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Example

Dry atmospheric air is actually a mixture of gases including oxygen, nitrogen, argon and trace amounts of other gases. Consider 1 m³ of air for which the **volumetric composition** is 21% O_2, 78% N_2 and 1% Ar. Initially the air is at 27°C and 100 kPa. It is then heated to 227°C at constant volume.

(a) Find the apparent molar mass and the ideal gas constant for the air.
(b) Find the mass of the air.
(c) Assuming variable specific heats,
  i. find the heat transfer added to the air during the process, and
  ii. calculate the entropy ~~generated~~ CHANGE during the process, in kJ/K.
(d) Repeat (c) by using the air tables instead of using the given mixture composition.

(a) Assume you have 1 kmole of mixture.

i	n_i [kmole]	M_i [kg/kmol]	$m_i = n_i M_i$ [kg]	
O_2	0.21	32.00	6.72	
N_2	0.78	28.01	21.85	
Ar	0.01	39.94	0.3994	
			28.97	

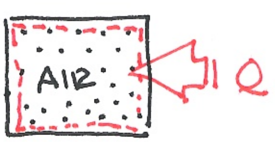
NOTE $M_{MIX} = \sum_i y_i M_i$ $= M_{MIX}$

$$R_{MIX} = R_u / M_{MIX} = \frac{8.314 \text{ kJ/kmol·K}}{28.97 \text{ kg/kmol}} = 0.287 \text{ kJ/kg·K}$$

(b) $P_1 V_1 = m R T_1$

$$m = P_1 V_1 / RT = \frac{(100 \cancel{\text{kPa}})(1 \cancel{\text{m}^3})}{\left(0.287 \frac{\cancel{\text{kJ}}}{\text{kg·}\cancel{\text{K}}}\right)(300 \cancel{\text{K}})} \left\langle \frac{\cancel{\text{kJ}}}{\cancel{\text{kPa·m}^3}} \right\rangle = 1.161 \text{ kg}$$

(c)



$W_{12} = \int_1^2 P\,\cancel{dV} \to 0$

CLOSED SYSTEM, FINITE TIME

$U_2 - U_1 = Q_{12} - \cancel{W_{12,out}} \to 0$

$Q_{12,IN} = m(u_2 - u_1)$

O_2: $\bar{u}_2 - \bar{u}_1 = 10,614 - 6,242 = 4372$ kJ/kmol

N_2: $\bar{u}_2 - \bar{u}_1 = 10{,}423 - 6229 = 4194 \text{ kJ/kmol}$

Ar: $\bar{u}_2 - \bar{u}_1 = \bar{c}_v (T_2 - T_1) = 12.5 \dfrac{\text{kJ}}{\text{kmol} \cdot \text{K}} (500 - 300)\text{K}$

$= 2500 \text{ kJ/kmol}$

$$(\bar{u}_2 - \bar{u}_1)_{MIX} = \sum_i y_i (\bar{u}_2 - \bar{u}_1)_i = (0.21)(4372) + (0.78)(4194) + (0.01)(2500) = 4214 \text{ kJ/kmol}$$

$$(u_2 - u_1)_{MIX} = \frac{(\bar{u}_2 - \bar{u}_1)_{MIX}}{M_{MIX}} = \frac{4214 \text{ kJ/kmol}}{28.97 \frac{\text{kg}}{\text{kmol}}} = 145.48 \frac{\text{kJ}}{\text{kg}}$$

$$Q_{12,IN} = (1.161 \text{ kg})\left(145.48 \frac{\text{kJ}}{\text{kg}}\right) = \boxed{169 \text{ kJ}}$$

ENTROPY CHANGES:

$$O_2: \quad (\bar{s}_2 - \bar{s}_1)_{O_2} = \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln\left(\frac{P_{O_2,2}}{P_{O_2,1}}\right)$$

$$= \bar{s}^\circ(T_2) - \bar{s}^\circ(T_1) - \bar{R} \ln\left(\frac{\cancel{y_{O_2,2}}}{\cancel{y_{O_2,1}}} \cdot \frac{P_2}{P_1}\right)$$

MUST USE PARTIAL PRESSURES

SINCE CONSTANT VOLUME:

$$P_2 = P_1 \frac{T_2}{T_1} = 100 \text{ kPa} \frac{500 \text{ K}}{300 \text{ K}} = 166.7 \text{ kPa}$$

$$(\bar{s}_2 - \bar{s}_1)_{O_2} = (220.589 - 205.213) - 8.314 \ln\left(\frac{5}{3}\right) = 11.129 \frac{\text{kJ}}{\text{kmol} \cdot \text{K}}$$

$$N_2: \quad (\bar{s}_2 - \bar{s}_1)_{N_2} = (206.63 - 191.682) - \text{ " } = 10.701 \text{ "}$$

$$Ar: \quad (\bar{s}_2 - \bar{s}_1)_{Ar} = \bar{c}_v \ln(T_2/T_1) + \bar{R} \ln(\cancel{v_2/v_1})$$

$$= 12.5 \ln(500/300) = 6.378 \text{ kJ/kmol} \cdot \text{K}$$

$$(\bar{s}_2-\bar{s}_1)_{MIX} = \sum y_i(\bar{s}_2-\bar{s}_1)_i = (0.21)(11.129) + (0.78)(10.701) + (0.01)(6.378)$$

$$= 10.748 \text{ kJ/kmol}\cdot\text{K}$$

$$(s_2-s_1)_{MIX} = \frac{(\bar{s}_2-\bar{s}_1)_{MIX}}{M_{MIX}} = \dots = 0.371 \text{ kJ/kg}\cdot\text{K}$$

$$(S_2-S_1)_{MIX} = m_{MIX}(s_2-s_1)_{MIX} = \dots = \boxed{0.742 \text{ kJ/K}}$$