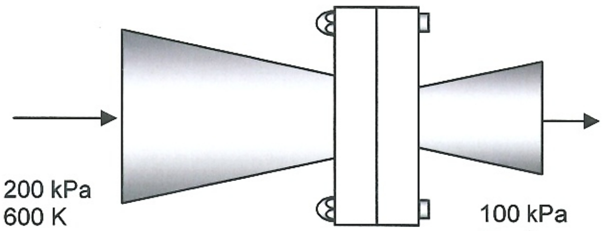
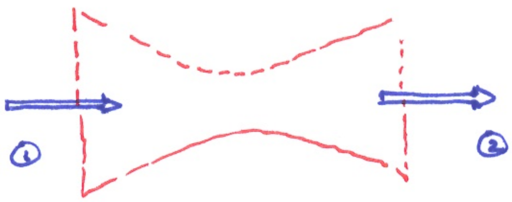
Example

Air flows steadily through a supersonic nozzle. The entering air has negligible velocity. If the process is **reversible and adaibatic,** (Everybody sing! That means it's also ________________!)

(a) find the exit air temperature, and
(b) the exit velocity.
(c) What do you think the *isentropic efficiency* of this nozzle is?[1]



(a)



Cons. of energy

$$\frac{dE_{sys}}{dt} = \dot{Q}_{in} + \dot{W}_{in} + \sum_{in} \dot{m}\left(h + \frac{V^2}{2} + ..\right) - \sum_{out} \dot{m}\left(h + \frac{V^2}{2} + \ldots\right)$$

$$0 = \dot{m}\left(h_1 + \frac{V_1^2}{2}\right) - \dot{m}\left(h_2 + \frac{V_2^2}{2}\right)$$

$$h_2 = h_1 - \frac{V_2^2}{2} \quad (1)$$

Acctng of S

$$\frac{d}{dt}(S_{sys}) = \sum_j \frac{\dot{Q}_j}{T_{b,j}} + \sum_{in} \dot{m}s - \sum_{out} \dot{m}s + \dot{S}_{gen}$$

$$0 = \dot{m}(s_1) - \dot{m}(s_2)$$

$$s_2 - s_1 = 0$$

$$0 = s^\circ(T_2) - s^\circ(T_1) - R\ln\left(\frac{P_2}{P_1}\right)$$

FROM IDEAL GAS TABLES FOR AIR

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[1] 4

$s^\circ(T_1 = 600\ K) = 2.4092\ kJ/kg\cdot K$

$$\therefore\ s^\circ(T_2) = s^\circ(T_1) + R\ln\left(\frac{P_2}{P_1}\right)$$

$$= 2.4092\ \frac{kJ}{kg\cdot K} + 0.287\ \frac{kJ}{kg\cdot K}\ln\left(\frac{100\ kPa}{200\ kPa}\right)$$

$$= 2.21\ \frac{kJ}{kg\cdot K}$$

$$\boxed{T_2 = 495.5\ K} \rightarrow h_2 = 498.2\ \frac{kJ}{kg}$$

(b) FROM (1)

$$V_2 = \sqrt{2(h_1 - h_2)}$$

$$= \sqrt{2(607.02 - 498.2)\ \frac{kJ}{kg}\left\langle\frac{1000\ J}{kJ}\right\rangle\left\langle\frac{m^2/s^2}{J/kg}\right\rangle}$$

$$= \boxed{467\ m/s}$$

(c) THE ISENTROPIC EFFICIENCY IS 100%.