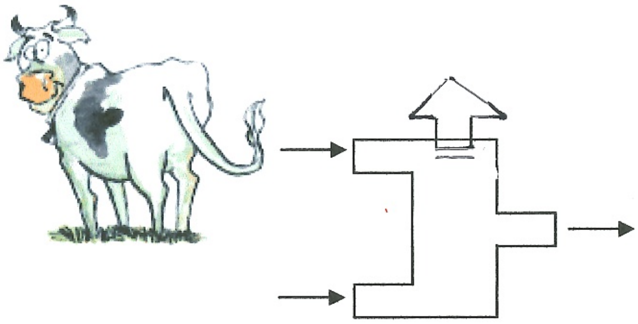
ROSE-HULMAN Institute of Technology
Department of Mechanical Engineering

Example

In his younger days, Dr. Thom got mixed up with a get-rich-quick scheme that proposed using bovine flatulence as an energy source. The proposed process was to combust methane (CH_4) with air in a steady-state reaction chamber. The process used sufficient air to produce 90% CO_2, 10% CO and no O_2 in the products. Both the methane and the air enter at 1 bar and 25°C. The products leave the chamber at 500 K and 1 bar. Find the heat transfer rate per unit molar flow rate of fuel for the process.



CHEM RXN (ASSUME MOLAR ANALYSIS of PRDTS)

$$CH_4 + a(O_2 + 3.76N_2) \rightarrow 0.9CO_2 + 0.10CO + bH_2O + cN_2$$

C: $1 = 0.9 + 0.1$ ✓

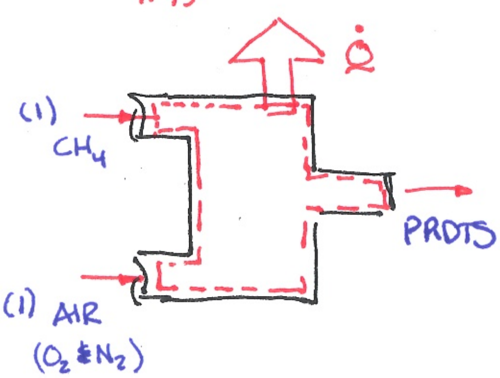
H: $4 = 2b \qquad b = \underline{2}$

O: $2a = (2)(0.9) + (0.10) + (\cancel{b})^{2} \qquad \underline{a = 1.95}$

N: $(\cancel{a})_{1.95}(3.76)(\cancel{2}) = (\cancel{2})(c) \qquad \underline{c = 7.332}$

$$\boxed{\therefore\ CH_4 + 1.95(O_2 + 3.76N_2) \rightarrow 0.9CO_2 + 0.10CO + 2H_2O + 7.332N_2}$$

BALANCED RXN



CoE

$$\cancel{\frac{dE}{dt}}^{\,0} = -\dot{Q}_{out} - \cancel{\dot{W}}^{\,0} + \underbrace{\sum \dot{n}\bar{h}}_{RXT} - \underbrace{\sum \dot{n}\bar{h}}_{PDT}$$

$$\dot{Q}_{out} = \dot{n}_{CH_4}\bar{h}_{CH_4})_1 + \dot{n}_{O_2}\bar{h}_{O_2})_1 + \dot{n}_{N_2}\bar{h}_{N_2})_1 - \dot{n}_{CO_2}\bar{h}_{CO_2})_2 - \dot{n}_{CO}\bar{h}_{CO})_2 - \dot{n}_{H_2O}\bar{h}_{H_2O})_2 - \dot{n}_{N_2}\bar{h}_{N_2})_2$$

$$\frac{\dot{Q}_{out}}{\dot{n}_{CH_4}} = \bar{h}_{CH_4})_1 + \cancel{\frac{\dot{n}_{O_2}}{\dot{n}_{CH_4}}}^{1.95}\bar{h}_{O_2})_1 + \cancel{\frac{\dot{n}_{N_2}}{\dot{n}_{CH_4}}}^{7.332}\bar{h}_{N_2})_1 - \cancel{\frac{\dot{n}_{CO_2}}{\dot{n}_{CH_4}}}^{0.9}\bar{h}_{CO_2})_2 - \cancel{\frac{\dot{n}_{H_2O}}{\dot{n}_{CH_4}}}^{2}\bar{h}_{H_2O})_2 - \cancel{\frac{\dot{n}_{N_2}}{\dot{n}_{CH_4}}}^{7.332}\bar{h}_{N_2})_2$$

$$\therefore \frac{\dot{Q}_{OUT}}{\dot{n}_{CH_4}} = \bar{h}_{CH_4}\Big|_1 + 1.95\,\bar{h}_{O_2}\Big|_1 + 7.332\,\bar{h}_{N_2}\Big|_1 - \left\{0.9\,\bar{h}_{CO_2}\Big|_2 + 2\,\bar{h}_{H_2O}\Big|_2 + 7.332\,\bar{h}_{N_2}\Big|_2\right\}$$

FOR EACH $\bar{h}$,

$$\bar{h} = \Delta\bar{h}_f^\circ + \left(\bar{h}(T) - \bar{h}(298\,K)\right)$$

i	T[K]	$\Delta\bar{h}_f^\circ$ [KJ/kg]	$\bar{h}(T)$ [KJ/kg]	$\bar{h}(298\,K)$ [KJ/kg]	$\bar{h}$ [KJ/kg]
$CO_2\|_2$	500	-393,520	17,678	9364	-385,206
$CO\|_2$	500	-110,530	14,600	8669	-104,599
$H_2O\|_2$	500	-241,820	16,828	9904	-234,896
$CH_4\|_1$	298	-74,850	?	SAME ?	-74,850
$O_2\|_1$	298	0	8682	8682	0
$N_2\|_1$	298	0	8669	8669	0
$N_2\|_2$	500	0	14,581	8669	5912

...

$$\frac{\dot{Q}_{OUT}}{\dot{n}_{CH_4}} = \left(-74{,}850\,\tfrac{KJ}{Kg}\right) + (1.95)(0) + (7.332)(0) - \left[(0.9)\left(-385{,}206\,\tfrac{KJ}{Kg}\right) + (2)\left(-234{,}896\,\tfrac{KJ}{Kg}\right) + (7.332)\left(5912\,\tfrac{KJ}{Kg}\right)\right]$$

$$= \boxed{708{,}752\ \frac{KJ}{Kmol\text{-}CH_4}}$$