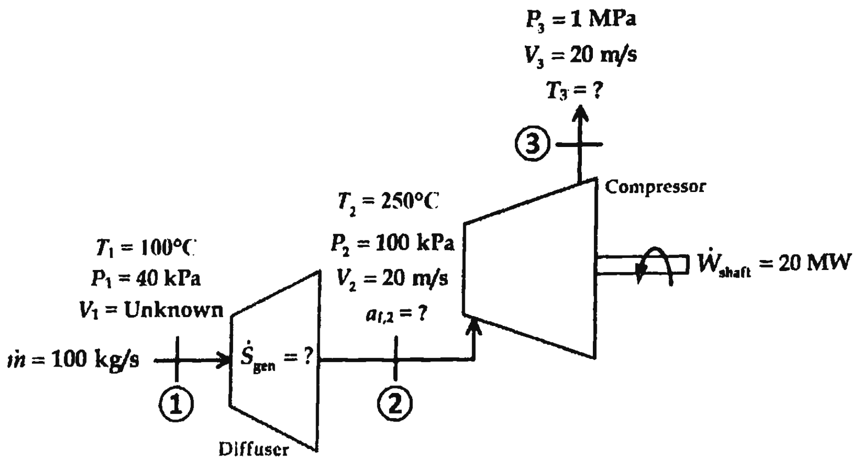
PROBLEM 2 [35 points]

Air flows through a diffuser and compressor at a rate of 100 kg/s and with the state properties as shown in the figure.

Model air as an ideal gas with variable specific heats and assume surrounding conditions of P_0 = 101.325 kPa and T_0 = 298 K.



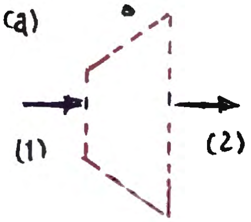
Determine:

10 (a) the <u>rate of entropy generation</u> in the diffuser, in kW/K,

12 (b) the <u>temperature of the air</u> at the exit of the compressor assuming that the compressor requires a power input of 20 MW, in K, and

12 (c) the <u>specific flow exergy</u> into the compressor, in kJ/kg.

(a)



AoS

$$\frac{d(S_{sys})}{dt} = \sum \frac{\dot{Q}_i}{T_{b,i}} + \dot{m}s_1 - \dot{m}s_2 + \dot{S}_{gen}$$

Steady

$$\dot{S}_{gen} = \dot{m}(s_2 - s_1)$$

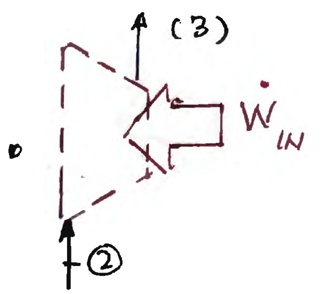
$$\begin{cases} s^\circ_{T_2} = 2.26588 \text{ kJ/kg·K} \\ s^\circ_{T_1} = 1.92112 \text{ ''} \end{cases}$$

$$\dot{S}_{gen} = (\dot{m})\left(s^\circ_{T_2} - s^\circ_{T_1} - R\ln\frac{P_2}{P_1}\right)$$

$$= \left(100\,\frac{kg}{s}\right)\left[(2.26588 - 1.92112)\frac{kJ}{kg\cdot K} - 0.287\frac{kJ}{kg\cdot K}\ln\left(\frac{100\,kPa}{40\,kPa}\right)\right]$$

$$= 8.18 \text{ kW/K}$$ ← ANS

(b)



CoE

$$\frac{d(E_{sys})}{dt} = \dot{Q}_{IN} + \dot{W}_{IN} + \dot{m}(h_2 + ..) - \dot{m}(h_3 + ..)$$

Steady ?

$$h_3 = h_2 + \frac{\dot{W}_{IN}}{\dot{m}} = 526.74\,\frac{kJ}{kg} + \frac{20000\,kW}{100\,\frac{kg}{s}}\left(\frac{kW\cdot s}{kJ}\right)$$

$$h_2 = h(T_2) = 526.74\,\frac{kJ}{kg}$$

$$h_3 = 726.74 \text{ kJ/kg}$$

$$T_3 = T(h_3) = 712.5 \text{ K}$$ ← ANS

(c) $a_{f,2} = h_2 - h_0 - T_0(s_2 - s_0) + ke$

$$= h_2 - h_0 - T_0\left[s^\circ_{T_2} - s^\circ_{T_0} - R\ln\left(\frac{P_2}{P_0}\right)\right] + \frac{V_2^2}{2}$$

$h(T_2) = 526.7$ kJ/kg

$h(T_0) = 298.18$ "

$s^\circ(T_2) = 2.26588$ kJ/kg·K

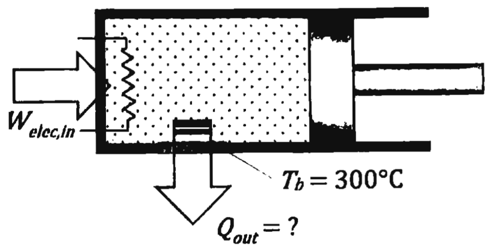
$s^\circ(T_0) = 1.69533$ "

$$a_{f,2} = (526.7 - 298.18)\frac{kJ}{kg} - (298\ K)\left[(2.26588 - 1.69533)\frac{kJ}{kg\cdot K} - 0.287\frac{kJ}{kg\cdot K}\ln\left(\frac{100\ kPa}{101.325\ kPa}\right)\right] + \frac{20^2\ m^2}{s^2}\left(\frac{kJ/kg}{1000\ m^2/s^2}\right)$$

$$= 59.6\ \frac{kJ}{kg} - 0.2\ \frac{kJ}{kg} = 59.4\ \frac{kJ}{kg}$$ ← ANS

Problem 3 [45 points]

A mass of m = 2.0 kg of water inside a piston-cylinder device is initially as a saturated vapor at a pressure of P_1 = 400 kPa. A resistance heater, placed inside the cylinder, is turned on and delivers 4 kW of electrical power for 10 minutes as the system expands under constant pressure until its volume reaches $\forall_2$ = 2.0 m^3.



Use T_0 = 25°C and P_0 = 101.325 kPa.

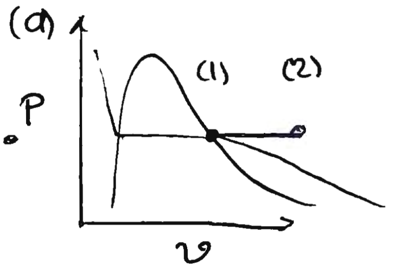
5 (a) Sketch the process path from the initial to the final state on a P–v diagram. Clearly indicate the relative position of the two-phase dome.

5 (b) Determine the final temperature of water, in °C.

20 (c) Determine the heat loss to the surroundings, in kJ.

15 (d) Assume all heat loss occurs at an average boundary temperature of 300°C, determine the exergy destruction for the entire process, in kJ.

(a)

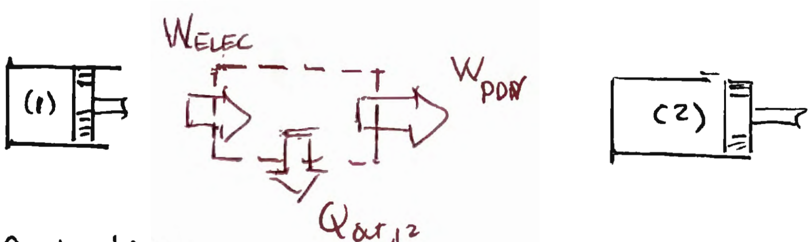


(b) $P_2 = P_1 = 400$ kPa

$v_2 = \forall_2 / m = 2.0\,m^3 / 2.0\,kg = 1.m^3/kg$

• $T_2 = T(P_2, v_2) = 595.2°C$ ← ANS

(c)



CoE, finite time,
closed system, no KE/PE

•• $U_2 - U_1 = -Q_{out,12} + W_{NET,IN} = -Q_{OUT,12} + W_{ELEC,IN} - W_{PDV}$

$m(u_2 - u_1) = -Q_{OUT,12} + W_{ELEC,IN} - W_{PDV}$

$Q_{OUT,12} = W_{ELEC,IN} - W_{PDV} - m(u_2 - u_1)$

• $W_{PDV} = \int_{\forall_1}^{\forall_2} P\,d\forall = P(\forall_2 - \forall_1) = mP_1(v_2 - v_1)$

• $v_1 = v(P_1, x_1 = 1) = 0.4627$ m3/kg

$$W_{PdV} = (2.0\,kg)(400\,kPa)(1.0 - 0.4627)\,m^3/kg \left\langle \frac{kJ}{kPa \cdot m^3} \right\rangle$$

$$= 430\ kJ$$

- $$W_{ELEC} = \dot{W}_{ELEC} \cdot \Delta t = (4.0\,kW)(10\ \text{minutes}) \left\langle \frac{60\,s}{min} \right\rangle \left\langle \frac{kJ}{kW \cdot s} \right\rangle$$

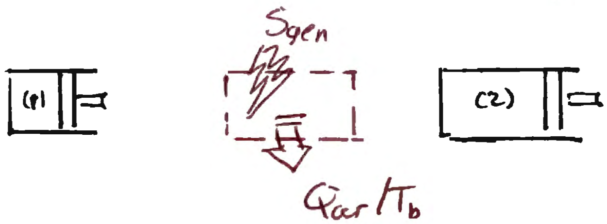
$$= 2400\ kJ$$

$$: \begin{cases} u_1 = u(P_1, x_1 = 1) = 2553.4\ kJ/kg \\ u_2 = u(P_2, v_2) = 3292.1\ \text{"} \end{cases}$$

- $$Q_{out,12} = 2400\,kJ - [illegible]\ kJ - 2.0\,kg(3292.1 - 2553.4)\,kJ/kg$$

$$= 491\ kJ \longleftarrow$$ ANS

(d)



AoS: closed, finite

$$(S_2 - S_1)_{sys} = \sum \frac{Q_{INi}}{T_{bi}} + S_{gen}$$

$$\therefore S_{gen} = m(s_2 - s_1) + Q_{out}/T_b$$

$$\begin{cases} s_1 = s(P_1, x_1) = 6.896\ kJ/kg \cdot K \\ s_2 = s(P_2, v_2) = 8.444\ kJ/kg \cdot K \end{cases}$$

$$= (2.0\,kg)(8.444 - 6.896)\frac{kJ}{kg \cdot K}$$

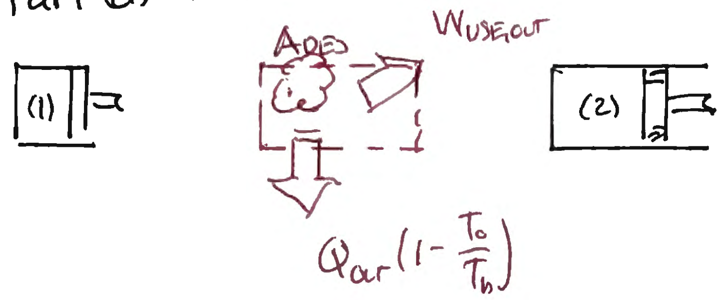
$$+ 464\,kJ/(300 + 273)\,K = 3.91\ kJ/K$$

- $$A_{DES} = T_0 \cdot S_{gen}$$

$$= (25 + 273)\,K\,(3.91\ kJ/K)$$

- $$= 1164\ kJ \longleftarrow$$ ANS

Part (d) via AoA:



AoA, finite, closed, no KE/PE

$$(A_2 - A_1)_{SYS} = \sum Q_{IN}(1 - T_o/T_b) - W_{USE,OUT} - A_{DES}$$

$$A_{DES} = (A_1 - A_2)_{SYS} - Q_{OUT}(1 - T_o/T_b) + W_{IN,ELEC} - \left[W_{OUT\,PDV} - P_o(V_2 - V_1)\right]$$

$$= m(a_2 - a_1) - \ldots$$

$$= m(u_2 - u_1 + P_o(v_2 - v_1) - T_o(s_2 - s_1)) - Q_{OUT}\left(1 - \frac{T_o}{T_b}\right) + W_{IN,ELEC} - W_{OUT\,PDV} + P_o(V_2 - V_1)$$

$$= (20\,kg)\left[(2553.4 - 3292.1)\frac{kJ}{kg} + 101.325\,kPa\,(0.4267 - 1.0)\frac{m^3}{kg} - (298\,K)(6.898 - 8.444)\frac{kJ}{kg \cdot K}\right]$$

$$- 464\,kJ\left(1 - \frac{298\,K}{(300 + 273)\,K}\right) + 2400\,kJ - 459\,kJ$$

$$+ 101.325\,kPa\,(1.0 - 0.4267)\,m^3/kg$$

$$= 1164\,kJ \quad \longleftarrow \text{ANS}$$