac{(1912)(0.0283\ W/m\cdot K)}{(5\ m)} = 10.8\ W/m^2\cdot K$$

$$\dot{Q} = hA(T_s - T_\infty) = 10.8\ \frac{W}{m^2\cdot K}(5\ m)(1\ m)(85-29)K$$

$$= \boxed{3{,}030\ W}$$