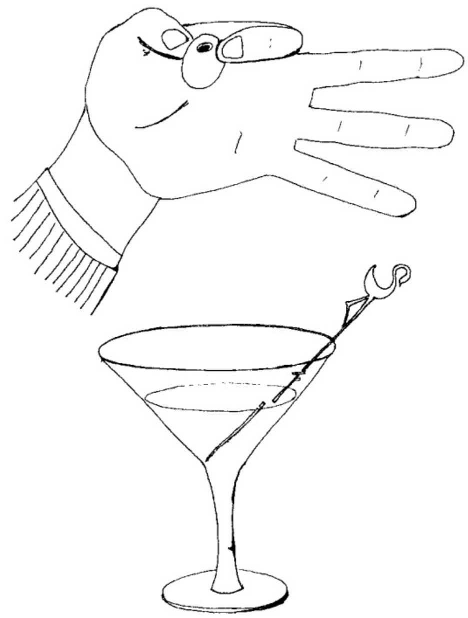
ACTIVE LEARNING EXERCISE: The lumped capacitance method

Consider a frozen olive initially at a temperature of T_i that is dropped into a martini at a temperature T_∞. We then stir the martini with a flamingo swizzle stick. We are interested in how the olive temperature changes with time, most notably how long it takes to warm up to T_∞.



Write **thermal energy balance** for the frozen olive for the time after is dropped into the martini. *Assume that the entire olive is at only one temperature at any point in time*. This is the **lumped capacitance assumption**.

What is the mode of heat transfer to the olive? ________________.

Rewrite the thermal energy balance.

This is a linear, non-homogeneous first order differential equation. We can make is homogeneous by letting

$$\theta = T - T_\infty$$

Do it!

Solve by direct integration:

Apply the initial condition:

The solution to this equation is given by

Rearrange a bit

$$\frac{T - T_\infty}{T_i - T_\infty} =$$

where

$$TC =$$

Now this model says that the olive never reaches T_∞, but it is generally accepted that 4τ is close enough. (At $4 \cdot TC$ you're 98% of the way there).

If the convective heat transfer coefficient between an olive and the martini is h = 100 W/(m^2·K) and the properties of a typical 2-cm diameter spherical olive are given by ρ = 850 kg/m^3 and c_p = 1780 J/(kg·K), we can calculate TC to be

$$TC =$$

which means that in about _________________ (or $4 \cdot TC$) the olive has reached T_∞.

In this, we *assumed* that the entire olive was at one temperature. In other words, we ignored any temperature gradients within the olive and therefore any _________________ heat transfer within it.[1] Was this a good assumption? Let's find out.

The _________________ _________________ is a measure of the internal resistance to conduction of an object to the external convection to which it is subject. It is defined as

$$Bi \equiv \underline{\hspace{6cm}} = \underline{\hspace{4cm}}$$

[1] Actually, we're not ignoring it as much as we are assuming that it is infinitely efficient!

If the Biot number is small (Bi << 1) then this assumption isn't too bad. With k_{olive} = 0.350 W/(m² ·C°) and $L_{char} = V_0/A = r/3$, for the macro-olive we get

$$Bi = \frac{100 \dfrac{\mathrm{W}}{\mathrm{m}^2 \cdot \mathrm{C}^{\mathrm{o}}} \cdot (0.01/3)\,\mathrm{m}}{0.350 \dfrac{\mathrm{W}}{\mathrm{m} \cdot \mathrm{C}^{\mathrm{o}}}} =$$

$Bi << 1$	$Bi = 1$	$Bi >> 1$