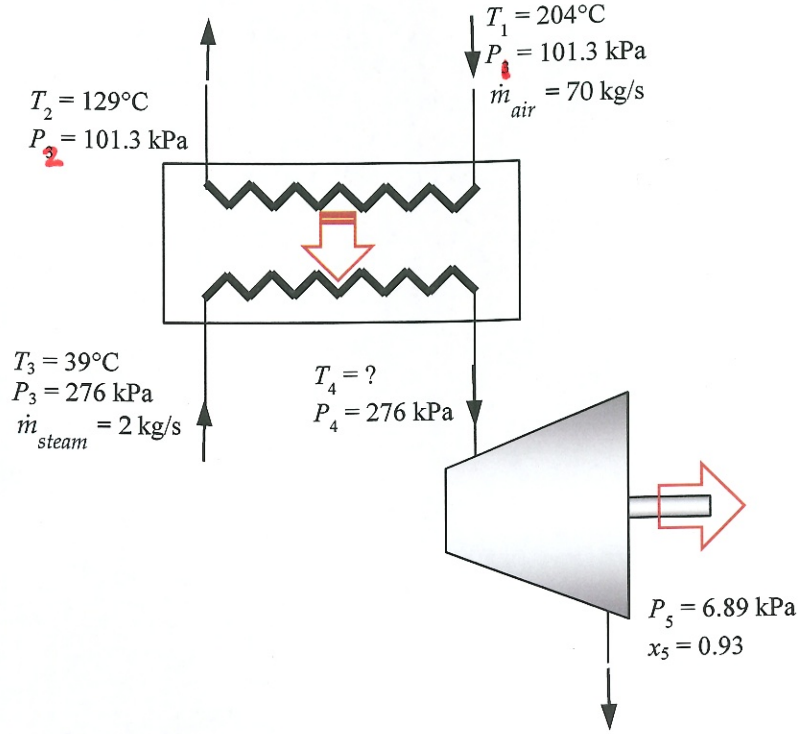
**ROSE-HULMAN Institute of Technology**
Department of Mechanical Engineering



## Example

Consider a heat exchanger and a steam turbine used as a waste heat recovery system. The heat exchanger takes hot combustion gases and uses them to heat steam, which in turn passes through a turbine. The gases can be modeled as air treated as an ideal gas with variable specific heats. The surroundings are at $T_0$ = 25°C and $P_0$ = 101 kPa.




(a) Find the power (in kW) delivered by the turbine.
(b) Find the isentropic (adiabatic efficiency) of the turbine.
(c) For the heat recovery system (heat exchanger and turbine combined) identify
   1. where inflows of exergy occur

      W/ AIR IN, W/ STEAM IN

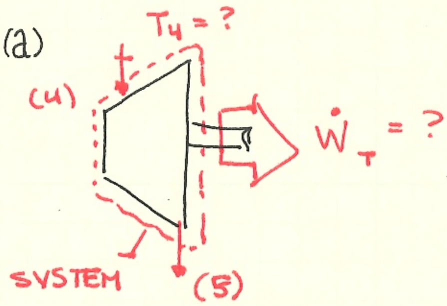
   2. outflows of exergy occur

      W/ AIR OUT, W/ STEAM OUT, W/ $\dot{W}_T$

   3. destruction of exergy occur

      HXR, IN TURBINE

(a)




Cons. of energy →

$$\frac{dE_{sys}}{dt} = \dot{Q}_{IN} - \dot{W}_{T,OUT} + \dot{m}_s(h_4 + ..) - \dot{m}_s(h_5 + ..)$$

$$\dot{W}_{T,OUT} = \dot{m}_s(h_4 - h_5) \quad (1)$$

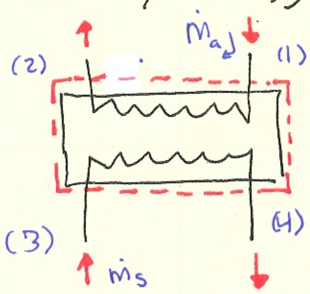
$$h_4 = h(\text{STEAM}, T_4, P_4)$$

$$h_5 = h(\text{STEAM}, P_5, x_5)$$

$$= \underline{2402 \text{ kJ/kg}}$$

NEED $h_4$.

Cons of energy on HXR →




$$0 = 0 - 0 + \sum_{IN} \dot{m}(h + ..) - \sum_{OUT} \dot{m}(h + ..)$$

$$0 = \dot{m}_a(h_1) + \dot{m}_s(h_3) - \dot{m}_a(h_2) - \dot{m}_s(h_4)$$

$$h_4 = h_3 + \frac{\dot{m}_a(h_1 - h_2)}{\dot{m}_s}$$

$$h_1 = h(\text{AIR}, T_1)$$ FUNC. of $T$ ONLY !!

$$= \underline{479.9 \text{ kJ/kg}}$$

$$h_2 = h(\text{AIR}, T_2) = \underline{403.4 \text{ kJ/kg}}$$

$$h_3 = h(\text{STEAM}, T_3, P_3) = \underline{163.6 \text{ kJ/kg}}$$

$$\therefore h_4 = 2838 \text{ kJ/kg} \Leftarrow$$

$$\boxed{\dot{W}_T = 872 \text{ kW}}$$

(b)

$$\eta_T = \frac{\dot{W}_{T,OUT}}{\dot{W}_{T,OUT,s}}$$

$$\dot{W}_{T,OUT,s} = \dot{m}_s(h_4 - h_{5s})$$

$$h_{5s} = h(\text{STEAM}, P_5, s_{5s} = s_4)$$

$$s_4 = s(\text{STEAM}, P_4, h_4) \;!!$$

$$= 7.291 \text{ kJ/kg-K}$$

$\therefore h_{5s} = 2263$ kJ/kg $\longrightarrow$ $\dot{W}_{T,s} = 1151$ kW

$$\eta_T = \frac{872 \text{ kW}}{1151 \text{ kW}} = \boxed{0.758}$$

(c) (1) EXERGY IN W/ AIR FLOW, STEAM FLOW

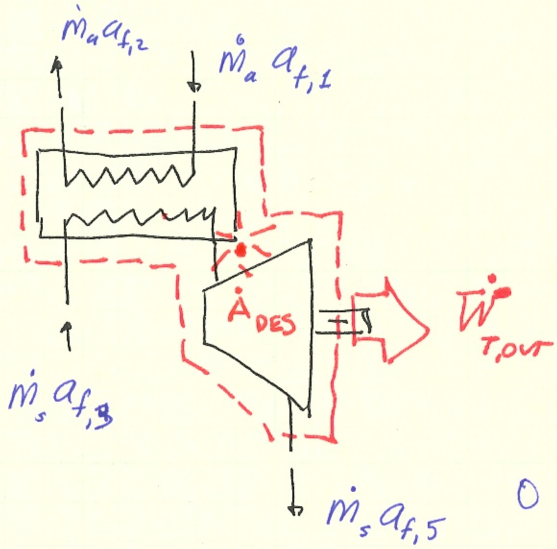
(2) " OUT " " " " "

& TURBINE POWER

(3) EXERGY DESTRUCTION IN

- HEAT EXCHANGER ($\dot{Q}$ THRU $\Delta T$)
- TURBINE ($\eta_T < 1$)

(d)




Acct of A

$$\frac{dA_{sys}}{dt} = \sum_i \left(1 - \frac{T_0}{T_i}\right)\dot{Q}_i - \dot{W}_{out} + \sum_{IN} \dot{m} a_f - \sum_{OUT} \dot{m} a_f - \dot{A}_{DES}$$

$$0 = -\dot{W}_T + \dot{m}_a a_{f,1} + \dot{m}_s a_{f,3} - \dot{m}_a a_{f,2} - \dot{m}_s a_{f,5} - \dot{A}_{DES}$$

$$\dot{A}_{DES} = \dot{m}_a (a_{f,1} - a_{f,2}) - \dot{m}_s (a_{f,5} - a_{f,3}) - \dot{W}_T \qquad (2)$$

NET INPUT DUE TO AIR

NET OUTFLOW DUE TO STEAM FLOW

USEFUL OUTPUT

$$a_{f,1} - a_{f,2} = \left\{ h_1 - h_{0,air} - T_0(s_1 - s_0) \right\} - \left[ h_2 - h_{0,air} - T_0(s_2 - s_{0,air}) \right]$$

$$= (h_1 - h_2) - T_0(s_1 - s_2)$$

$$s_1 - s_2 = s^\circ(T_1) - s^\circ(T_2) - R \ln\left(\frac{P_2}{P_1}\right)$$

$s^\circ \neq s_0$! THIS IS A TABLE THING FOR IDEAL GASES

OR USE EES:

$s_1 = s(\text{AIR}, T_1, P_1) = 6.171$ KJ/kg-K

$s_2 = s(\text{AIR}, T_2, P_2) = 5.997$ "

∴ NET INPUT

= NET AIR INPUT $= \dot{m}_a (a_{f,1} - a_{f,2}) = \ldots = 1722$ KW

$$a_{f,5} - a_{f,3} = (h_5 - h_3) - T_0(s_5 - s_3)$$

$h_3 = ✓$, $h_5 = ✓$

$s_3 = s(\text{STEAM}, T_3, P_3)$

$= 0.5588$ KJ/kg·K

$s_5 = s(\text{STEAM}, P_5, x_5)$

$= 7.739$ KJ/kg-K

∴ NET <u>OUTPUT</u> IN STEAM

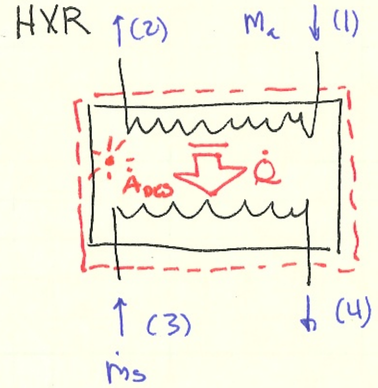
$= \dot{m}_s (a_{f,5} - a_{f,3}) = \ldots = 199$ KW

FROM (2)

$\dot{A}_{DES} = 1722 \text{ KW} - 199 \text{ KW} - 872 \text{ KW} = 651 \text{ KW}$

TO KNOW HOW MUCH IS DESTROYED IN HXR & TURBINE, MUST LOOK @ THOSE AS SEPARATE SYSTEMS.

HXR

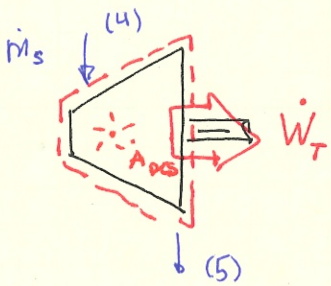



$$\frac{dA_{sys}}{dt} = \sum\left(1-\frac{T_0}{T_i}\right)\dot{Q}_i - \dot{W}_{out} + \sum_{IN}\dot{m}(a_f) - \sum_{OUT}\dot{m}\,a_f - \dot{A}_{DES,HXR}$$

$$\dot{A}_{DES,HXR} = \dot{m}_a(a_{f_1}) + \dot{m}_s(a_{f,3}) - \dot{m}_a(a_{f,2}) - \dot{m}_s(a_{f,4})$$

$$\dot{A}_{DES,HXR} = \dot{m}_a(a_{f_1} - a_{f,2}) + \dot{m}_s(a_{f,3} - a_{f,4}) = \ldots = 385 \text{ kW}$$

TURBINE:




$$\frac{dA_{sys}}{dt} = \sum\left(1-\frac{T_i}{T_0}\right)\dot{Q}_i - \dot{W}_{out} + \sum_{IN}\dot{m}(a_f) - \sum_{OUT}\dot{m}\,a_f - \dot{A}_{DES,TUR}$$

$$\dot{A}_{DES,TUR} = \dot{m}_s(a_{f,4} - a_{f,5}) - \dot{W}_{T,OUT}$$

$$= \dot{m}_s(h_4 - h_5 - T_0[s_4 - s_5]) - \dot{W}_{T,OUT}$$

$$= \ldots = 267 \text{ kW}$$

FINALLY

| | | |
|---|---|---|
| NET RATE EXERGY IN (AIR FLOW) | 1722 kW | (100%) |
| DISPOSITION of EXERGY | | |
| • RATE EXERGY OUT | | |
| - POWER OUT | 872 kW | (50.6%) |
| - WATER STREAM | 199 kW | (11.6%) |
| • RATE EXERGY DES. | | |
| - HXR | 385 kW | (22.4%) |
| - TURBINE | 267 kW | (15.5%) |
| | 1722 kW | (100 %) |