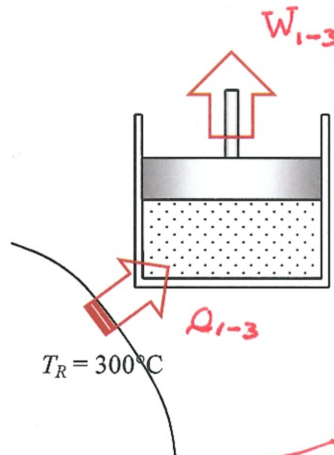
ROSE-HULMAN Institute of Technology
Department of Mechanical Engineering

ME 301 — Thermodynamics II

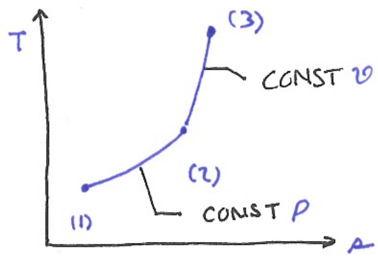
Example

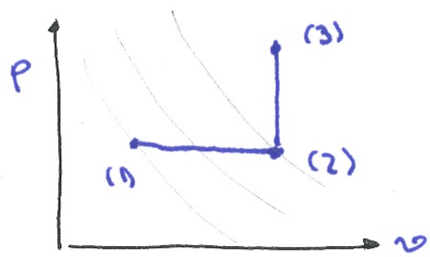
A mass of 0.25 kg of air (c_p = 1.005 kJ/kg·K, c_v = 0.718 kJ/kg·K) is contained in a piston cylinder initially at P_1 = 200 kPa and T_1 = 35°C. The air undergoes a two-step process consisting of a constant pressure process until the temperature reache~~d~~s 175°C followed by a constant volume process until the temperature reaches 250°C. All heat transfer to the air occurs from contact with a thermal reservoir at T_R = 300°C. The surroundings are at 100 kPa and 300 K.

(a) Sketch the two-step process on a T-s diagram. (& P-v too!)
(b) Using an energy conservation/entropy accounting approach, find
  1. the total useful work *out* of the air and
  2. the maximum possible useful work out of the air.
(c) Repeat (b) using an accounting of exergy approach.



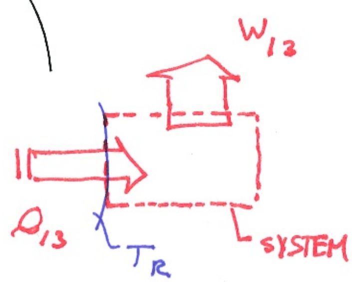
(a)





(b). Cons. of energy →

NO KE, PE, FINITE TIME, CLOSED SYS



$$\bar{U}_3 - \bar{U}_1 = Q_{13} - W_{OUT\,1-3} \quad (1)$$

$$\bar{W}_{1-3,OUT} = \int_1^2 P\,d\forall + \int_2^3 P\,\cancel{d\forall}^{\,0} = m\int_1^2 P\,dv = mP_1(v_2 - v_1)$$

$$v_2 = \frac{RT_2}{P_2} \xleftarrow{\text{(IN K!)}} = \ldots = 0.6431\ m^3/kg$$

$$v_1 = \frac{RT_1}{P_1} = \ldots = 0.4422\ m^3/kg$$

$$\bar{W}_{1-3,OUT} = \ldots = 10.04\ kJ$$

$$\bar{W}_{1-3,OUT,USE} = \bar{W}_{1-3,OUT} - P_0(\forall_3 - \forall_1) = \bar{W}_{1-3,OUT} - mP_0(\overset{= v_2!}{v_3} - v_1)$$

$$= \ldots = \boxed{5.02\ kJ}$$

FROM (1)

$$Q_{13} = m(u_3 - u_1) + W_{1-3,OUT} = mc_v(T_3 - T_1) + W_{1-3,OUT}$$

$$= \ldots = \boxed{48.6\ kJ}$$

Accounting of S ⟶

(FINITE TIME, CLOSED SYS)

$$(S_3 - S_1)_{SYS} = \frac{Q_{1-3}}{T_R} + S_{gen} \qquad S_{gen} = m(s_3 - s_1) - \frac{Q_{13}}{T_R}$$

$$S_{gen} = m\left(c_v \ln\frac{T_3}{T_1} + R\ln\frac{v_3}{v_1}\right) - \frac{Q_{13}}{T_R} \qquad (v_3 = v_2)$$

WHAT'S R? $R = c_p - c_v$!!!

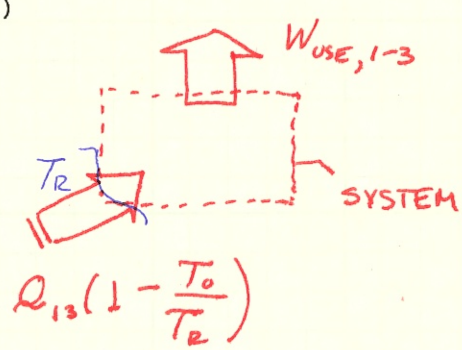
$$S_{gen} = (0.25\,kg)\left[0.718\frac{kJ}{kg\text{-}K}\ln\left(\frac{250+273}{35+273}\right) + 0.287\frac{kJ}{kg\text{-}K}\ln\left(\frac{v_2}{v_1}\right)\right]$$

$$- \frac{48.6\ kJ}{(300+273)K} = 0.03702\ \frac{kJ}{K}$$

$$I_{13} = T_0 S_{gen} = (300\ K)\left(0.03702\ \frac{kJ}{K}\right) = 11.1\ kJ$$

$$W_{1-3,MAX,USE} = W_{1-3,USE} + T_0 S_{gen} = \boxed{16.1\ kJ}$$

(c)



Acct of exergy ⟶

$$\frac{d}{dt}(A_{sys}) = \sum_i \dot{Q}_i\left(1 - \frac{T_0}{T_i}\right) + \sum_{IN} \cancel{\dot{m}}^{\,0}(a_f)$$

$$- \dot{W}_{OUT,USE} - \sum_{OUT} \cancel{\dot{m}}_{\,0}(a_f) - \dot{A}_{DES}$$

CLOSED

FINITE TIME

$$(A_3 - A_1) = Q_{13}\left(1 - \frac{T_0}{T_R}\right) - W_{1-3,USE} - \cancel{A_{DES,13}} \rightarrow 0 \text{ FOR MAX}$$

$$W_{13,USE,MAX} = Q_{13}\left(1 - \frac{T_0}{T_R}\right) - (A_3 - A_1)$$

$$= Q_{13}\left(1 - \frac{T_0}{T_R}\right) - m(a_3 - a_1)$$

$$= Q_{13}\left(1-\frac{T_0}{T_R}\right) - m\left[u_3-u_1 + p_0(v_3-v_1) - T_0(s_3-s_1)\right]$$

$$= Q_{13}\left(1-\frac{T_0}{T_R}\right) - m\left[c_v(T_3-T_1) + p_0(v_3-v_1) - T_0\left(c_v \ln\frac{T_3}{T_1} + R\ln\frac{v_3}{v_1}\right)\right]$$

$$= \ldots = \boxed{16.1\ \text{KJ}}$$