

## Economic Optimization of Heat Transfer Systems

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### Introduction

Designing heat transfer systems almost always involves trade-offs, most notably in the form of performance versus cost. Increasing the surface area of a fin array, for example, will tend to increase the heat transfer rate from it. Increasing the surface area of the fin array, however, also increases its expense, and as the fin array becomes increasingly large, the resulting expense may become prohibitive. The surface area of a fin array above which any increase in area no longer justifies the increase in cost represents one example of an economic optimum.

Adding a fin array to a surface increases the convective heat transfer from the surface by increasing the effective area for heat transfer. Another option to increase convective heat transfer is to increase the convective heat transfer coefficient  $h$ . In the case of forced convection this is most easily accomplished by increasing the flow rate of the fluid in contact with the surface. Increasing this flow rate, however, also involves an added expense, in this case in the form of increased power to a fan or pump. We again see the trade off between performance and cost, as well as the opportunity to define a economic optimum.

In the previous examples we saw that increased performance often comes at increased cost. However, two different kinds of costs were represented. In the case of the fin array, the cost of a larger surface area is a one time, up front cost, the **capital**

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**cost.** In the second example the cost of supplying power to a fan or a pump represents an ongoing expense, the **operating cost**. In some cases only one type of cost is significant and an economically optimum design may be simply defined as the one that minimizes this cost. Often both types of cost are important, however, and an additional trade off exists between capital and operating costs.

As an example consider a fin array used in conjunction with the forced convection of air to achieve a given rate of heat transfer. One possible design consists of a large fin array with a low flow rate of air. This design incurs a large capital cost (due to the large fins) with a low operating cost (due to the low fan power required). Another design may employ relatively small fins with a large flow rate of air. This design has a lower capital cost but a larger operating cost. In this case it is not at all clear that minimizing either capital or operating costs by themselves constitutes an optimum design. Rather, a more sophisticated economic analysis is required in order to define “optimum.”

This paper gives a brief introduction to simple engineering economics and outlines its use in optimizing a heat transfer system. Though a specific example is used for illustration, the methods employed here are general and can easily be extended to other applications. The method makes extensive use of computer software as a tool for optimization. Specifically, the software *Engineering Equation Solver* or EES (pronounced “Ease”) is used in the example. The software has been developed especially for thermal-fluid engineering applications and offers a versatility and ease of solution unavailable with more traditional optimization schemes such as linear programming or the method of Lagrangian Multipliers.

#### Simple Engineering Economics

In the previous section we saw that there are two basic types of costs, **capital costs** and **operating costs**. Capital costs represent the one time expenses usually associated with purchasing equipment, facilities, tools and/or land, whereas operating costs are ongoing expenses that occur repeatedly. From the perspective of designing a specific heat transfer system, the capital cost is how much the system costs to buy or build; the operating cost is how much it costs to run. (Operating costs are usually reported on a per year basis.) In general, heat transfer systems with large capital costs are less expensive to operate than systems with small capital costs.

Figure 1 shows a graph of possible designs for a given heat transfer objective, with capital cost on the vertical axis and operating cost on the horizontal axis. From Figure 1 we can see that Design *B* is better than Design *A*, as Design *B* has the same operating cost as Design *A* but has a smaller capital cost. Design *C* is also a better design than *A* since it offers less expensive operation than *A* for the same capital cost. The dashed line bounding the possible designs on the left and the bottom, then, represents a line of best designs. Designs to the left or below this line are impossible either due to physical laws or technological feasibility. The goal of economic optimization is to guarantee that a design falls along this line.

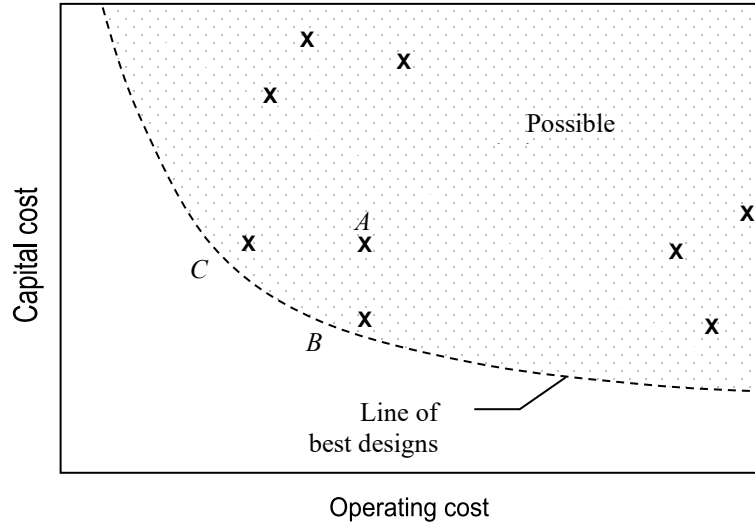


Figure 43.1: Competing effect of capital and operating costs

### Simple payback period

A number of economic parameters have been suggested to assess the competing nature of capital and operational costs. The most elementary parameter is known as the **simple payback period**, or more succinctly as the **simple payback (SPB)**. Mathematically, it is given as the derivative of capital cost with respect to annual savings:

$$SPB = \frac{d(Cap)}{d(Sav)} \quad (43.1)$$

where  $Cap$  is capital cost in dollars and  $Sav$  is yearly savings in dollars per year. If yearly savings  $Sav$  is given by a decrease in annual operating cost, then

$$SPB = -\frac{d(Cap)}{d(Oper)}. \quad (43.2)$$

where  $Oper$  is annual operating cost.

The interpretation of  $SPB$  as given in (43.1) is the amount of time in years required to recoup a capital investment in the form of savings brought about by a decrease in operating cost. Specifically, an *additional* capital dollar invested to improve an existing heat transfer design will be recovered in  $SPB$  years. Geometrically,  $SPB$  is given by the negative of the slope of the best design curve in Figure 1. (An  $SPB$  for a design not on the best design line is not considered here, as it does not represent an optimum.)

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Often the simple payback is cited as the time required to recover an *entire* capital investment in a new design over an existing, base design. Here we will refer to such a payback as the *average* simple payback,

$$SPB_{avg} = -\frac{\Delta(Cap)}{\Delta(Oper)}. \quad (43.3)$$

The difference between  $SPB$  and  $SPB_{avg}$  can be seen in Figure 2. Whereas  $SPB$  represents the instantaneous slope of the best design curve,  $SPB_{avg}$  represents only the slope of a straight line between two discrete designs falling on the curve.

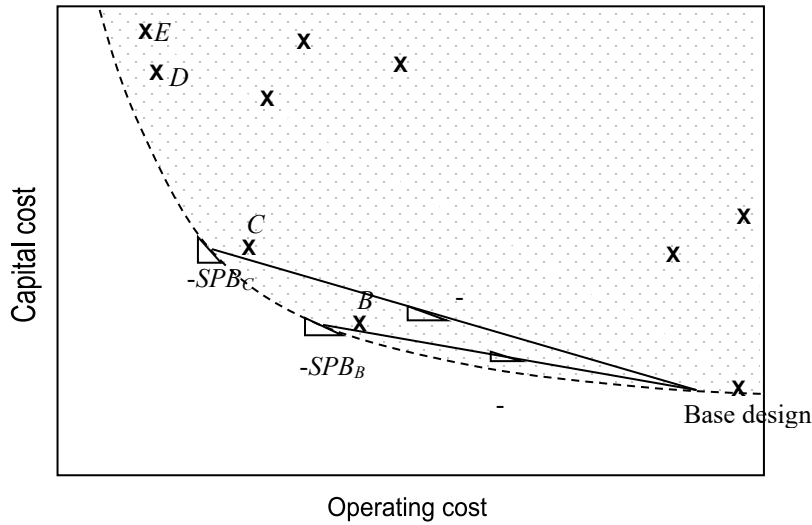


Figure 43.2: Simple payback

By considering only the simple payback as measured by  $SPB_{avg}$  we effectively ignore the design possibilities lying along the best design curve as bases of comparison. For example, Design C ostensibly has only a slightly longer simple payback than Design B as measured by  $SPB_{avg}$ . It is in error, however, to assume that the additional capital spent on Design C over Design B is recovered in  $(SPB_{avg,C} - SPB_{avg,B})$ . Rather, we see that  $SPB_C$  is significantly larger than  $SPB_B$ , indicating that *additional* capital spent on Design C over Design B takes a much longer period of time to recover than suggested by the average simple payback concept. This effect is most pronounced at very small operating costs where the best designs line becomes quite steep. It is difficult to argue that Design E is a better design than Design D, for instance, as additional capital spent on Design E over Design D will almost certainly

never be recovered in the lifetime of the design. This effect may go completely unnoticed when considering only  $SPB_{avg}$ .

In short, by defining the simple payback using (43.2) rather than the average simple payback of (43.3), we capture an effect not seen in the latter, namely the diminished returns encountered with increasingly large capital costs. For this reason, in this paper we will consider only simple payback as defined in (43.2).

### Present discounted value and return on investment

The advantage of using the simple payback period in the economic analysis of heat transfer systems lies in its simple conceptual and geometric interpretations. An inherent limitation, however, is that  $SPB$  does not take into account the **time value of money**. That is,  $SPB$  does not take into account the fact that rather than generating annual savings, one could make annual payments to interest bearing accounts and generate additional new revenue. And so the economic question becomes whether to invest capital in new/improved heat transfer equipment in order to generate future savings, or simply to invest money annually and earn interest. A parameter called the Present Discounted Value makes such a comparison.

The Present Discounted Value ( $PDV$ ) is defined by the following equation:

$$PDV = Sav \frac{(1+i)^n - 1}{i(1+i)^n} \quad (43.4)$$

where  $Sav$  is yearly savings,  $i$  is the annual interest rate and  $n$  is the number of years.  $PDV$  represents the *current* value of  $n$  years worth of future savings (decreases in operating costs) taking into account the fact that those savings could be invested at an interest rate of  $i$  instead. For example, consider a particular capital purchase of \$5000 that is expected to generate \$500 of savings per year over the next ten years. One may be tempted to claim that capital will be recovered in ten years, as \$500 of yearly savings over ten years gives 10 years x \$500/year = \$5000. This is not the case, however, as this \$5000 of *future* savings generated over ten years has a *present* value of only

$$PDV = \$500 \cdot \frac{(1+0.1)^{10} - 1}{0.1(1+0.1)^{10}} = \$3072 \quad (43.5)$$

where we have assumed an annual interest rate of  $i = 10\%$ . In other words, one could invest \$3072 today at  $i = 10\%$  and earn just as much at the end of ten years as if one had used \$3072 to purchase equipment that generated a \$500/year decrease in operating costs. This makes the prospect of investing \$5000 of capital in equipment to generate that same annual savings less attractive indeed. (Even if the time value of money were *not* considered here, we see that ten years of \$500 annual savings represents only  $SPB_{avg}$  and not the more meaningful  $SPB$ .)

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Another way to take into account the time value of money is to compute an effective interest rate for a capital cost  $Cap$  expected to produce future annual savings  $Sav$  over  $n$  years. This effective interest rate is known as the Return on Investment ( $ROI$ ), and is found by setting  $Cap$  equal to  $PDV$  and solving the resulting equation for the interest rate:

$$Cap = Sav \frac{(1 + ROI)^n - 1}{ROI(1 + ROI)^n} \quad (43.6)$$

Although the above equation must be solved numerically, the  $ROI$  gives a simple comparison of potential future savings to investment returns.

In our previous example, in which a capital purchase of \$5000 is expected to generate \$500 of savings per year over the next ten years, the  $ROI$  for that ten year period is 0%. Should the capital investment expected to generate a \$500/year savings over the next ten years be only \$4000, however, (43.6) gives

$$\$4000 = \$500 \cdot \frac{(1 + ROI)^{10} - 1}{ROI(1 + ROI)^{10}} \quad (43.7)$$

$$ROI = 0.043 = 4.3\% \quad (43.8)$$

If an interest bearing account can be found with  $i > 4.3\%$ , then our \$4000 might be better utilized by generating interest rather than purchasing new equipment. The simplicity of this type of comparison has made the Return on Investment perhaps the most popular economic parameter. (It should also be noted that  $ROI$  can take on negative values, indicating we are losing money over time for a particular capital expenditure.)

Numerous other economic parameters exist, but those given here represent three of the most commonly used parameters in engineering economics. The question of which parameter to use is largely a matter of taste. Luckily, the advent of modern software such as *EES* has made computing these quantities rather painless, and several different economic parameters can be calculated from the same general analysis.

### Case study: Optimization of a Water-Cooled Condenser Refrigeration Unit

As an example of the economic optimization of heat transfer systems, let us consider a case study in which a client of an engineering consulting firm is considering upgrading an existing air conditioning system. In particular, the client is interested in the economics of purchasing a new compressor/water-cooled condenser unit for an existing air-conditioning system. The client has little technical expertise in the area, but is familiar with economic terminology. Thus, the deliverable would most likely be a summary of economic predictions for the purchase of a new unit. This represents

an ideal opportunity to perform an economic analysis with the goal of recommending an optimized system.

One of the first tasks should be defining what constitutes an economic optimum. If only capital costs *or* operating costs, but not both are important, than the optimum design is the one which minimizes that cost. If both costs are important, however, then there are several ways to define an economic optimum, just as there is more than one economic parameter comparing the tradeoffs between capital and operating costs.

One way to optimize a design is to specify a simple payback period and find the best design meeting that criterion. Here the optimization process amounts to finding a design that lies along the line of best designs in Figure 1, where the local slope of the best design curve represents the negative of the specified *SPB*. Different specified *SPBs* result in different optimums. For example, the best design would be different for a client willing to accept a simple payback period of five years than for one willing to accept a simple payback of only three years. A design that maximizes Return on Investment represents another economic optimum. In the case study considered here, both of these approaches will be considered.

In order to perform an analysis, the problem definition must first be made more rigorous, and a number of modeling assumptions must be made as well. For our analysis, we will assume the following:

#### System requirements

- The client requires 1 ton (12000 Btu/hr) of cooling.
- The system uses R134-a as a refrigerant.
- The R134-a operates at an evaporator temperature of 45°F.

#### System reliability assumptions

- Evaporator exit superheating should be kept at 10°F.
- Condenser exit subcooling should be kept at 10°F.
- The hot to cold fluid temperature difference in any heat exchanger should be no less than 5°F.

#### Compressor modeling assumptions:

- Isentropic efficiency is 65%.
- Purchase cost is approximately \$800/horsepower.
- Operating cost is based on local utility rates at approximately \$0.07/kW-hour.

#### Condenser modeling assumptions

- The condenser is water-cooled with a purchase cost of approximately  $\$0.70/UA$ , where  $UA$  is in  $W/^\circ C$
- Operating costs for condensers are based on providing city water at a cost of approximately  $\$3.25/100 \text{ ft}^3$ . This includes sewer fee.
- City water is available at a temperature of 65°F.
- Fluid streams experience negligible pressure losses.

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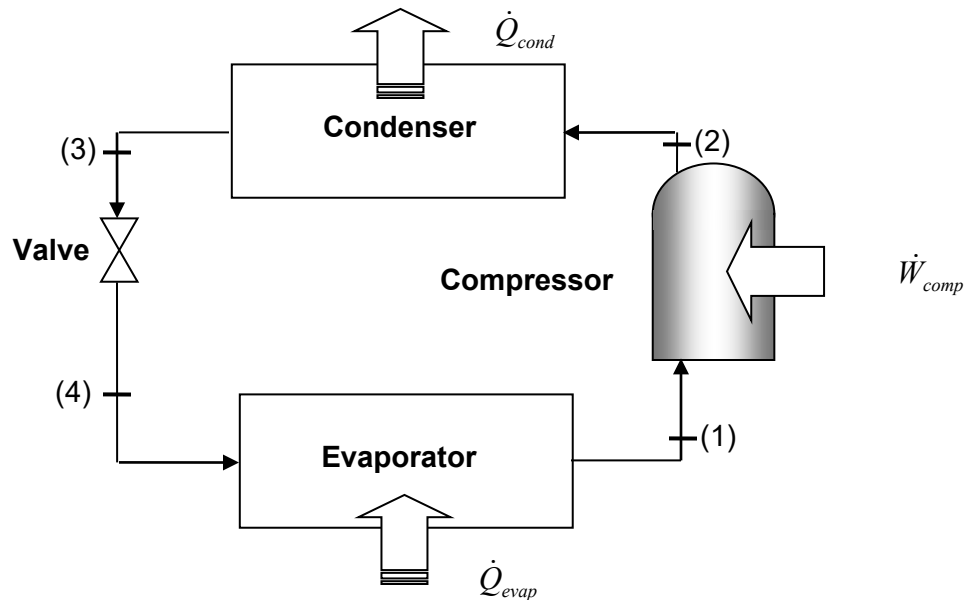
Other information

- Evaporator fluid streams experience negligible pressure losses.
- The existing system has an annual operating cost of \$2380.

This set of assumptions physically constrains some but not all of the system parameters. Choosing the remaining system parameter(s) in such a way that the additional constraint of a specified *SPB* or a maximized *ROI* is achieved is the heart of the optimization process.

The preceding assumptions regarding the capital costs and operating costs of the equipment are subject to frequent change and therefore possibly the most spurious of the modeling assumptions. It should be kept in mind that any model is better than none, however; furthermore, the versatility of a software model allows these figures to be easily changed should more reliable data become available.

Let us begin our analysis by analyzing the refrigerant side of the system only. A schematic of the basic vapor refrigeration cycle is shown in Figure 3.



**Figure 43.3:** Schematic diagram of the refrigerant side of the system

Conservation of energy applied to evaporator yields



$$\dot{Q}_{evap} = \dot{m}_{r134a} (h_1 - h_4) \quad (43.9)$$

The heat transfer rate into the evaporator is fixed at one ton via the problem statement. The exit enthalpy  $h_1$  is fixed by the refrigerant exit pressure and temperature, which is 10°F higher than the evaporation temperature of 45°F. Both the flow rate of refrigerant and the inlet enthalpy are unknown as of yet.

Conservation of energy for an isentropic compressor yields

$$\dot{W}_s = \dot{m}_{r134a} (h_{2s} - h_1) \quad (43.10)$$

where  $h_{2s}$  is the enthalpy of the refrigerant corresponding to condenser pressure and the compressor inlet entropy. Conservation of energy for the actual compressor is given by

$$\dot{W} = \dot{m}_{r134a} (h_2 - h_1). \quad (43.11)$$

The isentropic compressor efficiency relates the isentropic compressor power to the actual power.

$$\eta_c = \frac{\dot{W}_s}{\dot{W}} \quad (43.12)$$

Conservation of energy applied to condenser and throttling valve give, respectively

$$\dot{Q}_{cond} = \dot{m}_{r134a} (h_2 - h_3) \quad (43.13)$$

$$h_4 = h_3. \quad (43.14)$$

The enthalpy  $h_3$  is calculated at the condenser exit pressure and temperature, which is 10°F cooler than the condensing temperature.

Examination of the above equations will show that once a condensing temperature or pressure is chosen, all other property information can be found, and the resulting equations solved. This represents one degree of freedom in the cycle. Equivalently, assigning a value to the compression ratio, the ratio of the exit compressor pressure to the inlet compressor pressure, completes the equation set:

$$r_c = \frac{P_{cond}}{P_{evap}} \quad (43.15)$$

At this point let us examine the use of the *Engineering Equation Solver* software (EES) in the solution of the above equation set. It is not the intent here to give a detailed tutorial of EES, but rather to outline its major features and its utility in performing economic optimization. The two main features of EES include the ability to solve large numbers of simultaneous non-linear algebraic equations, and the ability

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to include thermodynamic and/or transport property look ups as part of the equation set. The interface is very intuitive, and once the syntax for writing a property look up in equation form is learned, a novice can literally start using the software in minutes.

Equations are entered into EES by typing them in the Equations window. A property function call has the following general form:

$$\text{Variable} = \text{Property}(\text{Substance}, \text{Prop1} = [\text{value}], \text{Prop2} = [\text{value}])$$

where `Property` is the desired property, `Substance`, is the substance name, and `Prop1` and `Prop2` are independent properties needed to fix the state for the desired property. For example, in order to calculate the value of  $h_1$  we would type

$$h\_1 = \text{enthalpy}(\text{R134a}, \text{T}=55, \text{P}=54.8)$$

The values of the supplied independent properties may be unknowns themselves, as is the case the enthalpy at state point 3:

$$h\_3 = \text{enthalpy}(\text{R134a}, \text{T}=\text{T}_3, \text{P}=\text{P\_cond})$$

This ability to couple unknown property information with the governing equations gives EES both its versatility and power.

Figure 4 shows a screen shot of the above equation set typed into the Equations window of EES. The reader will notice that the compression ratio  $r_c$  has been arbitrarily set to 3.

Once a complete equation set has been entered into the Equations window, choosing the `Solve` command from the `Calculate` menu will solve the equation set and list the results in the `Solutions` window. Figure 5 shows the solution to the above equation set.

Parametric trends are easily examined by changing values in the Equations window and recalculating. In our case, different values of the compression ratio can be set in the Equations window and its effect on the rest of the cycle assessed. The process can be streamlined by making use of the `Parametric Table` feature of EES. In this process, equations assigning values to parameters of interest are removed from the equations window and their values assigned to individual runs in a `Parametric Table`. The user then chooses `Solve Table` from the `Calculate` menu, at which point EES solves all the equations in the Equations window, using the assigned values of the parameter in the table as the remaining equation for each run. Figure 43.6 shows the results of this process for our examples with compression ratios ranging from two to seven. Any number of parameters can be assigned in the `Parametric Table` in this fashion.

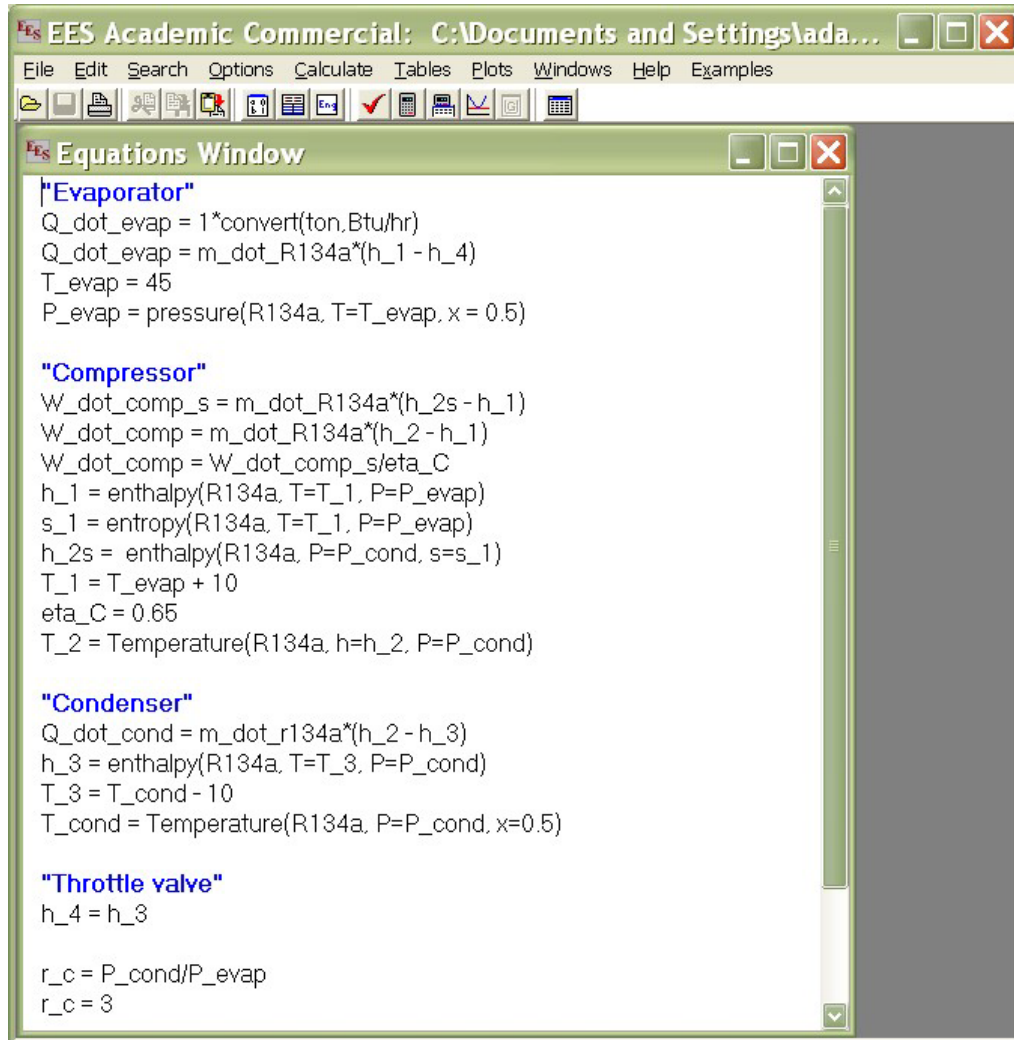


Figure 43.4: The EES Equations window

Examination of the table in Figure 43.6 shows that larger compression ratios always mean larger compressor sizes. This translates into both larger capital and operating costs for the compressor, which may lead one to believe that the optimum system is the one that utilizes the smallest possible compressor. However, we have yet to add the more detailed analysis of the water-cooled condenser. Larger compression ratios lead to larger condenser inlet refrigerant temperatures ( $T_2$ ). As the inlet water temperature of the condenser is set at 65°F, larger refrigerant-to-water fluid temperature differences result within the condenser, and therefore, a smaller required heat transfer surface area is required for the condenser heat exchanger.

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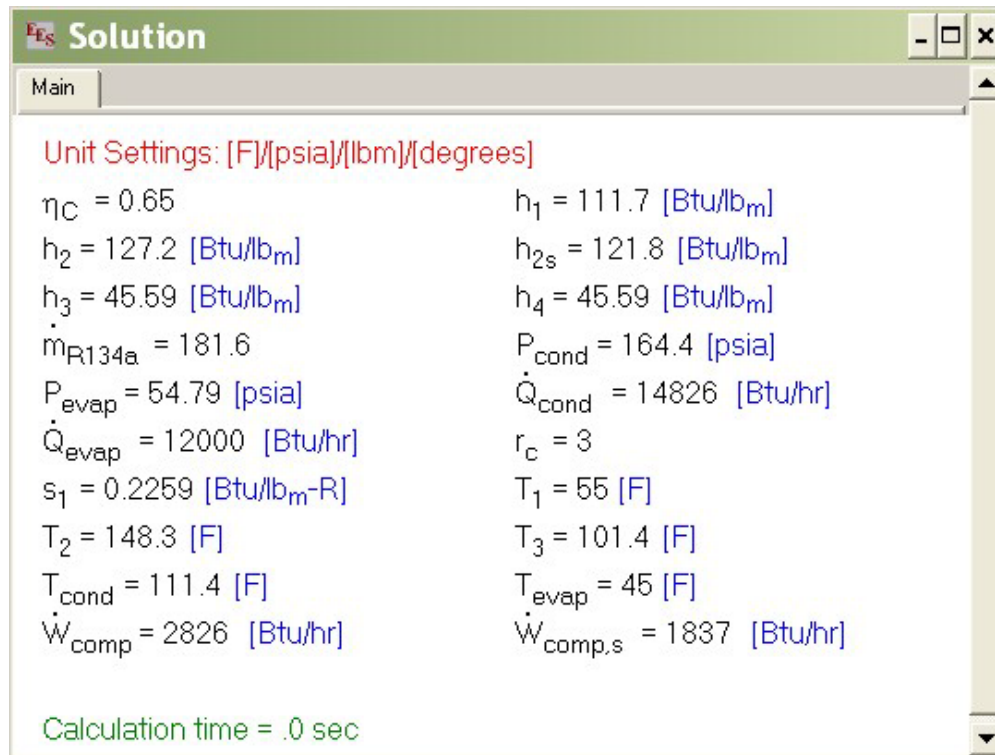


Figure 43.5: The EES Solution window

Thus, larger compressors mean smaller condensers. In economic terms, more capital invested in a compressor results in less capital invested in the condenser, and *vice versa*.

1..10	1 $r_c$	2 $\dot{W}_{\text{comp}}$ [Btu/hr]	3 $T_2$ [F]	4 $\dot{Q}_{\text{cond}}$ [Btu/hr]	5 $P_{\text{cond}}$ [psia]
Run 1	2.00	1560	113	13560	110
Run 2	2.56	2281	134	14281	140
Run 3	3.11	2960	152	14960	170
Run 4	3.67	3621	166	15621	201
Run 5	4.22	4280	179	16280	231
Run 6	4.78	4948	191	16948	262
Run 7	5.33	5637	201	17637	292
Run 8	5.89	6357	210	18357	323
Run 9	6.44	7119	219	19119	353

Figure 43.6: The EES Parametric Table window

It is unclear as to how changing compression ratio affects water flowrate within the condenser. Condensers with smaller surface areas may require larger flowrates of water to accomplish the same heat transfer rate, thereby increasing operating costs. And since the rate of heat transfer within the condenser increases with compression ratio, this effect may be exacerbated. On the other hand, the larger refrigerant to water temperature difference at large  $r_c$  may require smaller water flowrates. We see, then, that the trade-offs between capital and operating costs for the system become quite difficult to intuit. The use of software such as EES for modeling the system thus becomes indispensable in discerning these trends.

Figure 43.7 shows a schematic drawing of the condenser heat exchanger, modeled here as a simple counter-flow type. Figure 8 gives a temperature-area diagram of the same heat exchanger. Three distinct regions can be inferred from Figure 43.8, corresponding to those regions of the heat exchanger in which the refrigerant is desuperheated, condensed, and then subcooled.

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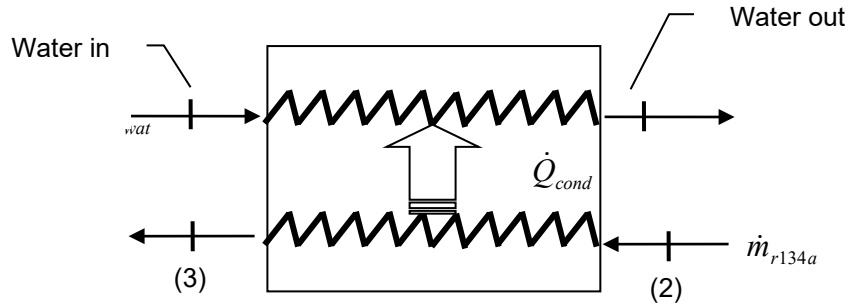


Figure 43.7: Schematic diagram of the water-cooled condenser

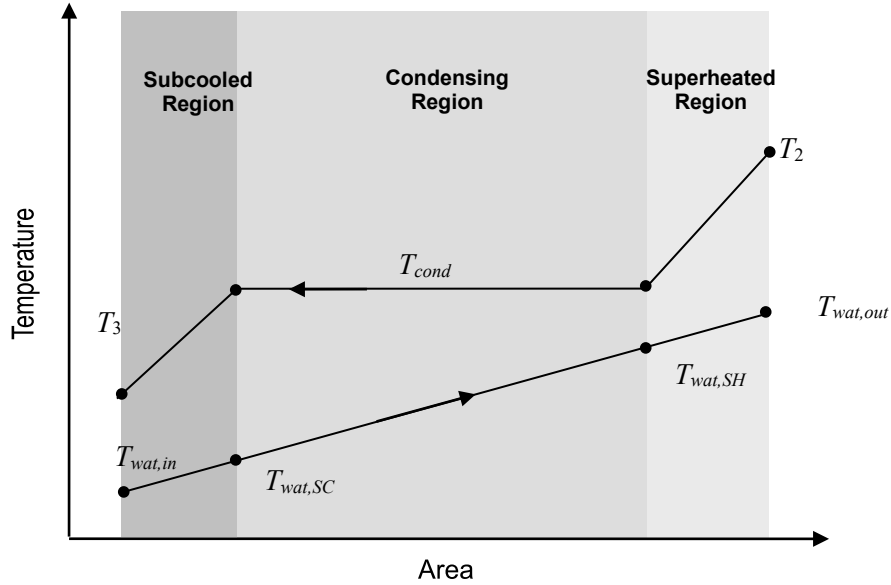


Figure 8: Variation of fluid temperatures within the condenser heat exchanger

An energy balance on the superheated region of the condenser gives

$$\dot{Q}_{sh} = \dot{m}_{r134a} (h_2 - h_g) = \dot{m}_{wat} c_{p,wat} (T_{wat,out} - T_{wat,sh}) \quad (43.16)$$

The  $UA$  for the superheated section is related to the heat transfer rate by

$$\dot{Q}_{sh} = UA_{sh} \frac{(T_2 - T_{wat,out}) - (T_{cond} - T_{wat,out})}{\ln \frac{(T_2 - T_{wat,out})}{(T_{cond} - T_{wat,out})}} \quad (43.17)$$

The analogous equations for the condensing and subcooled regions, respectively, are

$$\dot{Q}_{sat} = \dot{m}_{r134a} (h_g - h_f) = \dot{m}_{wat} c_{p,wat} (T_{wat,sh} - T_{wat,sc}) \quad (43.18)$$

$$\dot{Q}_{sat} = UA_{sat} \frac{(T_{cond} - T_{wat,sh}) - (T_{cond} - T_{wat,sc})}{\ln \frac{(T_{cond} - T_{wat,sh})}{(T_{cond} - T_{wat,sc})}} \quad (43.19)$$

$$\dot{Q}_{sc} = \dot{m}_{r134a} (h_f - h_3) = \dot{m}_{wat} c_{p,wat} (T_{wat,sc} - T_{wat,in}) \quad (43.20)$$

$$\dot{Q}_{sc} = UA_{sc} \frac{(T_{cond} - T_{wat,sc}) - (T_3 - T_{wat,in})}{\ln \frac{(T_{cond} - T_{wat,sc})}{(T_3 - T_{wat,in})}} \quad (43.21)$$

The total  $UA$  for the entire heat exchanger is simply the sum of the  $UAs$  for the separate regions

$$UA = UA_{sh} + UA_{sat} + UA_{sc} \quad (43.22)$$

Figure 8 shows that the minimum temperature difference between the refrigerant and the water most likely occurs as the refrigerant enters the condensing region of the heat exchanger. The problem statement requires that this difference be no less than 5°F. Thus,

$$T_{wat,sh} = T_{cond} - 5^\circ\text{F} \quad (43.23)$$

The operating cost of the condenser is governed by the required volumetric flow rate of water. This is easily found from knowledge of the water mass flow rate,

$$\dot{m}_{wat} = \rho \dot{V} \quad (43.24)$$

where  $\rho$  and  $\dot{V}$  are the density and volumetric flow rate of water, respectively.

Once a compression ratio is chosen, the previous refrigerant side analysis fixes  $T_2$ . Equations 14-22 can then be solved to size the condenser heat exchanger. In other words, equations 14-22 can be added to our previous analysis with the resulting degrees of freedom remaining at one, that being the compression ratio  $r_C$ .

To find the economic optimum, the cost data must also be included. Per our modeling assumptions, the capital cost of the system is governed by the purchase of a compressor and condenser, and is given by the equations

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$$Cap_{comp} = \dot{W}_{comp} \cdot 800 \frac{\$}{\text{hp}} \quad (43.25)$$

$$Cap_{cond} = UA \cdot 0.70 \frac{\$}{\text{W}/^\circ\text{C}} \quad (43.26)$$

where  $\dot{W}_{comp}$  is in horsepower and  $UA$  is in  $\text{W}/^\circ\text{C}$ . The operating cost of the system is governed by the same two pieces of equipment, and is given by

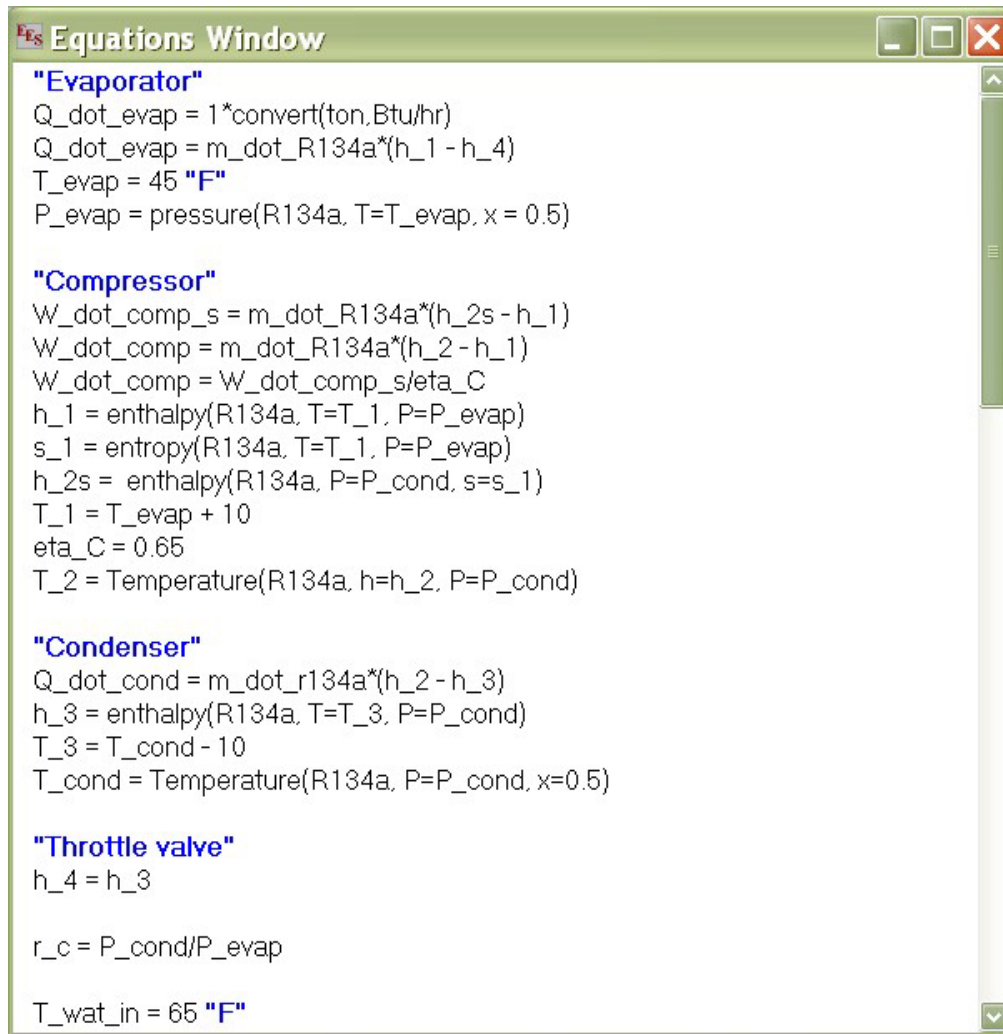
$$Oper_{comp} = \dot{W}_{comp} \cdot 210 \frac{\text{days}}{\text{year}} \cdot 0.07 \frac{\$}{\text{kW} - \text{hr}} \quad (43.27)$$

$$Oper_{cond} = \dot{V} \cdot 210 \frac{\text{days}}{\text{year}} \cdot 0.0325 \frac{\$}{\text{ft}^3} \quad (43.28)$$

where we have assumed operation of the unit for 210 days per year. (Care must be taken in handling the units in the above two equations. EES facilitates this as well by inclusion of a unit conversion function call.)

Figures 43.9 and 43.10 show the Equations window for the complete equation set outlined above. By using the Parametric Table feature for various values of  $r_C$ , the overall system capital and operating costs are calculated as a function of  $r_C$ . Figures 43.11 and 43.12 show the results of this analysis. (The figures were created using the graphing capabilities in EES.)





```

"Evaporator"
Q_dot_evap = 1*convert(ton.Btu/hr)
Q_dot_evap = m_dot_R134a*(h_1 - h_4)
T_evap = 45 "F"
P_evap = pressure(R134a, T=T_evap, x = 0.5)

"Compressor"
W_dot_comp_s = m_dot_R134a*(h_2s - h_1)
W_dot_comp = m_dot_R134a*(h_2 - h_1)
W_dot_comp = W_dot_comp_s/eta_C
h_1 = enthalpy(R134a, T=T_1, P=P_evap)
s_1 = entropy(R134a, T=T_1, P=P_evap)
h_2s = enthalpy(R134a, P=P_cond, s=s_1)
T_1 = T_evap + 10
eta_C = 0.65
T_2 = Temperature(R134a, h=h_2, P=P_cond)

"Condenser"
Q_dot_cond = m_dot_r134a*(h_2 - h_3)
h_3 = enthalpy(R134a, T=T_3, P=P_cond)
T_3 = T_cond - 10
T_cond = Temperature(R134a, P=P_cond, x=0.5)

"Throttle valve"
h_4 = h_3

r_c = P_cond/P_evap

T_wat_in = 65 "F"

```

Figure 43.9: Case study equation set within EES Equations window

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```
EES Equations Window
"Condenser HXR analysis"
m_dot_wat*c_p_wat*(T_wat_out - T_wat_sh) = m_dot_R134a*(h_2 - h_g_cond) "Superheated section"
h_g_cond = enthalpy(R134a,T=T_cond,x=1)
c_p_wat = specheat(water,T=T_wat_in,P=14.7)
Q_dot_sh = m_dot_R134a*(h_2 - h_g_cond)
Q_dot_sh = UA_sh*LMTD_sh
LMTD_sh = ((T_2 - T_wat_out) - (T_cond - T_wat_sh))/ln((T_2 - T_wat_out)/(T_cond - T_wat_sh))

m_dot_wat*c_p_wat*(T_wat_sh - T_wat_sc) = m_dot_R134a*(h_g_cond - h_f_cond) "Condensing section"

Q_dot_sat = m_dot_R134a*(h_g_cond - h_f_cond)
Q_dot_sat = UA_sat*LMTD_sat
LMTD_sat = ((T_cond - T_wat_sh) - (T_cond - T_wat_sc))/ln((T_cond - T_wat_sh)/(T_cond - T_wat_sc))

m_dot_wat*c_p_wat*(T_wat_sc - T_wat_in) = m_dot_R134a*(h_f_cond - h_3) "Subcooled section"
h_f_cond = enthalpy(R134a,T=T_cond,x=0)
Q_dot_sc = m_dot_R134a*(h_f_cond - h_3)
Q_dot_sc = UA_sc*LMTD_sc
LMTD_sc = ((T_cond - T_wat_sc) - (T_3 - T_wat_in))/ln((T_cond - T_wat_sc)/(T_3 - T_wat_in))

UA = UA_sc + UA_sat + UA_sh
T_wat_sh = T_cond - 5
m_dot_wat = density(water,T=T_wat_in,P=14.7)*(Vol_dot_wat*convert(gpm,ft^3/hr))

Cap_comp = 800*W_dot_comp*convert(Btu/hr,HP) "Econ Anal"
Cap_HXR = 0.7*(UA*convert(Btu/(hr*R),W/K))
Cap = Cap_HXR + Cap_comp
Oper_comp = W_dot_comp*convert(Btu/hr, kW*hr/day)*210*0.07 "$/yr"
Oper_HXR = Vol_dot_wat*convert(gpm,ft^3/day)*210*0.0325 "$/yr"
Oper = Oper_HXR + Oper_comp
```

Figure 43.10: Case study equation set within EES Equations window (continued)

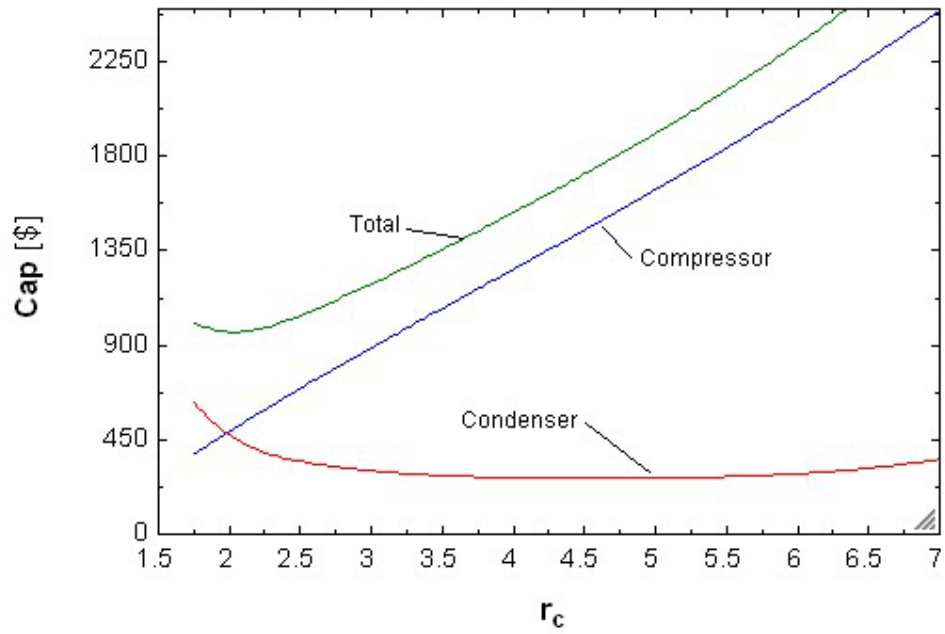


Figure 43.11: System capital costs

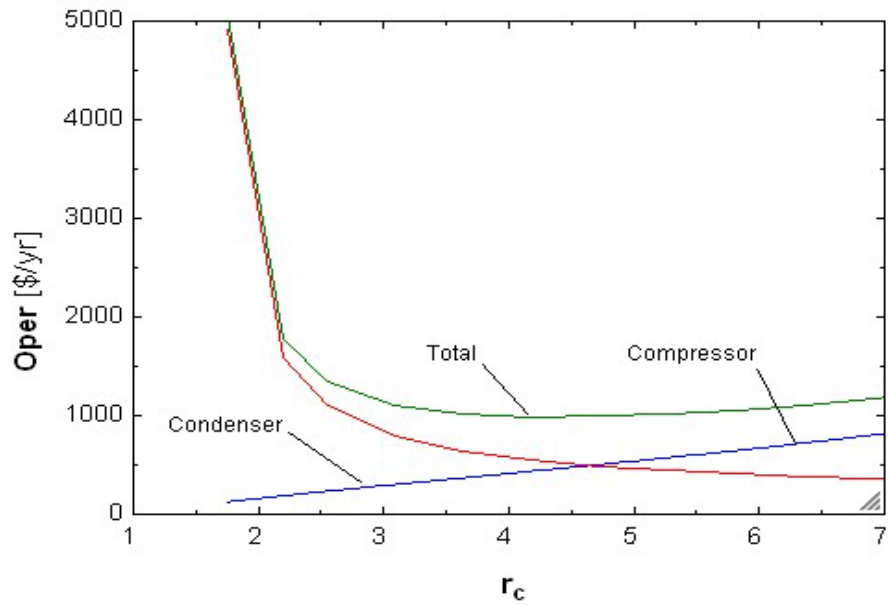


Figure 43.12: System operating costs

### 43.22 Economic Optimization

Figures 43.11 and 43.12 show that increased compression ratio indeed has the opposite effect on the costs of the compressor and the condenser. As compression ratio increases, both the capital and operating costs of the compressor steadily rise, whereas both costs decrease for the condenser. Figures 11 and 12 also show that the total capital cost and the total operating cost for the system occur at very different compression ratios of approximately 2 and 4.5, respectively. As previously stated, in a case where only capital or operating costs, but not both, are important, this analysis alone is sufficient to determine the best design. In our example, if the salvage value of the existing system is significant, capital costs may not be important. Thus the best design would be the one that minimizes total operating cost at a compression ratio of two.

For the case in which both of these costs are important, let us first determine the optimum system by optimizing for a specified *SPB*. To find this optimum, the following figure of merit (*FOM*) is minimized

$$FOM = Cap + SPB \cdot Oper \quad (43.29)$$

The rationale for minimizing this term is seen by taking the first derivative of *FOM* with respect to *Oper* and setting it equal to zero.

$$\frac{d(FOM)}{d(Oper)} = \frac{d(Cap)}{d(Oper)} + SPB = 0 \quad (43.30)$$

Solving for *SPB* we find

$$SPB = -\frac{d(Cap)}{d(Oper)} \quad (43.31)$$

identically, as it should be. The interpretation of *FOM* is the total cost of a design for a period of time equal to the simple payback, including both capital and operating costs. By minimizing this cost, we ensure that our design falls along the line of best designs in Figure 1.

Adding (43.29) for *FOM* to our equation set and setting *SPB* = 4 years generated the numbers used to create Fig. 43.13. From Figure 43.13 we see that the best design for a simple payback of four years corresponds to a compression ratio of approximately 3.65. This process can be repeated for a range of simple paybacks, which would allow the client choose the optimum design for the *SPB* they are willing to accept. Such an analysis is presented in Figure 43.14. For our case, it is interesting to note that the compression ratio for the best system varies only slightly for a wide range of *SPB*.

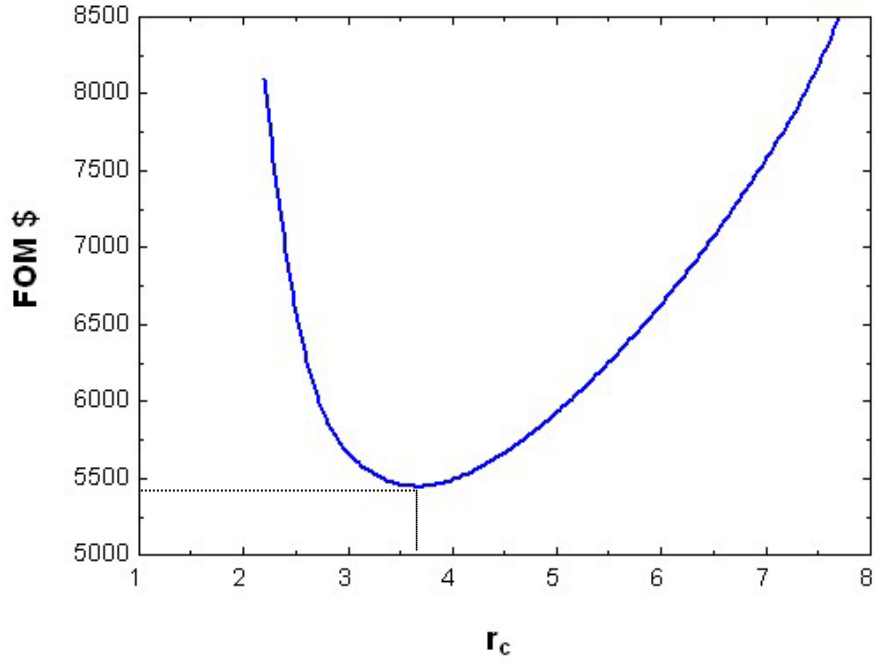


Figure 43.13: Determination of best system size for  $SPB = 4$  years

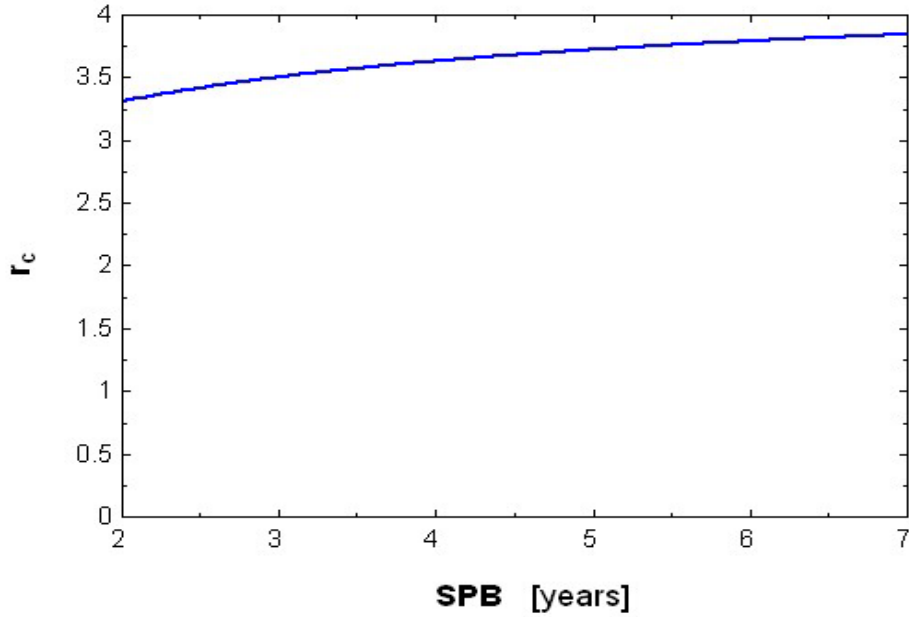


Figure 43.14: Size of best system for given simple payback period

#### 43.24 Economic Optimization

Optimizing for a specified *SPB* allows for the competing effects of capital and operating costs to be taken into account, but neglects the time value of money. By defining the optimum design as the one that maximizes the *ROI* both effects are considered. In our example this is accomplished by including the defining equation for *ROI*, (43. 6, in our equation set. Here the savings is given by the decrease in annual operating costs for the proposed design over the existing system.

$$Sav = \$2083 - Oper \quad (43.32)$$

Figure 15 shows the *ROI* over a four year period as a function of  $r_c$ . The optimum system using this criterion occurs at a compression ratio of just under three, with a *ROI* reaching close to 100%. As interest bearing accounts do not have interest rates anywhere close to this value, this represents a very attractive design indeed.

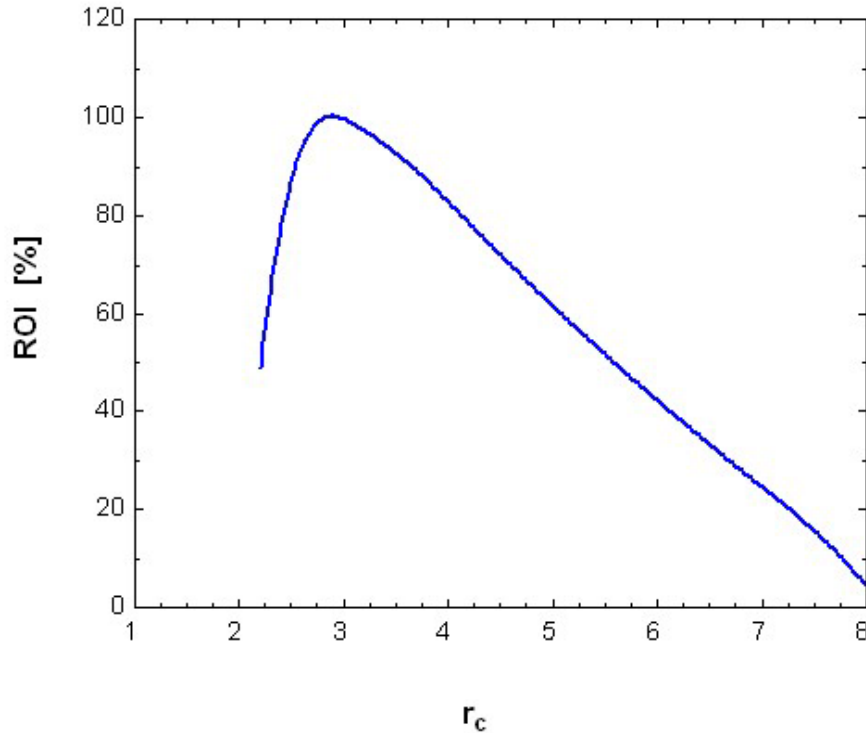


Figure 43.15: Return on investment analysis results

At this point in our case study, we can either remain satisfied with our analysis or add more detail as needed. For example, rather than treating the condenser heat exchanger as something of a “black box” for which a required *UA* value was

calculated but no physical dimensions, the individual convective heat transfer coefficients (the  $h$ s) could be calculated for specific heat exchanger types and geometries. This would undoubtedly give us a more accurate estimate of both the capital cost and operating cost of the condenser. Pressure drops within the fluids may also be considered. Whether or not we should spend the extra time and effort in this more detailed analysis, of course, should be dictated by the time and budget restraints of the project itself.

Even without an extremely detailed model of the system, however, we can easily distinguish the major trends involved. It is clear, for example, that the economically optimum system, however it is defined, should have a compression ratio between three and four. Furthermore, we see from Figure 11 that the capital cost of the compressor dominates the other costs in this range of compression ratios. Therefore, we may want to spend more time in a future analysis exploring the effect of compressor isentropic efficiency on system performance rather than the more tedious heat exchanger analysis of the condenser.

## Conclusion

We have introduced simple engineering economic parameters as a way of assessing the competing costs associated with the design of heat transfer systems. Through a case study, we have coupled the physical governing equations of a particular heat transfer system with these parameters in order to assess the economic trends, defining a economic optimum as the best design for a specified simple payback, or one that maximizes return on investment. In the process, we made extensive use of the *Engineering Equation Solver* software (EES), as its versatility and functionality makes it especially suited for such applications. The methods outlined in this example are easily extended to a very large variety of heat transfer systems.

## 43.26 Economic Optimization

### Nomenclature

$Cap$	capital cost, \$
$c_p$	specific heat, B/lbm F°
$h$	enthalpy, B/lbm
$i$	interest rate
$n$	number of years
$Oper$	operating cost, \$/year
$PDV$	present discounted value, \$
$Q$	rate of heat transfer, B/hour
$P$	pressure, psia
$r_c$	compression ratio
$ROI$	return on investment
$Sav$	annual savings, \$/year
$SPB$	simple payback, years
$UA$	heat exchanger combined heat transfer coefficient, B/hour F°
$\dot{V}$	volumetric flowrate, gpm
$\dot{W}$	power, B/hour

### Greek

$\rho$	density, lbm/ft <sup>3</sup>
$\eta$	efficiency

### Subscripts

$cond$	condenser
$evap$	evaporator
$f$	saturated liquid
$g$	saturated vapor
$r134a$	refrigerant R134a
$s$	isentropic
$sat$	saturated
$sc$	subcooled
$sh$	superheated
$wat$	water