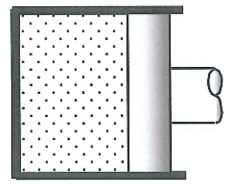
ROSE-HULMAN Institute of Technology
Department of Mechanical Engineering

ME 301 | Thermodynamics II

Example

An **air-standard** Diesel cycle has a compression ratio of 15. Before the adiabatic compression, the air is at 100 kPa and 20°C. 1600 kJ/kg of thermal energy is added during the constant pressure heat addition.

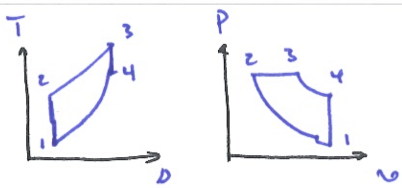


(a) Sketch the cycle on *P-v* and *T-s* diagrams.
(b) Find the heat transfer and work (per unit mass) for each process in kJ/kg.
(c) Find cut-off ratio for the cycle.
(d) Find the cycle efficiency. For a **cold**-air-standard, the cycle efficiency is given by

$$\eta_{cold-air} = 1 - r^{1-k}\,\frac{r_c^{\,k} - 1}{k(r_c - 1)}.$$

How does your value compare to this?

(a)



b)

CoE, FINITE TIME, CLOSED SYS

$$U_2 - U_1 = \cancel{Q_{12}}^{\,0} + W_{IN,12}$$

$$m(u_2 - u_1) = W_{12,IN}$$

$$w_{IN,12} = \frac{W_{IN,12}}{m} = u_2 - u_1$$

$$= u(\overset{?}{T_2}) - u(T_1)$$

$u_1 = 209.3$ kJ/kg

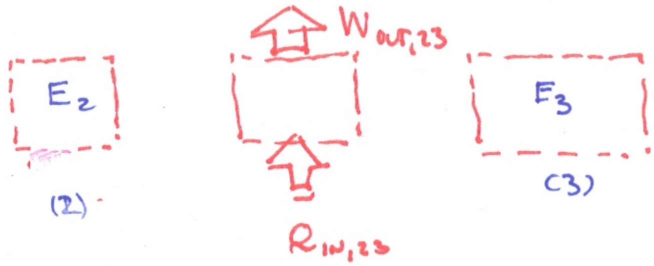
SINCE $s_2 - s_1 = 0$

$$\frac{v_2}{v_1} = \frac{v_{r2}(T_2)}{v_{r1}(T_1)} = \frac{1}{r} \Rightarrow \quad v_{r2} = \frac{v_{r1}(T_1)}{r} = \frac{659.2}{15} = 43.94$$

$\therefore\ T_2 = 826$ K

& $u(T_2) = 614$ kg

$$w_{IN,12} = \ldots = \boxed{405 \text{ kJ/kg}}$$



CoE FINITE TIME, CLOSED

$$U_3 - U_2 = Q_{23,IN} - W_{23,OUT}$$

$$u_3 - u_2 = q_{23,IN} - w_{23,OUT}$$

A TRICK

$$W_{OUT,23} = \int_2^3 p\,dV$$

B/C CONST p

$$w_{OUT,23} = \int_2^3 p\,dv = p(v_3 - v_2) = pv_3 - pv_2$$

$$\therefore \quad u_3 - u_2 = q_{23,IN} - (pv_3 - pv_2)$$

$$q_{23,IN} = \underbrace{(u_3 + p_3 v_3)}_{\equiv h_3!} - \underbrace{(u_2 + p_2 v_2)}_{\equiv h_2!} = h_3 - h_2$$

$$h_2 = h(T_2) = 851 \text{ kJ/kg}$$

$$\therefore \quad 1600 \frac{\text{kJ}}{\text{kg}} = h_3 - 851 \text{ kJ/kg} \qquad h_3 = 2451 \text{ kJ/kg}$$

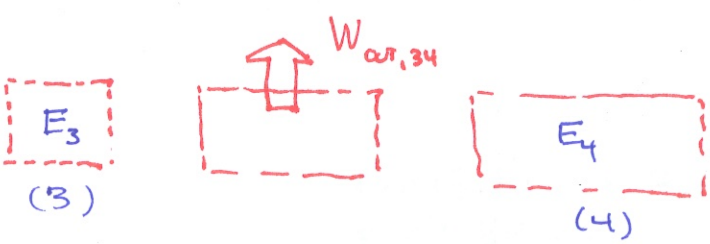
$$\Rightarrow T_3 = 2158 \text{ K}$$

$$u_3 = 1832 \text{ kJ/kg}$$

BACK TO WORK (HA!)

$$w_{OUT,23} = p_2(v_3 - v_2)$$ OR FROM CoE $$= q_{23,IN} + u_2 - u_3$$

$$= \dots = \boxed{382 \text{ kJ/kg}}$$



CoE, FINITE TIME, CLOSED

$$U_4 - U_3 = \cancel{Q_{34}} - W_{OUT,34}$$

$$w_{34,OUT} = u_3 - \overset{?}{u_4}$$

SINCE $s_4 - s_3 = 0$

$$v_{r4} = v_{r3}\left(\frac{v_4}{v_3}\right) \qquad v_4 = v_1 = \frac{R_{AIR}\, T_1}{P_1}$$

$$v_3 = \frac{R_{AIR}\, T_3}{P_3}$$

?

WHAT IS $P_3 = ?$ $\quad P_3 = P_2$.

BUT WHAT IS $P_2 = ?$ MUST GO BACK TO $(1) \rightarrow (2)$.

SINCE $s_2 - s_1 = 0$

$$\frac{P_2}{P_1} = \frac{P_{r2}}{P_{r1}} \qquad P_2 = P_1 \frac{P_r(T_2)}{P_r(T_1)} = (100\ \text{kPa})\left[\frac{54.1}{1.2765}\right] = 4230\ \text{kPa}!!$$

NOW

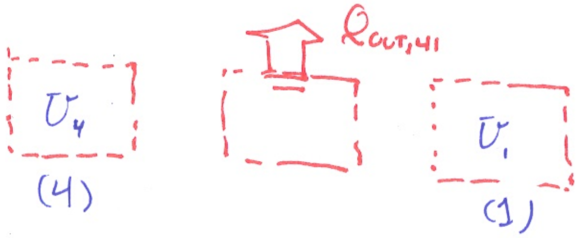
$$v_{r4} = v_{r3}\left(\frac{v_4}{v_3}\right) = v_{r3}\left(\frac{\cancel{R_{AIR}}\, T_1 / P_1}{\cancel{R_{AIR}}\, T_3 / P_2}\right) = (2.1482)\left[\frac{293\cancel{K}/100\cancel{kPa}}{2158\cancel{K}/4230\cancel{kPa}}\right]$$

$$= 12.36$$

$$\Rightarrow T_4 = 1263\ \text{K}$$

$$u_4 = 989.5\ \text{K}$$

$$w_{34,out} = \ldots = \boxed{842.\ \text{kJ/kg}}$$



CoE, FINITE TIME, CLOSED

$$U_1 - U_4 = -Q_{OUT,41} + \cancel{W}^{\,0}$$

$$q_{OUT,41} = \frac{Q_{OUT,41}}{m} = u_4 - u_1 = u(T_4) - u(T_1) = \ldots = \boxed{780\ \text{kJ/kg}}$$

(d)

$$\eta = \frac{w_{NET,OUT}}{q_{IN}} = \frac{w_{OUT,23} + w_{OUT,34}}{q_{IN,23}} = \ldots = 0.513$$

$$= \boxed{51.3\%}$$

(c) $r_{co} = \frac{v_3}{v_2} = \left(\frac{\cancel{R_{AIR}} T_3 / \cancel{P_3}}{\cancel{R_{AIR}} T_2 / \cancel{P_2}} \right)$ $(P_3 = P_2)$

$= \left(\frac{2158 \cancel{K}}{826 \cancel{K}} \right) = \boxed{2.61}$

(e) $\eta_{COLD} = 1 - (r)^{1-k} \frac{(r_{co}^{k} - 1)}{k(r_{co} - 1)}$

$= 1 - (15)^{1-1.4} \frac{(2.61^{1.4} - 1)}{1.4(2.61 - 1)} = 0.575$

$= \boxed{57.5\%}$

MUCH HIGHER THAN $\eta_{AIR\ STANDARD.}$