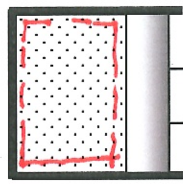
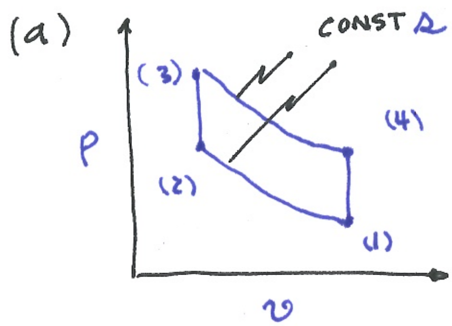
EXAMPLE

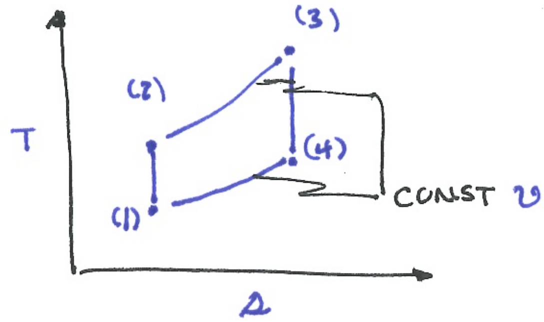
A reciprocating device operating at 6000 rpm is modeled as a **cold-air-standard** Otto cycle with a compression ratio of 8.5 and a displacement of 0.005 m³. Before the adiabatic compression, the air is at 120 kPa and 40°C. After the constant volume heat addition, the air is at 950°C. Use c_v = 0.713 kJ/kg-K and c_p = 1.001 J/kg-K.

(a) Sketch the cycle on *P-v* and *T-s* diagrams.
(b) Find the heat transfer and work (per unit mass) for each process in kJ/kg.
(c) Find the net work (per unit mass) and the efficiency of the cycle.
(d) Find the power delivered by the device in kW.

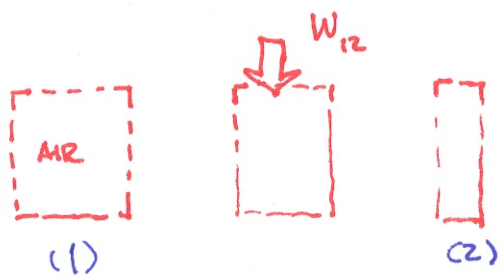


(a)





(b)



CoE (1) → (2), CLOSED/FINITE TIME/NO KE-PE

$$U_2 - U_1 = Q_{12,IN} + W_{IN,12}$$

$$m(u_2 - u_1) = Q_{12,IN} + W_{IN,12}$$

$Q_{12,IN} \to 0$ BY DEFN of OTTO CYCLE

COLD-AIR STND:

$$u_2 - u_1 = c_v(T_2 - T_1)$$

└ VALUE @ 298K

$$\therefore \quad W_{IN,12} = m c_v (T_2 - T_1)$$

$$w_{IN,12} = \frac{W_{IN,12}}{m} = c_v(T_2 - T_1)$$

✓ ? ✓

(1) → (2) ISENTROPIC

WITH CONSTANT c_v, c_p THIS MEANS

$$s_2 - s_1 = c_v \ln\left(\frac{T_2}{T_1}\right) + R \ln\left(\frac{v_2}{v_1}\right)$$

$$0 = \quad '' \quad\quad '' $$

$$\frac{T_2}{T_1} = \left(\frac{v_2}{v_1}\right)^{-R/c_v}_{s=\text{CONST}} = \left(\frac{v_2}{v_1}\right)^{-(c_p - c_v)/c_v}_{s=\text{CONST}} = \left(\frac{v_2}{v_1}\right)^{1-k}_{s}$$

WHERE $k \equiv c_p / c_v$

CAN USE $pv = RT$ TO SHOW

$$\left.\frac{P_2}{P_1} = \frac{T_2}{T_1}\right)^{k/k-1}_{s=\text{CONST}} \quad \& \quad \left.\frac{P_2}{P_1} = \frac{v_1}{v_2}\right)^{k}_{s=\text{CONST}}$$

SAME AS $pv^k = \text{CONST}$

ANYWAY

$$T_2 = T_1 \left(\frac{v_2}{v_1}\right)^{1-k} = T_1 \left(\frac{1}{r_c}\right)^{1-k} = (313\ \text{K})\left(\frac{1}{8.5}\right)^{1-1.4}$$

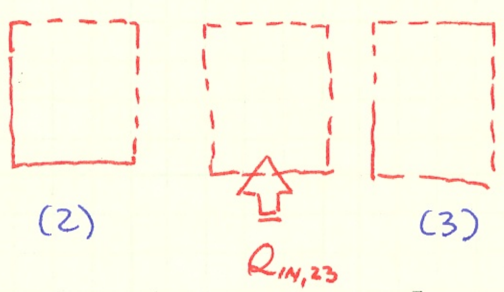
$$= 737\ \text{K}$$

$$w_{12} = c_v (T_2 - T_1) = 0.713 \frac{\text{kJ}}{\text{kg}\cdot\text{K}} [737 - 313] = \boxed{302.3\ \text{kJ/kg}}$$

COULD WE HAVE USED

$$w_{IN} = -\int_1^2 P\,dv \ ?$$

SURE, BUT NOT UNTIL WE KNEW THAT $pv^k = \text{CONST}$.



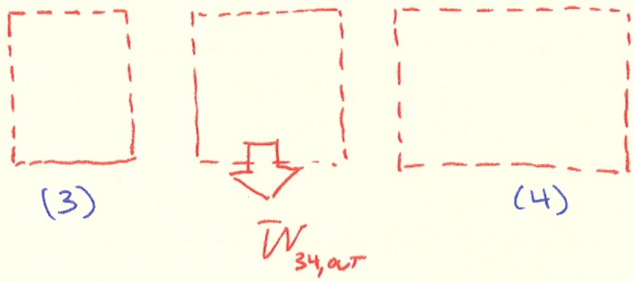
C.O.E. (2) → (3)

$$U_3 - U_2 = Q_{23,IN} + W_{23,IN}$$

$$W_{23,IN} = -\int_2^3 P\,dV \quad (dV = 0)$$

$$\frac{Q_{23,IN}}{m} = q_{23} = (u_3 - u_2) = c_v(T_3 - T_2)$$

$$= \left(0.713 \frac{kJ}{kg \cdot K}\right)(1223 - 737)\,K = \boxed{349 \frac{kJ}{kg}}$$



C.O.E. (3) → (4)

$$U_4 - U_3 = Q_{34} - \overline{W}_{34,OUT}$$

Q_{34} = 0 FOR OTTO CYCLE

$$m(u_4 - u_3) = -\overline{W}_{34,OUT}$$

$$w_{OUT,34} = (u_4 - u_3) = c_v(T_3 - T_4)$$

(3) → (4) ISENTROPIC

$$\frac{T_4}{T_3} = \left(\frac{v_4}{v_3}\right)^{1-k} = \left(r_c\right)^{1-k}$$

$$T_4 = \left(r^{1-k}\right) T_3$$

$$= (8.5)^{1-1.4}(1223\,K)$$

$$= 520\,K$$

$$w_{OUT,34} = c_v(T_3 - T_4)$$

$$= \left(0.713 \frac{kJ}{kg \cdot K}\right)(1223 - 520)\,K = \boxed{501\ kJ/kg}$$

(4) → (1) PROCEEDS IN SAME WAY. RESULT IS ...

$$\ldots\ q_{41,out} = c_v(T_4 - T_1) = 0.713 \frac{kJ}{kg \cdot K}(520 - 313)\,K$$

$$= \boxed{148\ kJ/kg}$$

(c)

$$w_{NET,OUT} = -w_{IN\,12} + \cancel{w_{23}}^{\,0} + w_{34\,OUT} + \cancel{w_{41}}^{\,0}$$

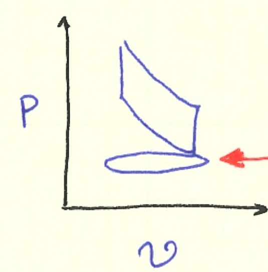
$$= -302.3 - 501 = \boxed{199\ kJ/kg}$$

$$\eta = \frac{W_{OUT,NET}}{Q_{IN}} = \frac{w_{OUT,NET}}{q_{IN}} = \frac{199\ kJ/kg}{349\ kJ/kg} = \boxed{0.57}$$

(d)

$$\dot{W}_{OUT} = W_{CYCLE\ NET,OUT} * \frac{CYCLES}{SEC} = W_{CYCLE} * RPM * \left\langle \frac{min}{60\ s} \right\rangle * \frac{1}{2}$$

ONLY HALF THE CYCLES PRODUCE POWER. (REMEMBER WE HAVE IGNORED INTAKE & EXHAUST STROKES



$$W_{CYCLE\ NET,OUT} = m\, w_{NET,OUT}$$

$$m = \frac{\Delta V}{\Delta v} = \frac{V_1 - V_2}{v_1 - v_2} = \frac{\Delta V}{v_1 - \frac{v_1}{r_c}}$$

$$v_1 = \frac{RT_1}{P_1} = \frac{\left(0.287 \frac{kJ}{kg \cdot K}\right)(313\ K)}{120\ kPa \left\langle \frac{kJ}{kPa \cdot m^3} \right\rangle} = 0.7486\ m^3/kg$$

$$= \frac{0.005\ m^3}{\left[0.7486 - \frac{0.7486}{8.5}\right] \frac{m^3}{kg}}$$

$$= 0.00757\ kg$$

$$\therefore\ W_{CYCLE\ NET\ OUT} = 1.52\ kJ$$

$$\dot{W}_{OUT} = 1.52\ kJ * 6000 \frac{rev}{min} \left\langle \frac{min}{60\ s} \right\rangle \frac{1}{2} = \boxed{76\ kW}$$