Known Gasoline is draining.

Find

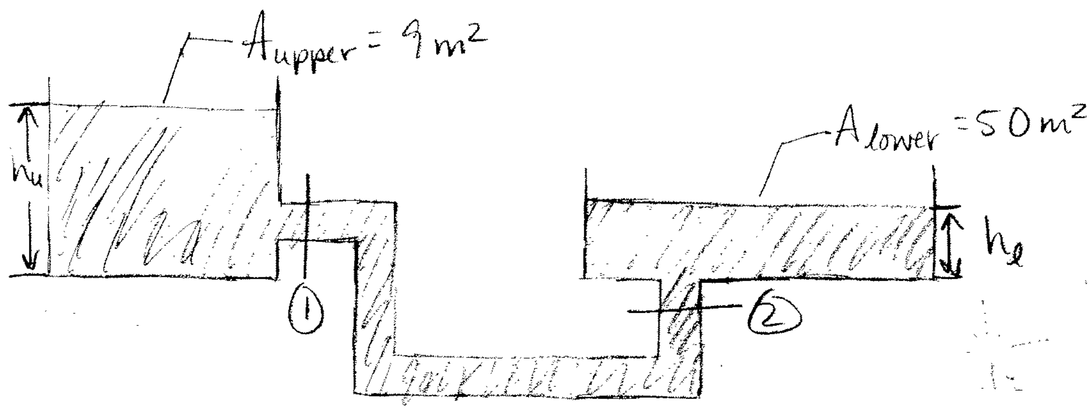
(a) Starting w/ the rate form of the Cons Mass equation and an an appropriate OPEN system, determine the volumetric flow rate ($\dot{V}$) and speed of the fluid (V) in the connecting pipe, in m³/min and m/s, respectively.

(b) Using Cons Mass and an OPEN system, find the rate at which the fluid level in the lower tank changes, in m/min. Indicate whether increasing or decreasing.

(c) Repeat part (b) but instead use a CLOSED system (all of the fluid in both tanks and the connecting pipe).

Given

h_u is decreasing at a rate of 0.333 m/min



$l_{pipe} = 50m$

$d_{pipe} = 0.10m$

$r_{pipe} = 0.05m$

A_{pipe} = cross-sectional area of pipe

V_{pipe} = velocity of fluid through pipe

$\dot{V}_{pipe}$ = volumetric flow rate of fluid through pipe

Define Variables

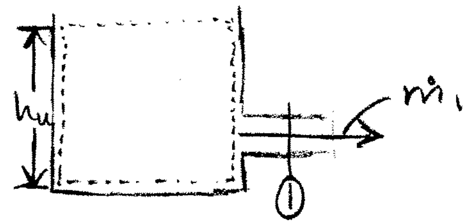
$v_{gas} = \frac{1}{\rho_{gas}} = 0.7$

$v \rightarrow$ specific gravity

$\rho \rightarrow$ density

Analysis "Find" suggests Cons of Mass, so begin w/that. May need to integrate later.

(a) System: Oil in upper tank



Cons Mass: $\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

0 b/c no mass entering system

$\frac{dm_{sys}}{dt} = \sum \cancel{\dot{m}_{in}} - \sum \dot{m}_{out}$ (Equation 1)

Important plug-in relationships:

$m_{sys} = \rho_{gas} A_{upper} h_u$

$\dot{m}_{out} = \dot{m}_1 = \rho_{gas} A_{pipe} V_{pipe} = \rho_{gas} \dot{V}_{pipe}$

$\dot{V}_{pipe} = A_{pipe} V_{pipe}$

$\frac{d(\rho_{gas} A_{upper} h_u)}{dt} = 0 - \rho_{gas} A_{pipe} V_{pipe}$ (Equation 1 w/plug-ins)

*Since ρ_{gas} and A_{upper} are constants in this situation (only h_u is changing as the fluid leaves the tank) they are moved outside the $\frac{d()}{dt}$ so becomes: (Calculus rule)

$\rho_{gas} A_{upper} \left(\frac{d(h_u)}{dt}\right) = 0 - \rho_{gas} A_{pipe} V_{pipe}$

*Divide everything by ρ_{gas}

$A_{upper} \frac{d(h_u)}{dt} = 0 - A_{pipe} V_{pipe}$

Solve for $A_{pipe} V_{pipe}$, or $\dot{V}_{pipe}$ →

$$-A_{upper}\frac{d(h_u)}{dt} = A_{pipe}V_{pipe}$$

$$-A_{upper}\frac{d(h_u)}{dt} = \dot{V}_{pipe}$$

$$\dot{V}_{pipe} = -(9\,m^2)\underbrace{(-0.333\,m/min)}_{\text{negative b/c } h_u \text{ is decreasing}}$$

$$\boxed{\dot{V}_{pipe} = 3\,m^3/min} \quad (a)$$

Since $\dot{V}_{pipe} = A_{pipe}V_{pipe}$ still need to find V_{pipe}

$$V_{pipe} = \frac{\dot{V}_{pipe}}{A_{pipe}}$$

$A_{pipe} = \pi r_{pipe}^2$ Substitute

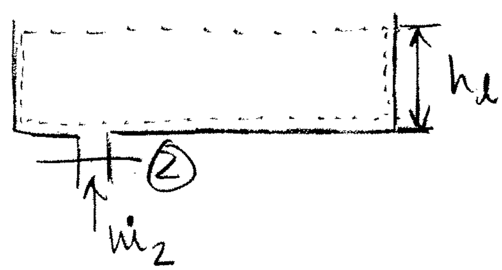
$$V_{pipe} = \frac{\dot{V}_{pipe}}{\pi r_{pipe}^2}$$

$$V_{pipe} = \frac{3\,m^3/min}{\pi(0.05\,m^2)}$$

$V_{pipe} = 382\,m/min \Rightarrow$ convert to m/s

$$\frac{382\,m}{\cancel{min}}\cdot\frac{1\,\cancel{min}}{60\,sec} = \boxed{6.37\,m/s = V_{pipe}} \quad (a)$$

(b) System: Oil in lower tank



Cons Mass: $\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

$$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \cancel{\sum \dot{m}_{out}}^{\,0} \quad \text{b/c no mass leaving system}$$

(Equation 2)

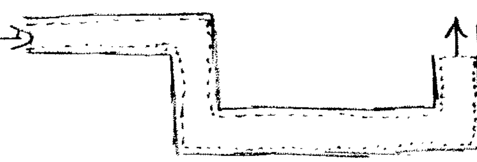
Important relationships:

$m_{sys} = \rho_{gas} A_{lower} h_\ell$

$\dot{m}_{in} = \underbrace{\dot{m}_2 = \dot{m}_1}$ → because Cons Mass on pipe,

System: Pipe

$\dot{m}_1$



$\uparrow \dot{m}_2$

$$\frac{\cancel{dm_{sys,pipe}}}{\cancel{dt}} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

0 because assume that amount of mass inside the pipe is constant

$0 = \dot{m}_1 - \dot{m}_2$

$\dot{m}_2 = \dot{m}_1 = \rho_{gas} \dot{V}_{pipe}$

Equation 2 becomes:

$$\frac{d(\rho_{gas} A_{lower} h_\ell)}{dt} = \dot{m}_2 - 0$$

$$\cancel{\rho_{gas}} A_{lower} \frac{d(h_\ell)}{dt} = \cancel{\rho_{gas}} \dot{V}_{pipe} - 0$$

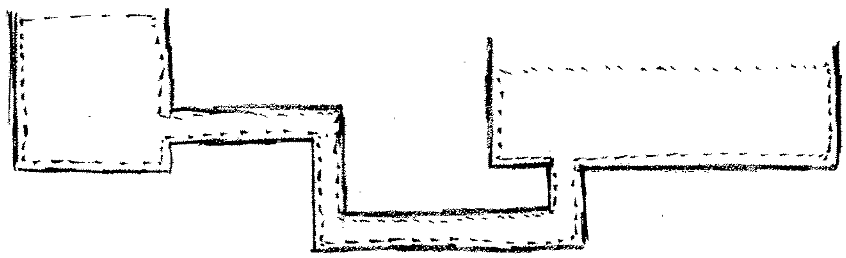
$$A_{lower} \frac{d(h_\ell)}{dt} = \dot{V}_{pipe} \Rightarrow \text{Solve for } \frac{d(h_\ell)}{dt}$$

$$\frac{d(h_\ell)}{dt} = \frac{\dot{V}_{pipe}}{A_{lower}}$$

(b)

$$\frac{dh_\ell}{dt} = \frac{3\ m^3/min}{50\ m^2} = \boxed{0.06\ m/min}$$ Height is increasing

(c) System: all oil in both tanks & pipe



Cons mass: $\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

$$\frac{dm_{sys}}{dt} = \underbrace{\sum \cancel{\dot{m}}_{in}}_{0} - \underbrace{\sum \cancel{\dot{m}}_{out}}_{0} \quad \text{(Equation 3)}$$

both 0 b/c closed system; nothing in or out

Important relationships:

$m_{sys} = m_{upper} + m_{pipe} + m_{lower}$ *NO DOT!!

$m_{sys} = \rho_{gas} A_{upper} h_u + \rho_{gas} V_{pipe} + \rho_{gas} A_{lower} h_l$

$m_{sys} = \rho_{gas} A_{upper} h_u + \rho_{gas} A_{pipe} l_{pipe} + \rho_{gas} A_{lower} h_l$

So Equation 3 becomes:

$$\frac{d}{dt}\left(\underbrace{\rho_{gas} A_{upper}}_{constant} h_u + \underbrace{\rho_{gas} A_{pipe} l_{pipe}}_{constant} + \underbrace{\rho_{gas} A_{lower}}_{constant} h_l\right) = 0$$

$$\cancel{\rho_{gas}} A_{upper} \frac{d(h_u)}{dt} + 0 + \cancel{\rho_{gas}} A_{lower} \frac{d(h_l)}{dt} = 0 \quad \text{Solve for } \frac{d(h_l)}{dt}$$

$$\frac{d(h_l)}{dt} = -\frac{A_{upper}}{A_{lower}} \frac{d(h_u)}{dt} = -\left(\frac{9\,m^2}{50\,m^2}\right)(-0.333\,m/min)$$

$$\frac{d(h_l)}{dt} \boxed{= 0.06\,m/min}$$ It checks!! :)

Comments: This is a case where Cons Mass on a closed system was helpful. Also, make sure you keep your symbols/notations consistent!!

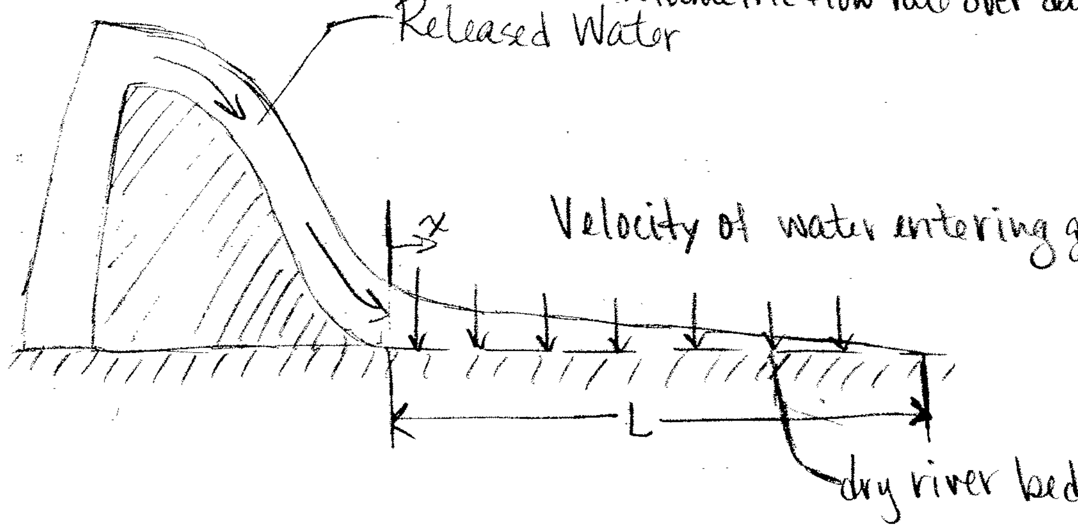
Known Water was released from a dam

Find Length of riverbed that absorbs water

Given

Width of dry river bed, $w = 50m$

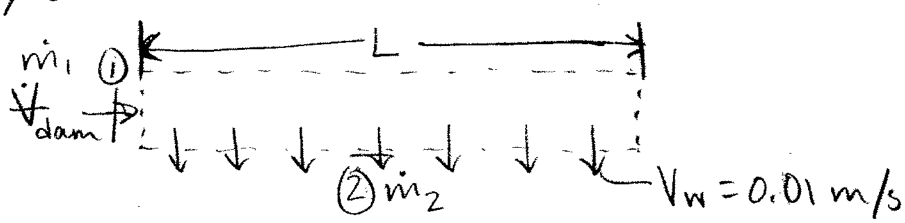
Volumetric flow rate over dam, $\dot{V}_{dam} = 200m$



Velocity of water entering ground, $V_w = 0.01m$

Analysis

System → Riverbed



Cons Mass $\quad \frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

start w/ Cons Mass b/c it has to do w/ your "Find", + there is clearly a mass component to be analyzed by the system

→

$$\frac{d\cancel{m_{sys}}}{\cancel{dt}} = \sum \dot{m}_{in} - \sum \dot{m}_{out} \quad \text{(Equation 1)}$$

0 b/c assuming system is steady state; ☆why?? ☆

Important relationships

$$\dot{m} = \rho \dot{\forall} = \rho A V \quad \text{sub into Equation 1}$$

$$0 = \dot{m}_1 - \dot{m}_2$$

$$0 = \rho_{water} \dot{\forall}_{dam} - \rho_{water} A_{bed} V_w$$

$$0 = \cancel{\rho_{water}} \dot{\forall}_{dam} - \cancel{\rho_{water}} (L \cdot w) V_w \quad \text{*Solve for L*}$$

$$0 = \dot{\forall}_{dam} - (L \cdot w) V_w$$

$$(L \cdot w) V_w = \dot{\forall}_{dam}$$

$$L = \frac{\dot{\forall}_{dam}}{w V_w}$$

$$L = \frac{200 \, \cancel{m^3}/\cancel{s}}{(50 \cancel{m})(0.01 \cancel{m}/\cancel{s})} \boxed{= 400 \text{ m}}$$

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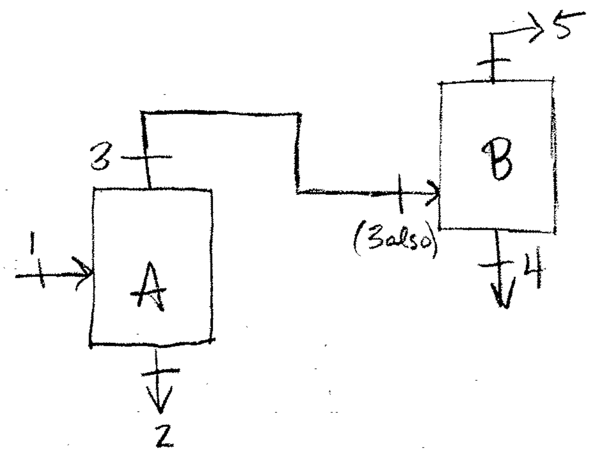
Known Co-mingling fluids

Find A set of independent Eqns that can be solved for all unknowns.

Given



98% (by weight) of xylene that enters A in feed stream 1 leaves through stream 3

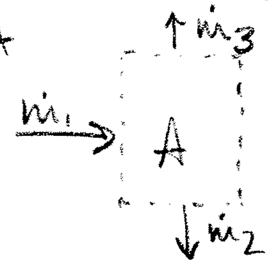
96% (by weight) of benzene that enters A in feed stream 1 leaves through stream 4.

Stream	Mass flow rate (kg/h)	Mass fraction % Benzene	Mass frac % Toluene	Mass fraction % Xylene
1 (Feed)	1275	30.0	25.0	45.0
2 (leaves A)				
3 (Intermed)				
4 (leaves B)		99.0	1.0	0.0
5 (leaves B)		0.0	1.0	99.0

*Assume Steady State system

Analysis Begin w/ defining system(s) and developing Equations

System 1: A



Deal w/ each chemical (benzene, toluene, xylene) separately at first for each system.

So to begin, only look at benzene in System A:

Cons Mass $\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

Benzene A: $\cancel{\frac{dm_A}{dt}}^{\,0 \text{ b/c steady state}} = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$ (Equation 1)

Important relationships

$\dot{m}_B$ = mass flow rate of Benzene = $\dot{m}$(from table) x Mass fraction$_{Be}$

$\dot{m}_B = \dot{m} \cdot mf_B = (1275 \text{ kg/h})(0.3)$

Sub these relationships into Equation 1

Benzene A: $0 = \dot{m}_{1B} - \dot{m}_{2B} - \dot{m}_{3B}$

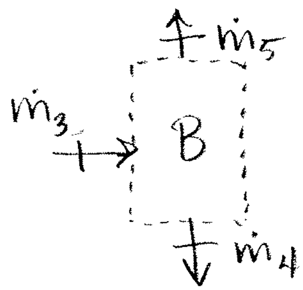
$$0 = (1275 \text{ kg/h})(0.3) - \dot{m}_2 mf_{B2} - \dot{m}_3 mf_{B3} \quad \text{Eq}\ (1)$$

Apply same ideas to Toluene & Xylene

Toluene A: $0 = (1275 \text{ kg/h})(0.25) - \dot{m}_2 mf_{T2} - \dot{m}_3 mf_{T3}$ (2)

Xylene A: $0 = (1275 \text{ kg/h})(0.45) - \dot{m}_2 mf_{X2} - \dot{m}_3 mf_{X3}$ (3)

Now apply Cons Mass in the same way, but use "B" as the system:



Cons Mass $\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

0 b/c steady state

Benzene B: $\cancel{\frac{dm_B}{dt}} = \dot{m}_3 - \dot{m}_4 - \dot{m}_5$ (Equation 4)

Same "Important Relationships" as before, so (4) becomes

Benzene B: $0 = \dot{m}_{3B} - \dot{m}_{4B} - \dot{m}_{5B}$

$0 = \dot{m}_3\, mf_{B3} - \dot{m}_4\, mf_{B4} - \dot{m}_5\, mf_{B5}$

$0 = \dot{m}_3\, mf_{B3} - \dot{m}_4(0.99) - 0$

$$0 = \dot{m}_3\, mf_{B3} - \dot{m}_4(0.99) \qquad (4)$$

Toluene B: $0 = \dot{m}_{3T} - \dot{m}_{4T} - \dot{m}_{5T}$

$0 = \dot{m}_3\, mf_{T3} - \dot{m}_4\, mf_{T4} - \dot{m}_5\, mf_{T5}$

$$0 = \dot{m}_3\, mf_{T3} - \dot{m}_4(0.01) - \dot{m}_5(0.01) \qquad (5)$$

Xylene B: $0 = \dot{m}_{3X} - \dot{m}_{4X} - \dot{m}_{5X}$

$0 = \dot{m}_3\, mf_{X3} - \dot{m}_4\, mf_{X4} - \dot{m}_5\, mf_{X5}$

$0 = \dot{m}_3\, mf_{X3} - \dot{m}_4(0) - \dot{m}_5(0.99)$

$$0 = \dot{m}_3\, mf_{X3} - \dot{m}_5(0.99) \qquad (6)$$

Composition Equations

We need more independent equations, so let's look at other ways to get them.

We know that Benzene, Toluene, and Xylene are the only fluids in the entire setup, so maybe we can make an equation or 2 from the mass fraction information we have about each one.

In stream 1, we ~~see~~ know that $mf_B + mf_T + mf_X = 100\%$ of the mass, or 1. Since we already ~~kn~~ know the values for the mass fractions in Stream 1, we should not use it as an equation, but we can do streams 2 & 3.

$$\text{Stream 2: } mf_{B2} + mf_{T2} + mf_{X2} = 1 \qquad (7)$$

$$\text{Stream 3: } mf_{B3} + mf_{T3} + mf_{X3} = 1 \qquad (8)$$

We still need 2 more equations b/c we have 10 unknowns →

Unknowns

$\dot{m}_2, \dot{m}_3, mf_{B2}, mf_{B3}, mf_{T2}, mf_{T3}, mf_{X2}, mf_{X3}, \dot{m}_4, \dot{m}_5$

Let's look at other information given in the problem statement.

98% Xylene that enters A via stream 1 leaves via stream 3; so

$$(0.98)\dot{m}_1 mf_{X1} = \dot{m}_3 mf_{X3}$$

$$(0.98)(1275\ kg/h)(0.45) = \dot{m}_3 mf_{X3} \qquad (9)$$

96% of Benzene enters A via stream 1 leaves via stream 4; so

$$(0.96)\dot{m}_1 mf_{B1} = \dot{m}_4 mf_{B4}$$

$$(0.96)(1275\ kg/h)(0.30) = \dot{m}_4 (0.94) \qquad (10)$$

Now we have 10 (numbered) Equations and 10 (listed) Unknowns.
Life is Good!!

Maple wksht used to solve is attached. Please ALWAYS include your Maple, Excel, etc to show all your work.

Comments: When dealing w/ mixture problems be sure to deal w/ each component (in this case, fluid) separately first.

Then look for alternate equations.

```
> restart:
> mdot[1]:=1275;
  mf[B1]:=0.3;
  mf[T1]:=0.25;
  mf[X1]:=0.45;
  mf[B4]:=0.99;
  mf[T4]:=0.01;
  mf[T5]:=0.01;
  mf[X5]:=0.99;
```

$$mdot_1 := 1275$$

$$mf_{B1} := 0.3$$

$$mf_{T1} := 0.25$$

$$mf_{X1} := 0.45$$

$$mf_{B4} := 0.99$$

$$mf_{T4} := 0.01$$

$$mf_{T5} := 0.01$$

$$mf_{X5} := 0.99$$

```
> e1:=0=(1275*0.3)-(mdot[2]*mf[B2])-(mdot[3]*mf[B3]);
  e2:=0=(1275*0.25)-(mdot[2]*mf[T2])-(mdot[3]*mf[T3]);
  e3:=0=(1275*0.45)-(mdot[2]*mf[X2])-(mdot[3]*mf[X3]);
  e4:=0=(mdot[3]*mf[B3])-(mdot[4]*0.99);
  e5:=0=(mdot[3]*mf[T3])-(mdot[4]*0.01)-(mdot[5]*0.01);
  e6:=0=(mdot[3]*mf[X3])-(mdot[5]*0.99);
  e7:=1=mf[B2]+mf[T2]+mf[X2];
  e8:=1=mf[B3]+mf[T3]+mf[X3];
  e9:=0=(0.98*mdot[1]*mf[X1])-(mdot[3]*mf[X3]);
  e10:=0=(0.96*mdot[1]*mf[B1])-(mdot[4]*mf[B4]);
```

$$e1 := 0 = 382.5 - mdot_2\, mf_{B2} - mdot_3\, mf_{B3}$$

$$e2 := 0 = 318.75 - mdot_2\, mf_{T2} - mdot_3\, mf_{T3}$$

$$e3 := 0 = 573.75 - mdot_2\, mf_{X2} - mdot_3\, mf_{X3}$$

$$e4 := 0 = mdot_3\, mf_{B3} - 0.99\, mdot_4$$

$$e5 := 0 = mdot_3\, mf_{T3} - 0.01\, mdot_4 - 0.01\, mdot_5$$

$$e6 := 0 = mdot_3\, mf_{X3} - 0.99\, mdot_5$$

$$e7 := 1 = mf_{B2} + mf_{T2} + mf_{X2}$$

$$e8 := 1 = mf_{B3} + mf_{T3} + mf_{X3}$$

$$e9 := 0 = 562.2750 - mdot_3\, mf_{X3}$$

$$e10 := 0 = 367.200 - 0.99\, mdot_4$$

```
> Solution:=
  solve({e1,e2,e3,e4,e5,e6,e7,e8,e9,e10},{mdot[2],mdot[3],mdot[4],md
  ot[5],mf[B2],mf[B3],mf[T2],mf[T3],mf[X2],mf[X3]});
```

$Solution := \{ mdot_5 = 567.9545455, mdot_4 = 370.9090909, mdot_3 = 938.8636364,$

$mdot_2 = 336.1363636, mf_{X3} = 0.5988888889, mf_{X2} = 0.03413793103, mf_{T3} = 0.01000000000,$

$mf_{T2} = 0.9203448276, mf_{B3} = 0.3911111111, mf_{B2} = 0.04551724138 \}$

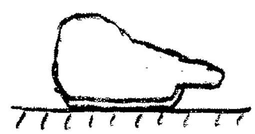
```
>
```

Known: Rocket sled slowed by air drag

Find: (a) Value of k if $F_{drag} = k V^2$

(b) Time to slow to 70 ft/s after engine shuts off. And distance travelled !!

Given:



$W_{sled} = 3220$ lbf

$V_1 = 700$ ft/s (max)

$F_T = 8000$ lbf (Thrust)

$F_{drag} = k V^2$

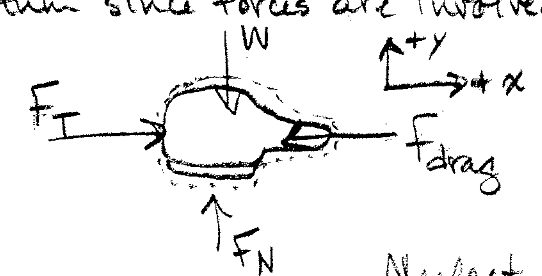
Analysis:

Strategy: Try linear momentum since forces are involved.

System → Sled (Closed)

Time → Finite

Cons Lin Mom (LM)



Neglect friction

$$\frac{dP_{sys}}{dt} = \sum F_{ext} + \underbrace{\cancel{\sum \dot{m}_{in} V_{in}}}_{\text{closed system}} - \underbrace{\cancel{\sum \dot{m}_{out} V_{out}}}_{\text{closed system}}$$

Don't forget: $P = mV$ ⇒ in general terms

$$\frac{d(\overset{\text{not changing}}{m} V)_{sys}}{dt} = \sum F_{ext}$$

$$m \frac{dV_{sys}}{dt} = \sum F_{ext}$$

→

Look at x-direction alone

$$\xrightarrow{+} \quad \frac{dP_x}{dt} = F_T - F_{drag} \qquad \text{Sub in } P = mV \;\&\; F_{drag} = kV^2$$

$$m_{sys}\frac{dV_x}{dt} = F_T - kV^2$$

Assume $V_c = 700$ ft/s constant to find "k"

$$\text{So } \frac{dV_x}{dt} = 0$$

$$0 = F_T - kV_c^2 \qquad \text{Solve for } k$$

$$kV^2 = F_T$$

$$k = \frac{F_T}{V_c^2}$$

$$\boxed{k = \frac{8000 \text{ lbf}}{(700 \text{ ft/s})^2} = 16.33 \times 10^{-3} \text{ lbf}\cdot\text{s}^2/\text{ft}^2} \quad (a)$$

(b) From before: $m_{sys}\dfrac{dV_x}{dt} = F_T - kV^2$

If $F_T = 0$ suddenly (engine cuts out) when $V = V_c$

$$m_s\frac{dV_x}{dt} = -kV^2 \qquad \text{rearrange}$$

$$\frac{dV_x}{V^2} = -\frac{k}{m_s}dt$$

$$\int \frac{dV_x}{V^2} = -\int \frac{k}{m_s}\,dt$$

$$-\frac{1}{V_x} = -\frac{k}{m_s}t + C$$

At $t=0$ $\quad V_x = V_{ss} = 700$ ft/s

$$C = -\frac{1}{V_x}$$

$$C = -\frac{1}{V_{ss}}$$ Plug this in to

$$-\frac{1}{V_x} = -\frac{k}{m_s}t - \frac{1}{V_{ss}}$$

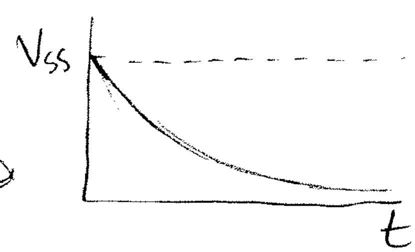
$$V_{ss}\left(\frac{1}{V_{ss}} - \frac{1}{V_x}\right) = \left(-\frac{k}{m_s}t\right)V_{ss}$$ multiply both sides by V_{ss}

$$1 - \frac{V_{ss}}{V_x} = -\frac{kV_{ss}}{m_s}t$$ Solve for $\frac{V_x}{V_{ss}}$

$$(-1)\left(-\frac{V_{ss}}{V_x}\right) = \left(-\frac{kV_{ss}}{m_s}t - 1\right)(-1)$$

$$\frac{V_{ss}}{V_x} = 1 + \frac{kV_{ss}}{m_s}t$$

$$\boxed{\frac{V_x}{V_{ss}} = \frac{1}{1 + \frac{kV_{ss}}{m_s}t}}$$



Get a number for $\frac{K\,V_{ss}}{m_s}$ term (makes things easier)

$$\frac{K\,V_{ss}}{m_s} = \frac{\left(16.33\times10^{-3}\ \frac{lbf\cdot s^2}{ft^2}\right)\ 700\ ft/s}{(3220\ lbf)\left(\frac{1\ lbf}{32.174\ ft/s^2}\right)}$$

$$\frac{K\,V_{ss}}{m_s} = \frac{1}{8.7552\,s}$$

Solve for t for $V_x = 700\ ft/s \rightarrow 70\ ft/s$

$$\frac{K\,V_{ss}}{m_s}t = \frac{V_{ss}}{V_x} - 1$$

$$t = \frac{\frac{V_{ss}}{V_x} - 1}{\frac{K\,V_{ss}}{m_s}} \xrightarrow{\text{Plug In}} \frac{\frac{700\ ft/s}{70\ ft/s} - 1}{(1/8.7552\ s)} - \cancel{\frac{\frac{700\ ft/s}{700\ ft/s} - 1}{(1/8.7552\ s)}}$$

0

☆ $t = 78.8\ s$ ☆ Does not agree w/ Soln

Notice that $\frac{K\,V_{ss}}{m_s} = \frac{1}{\tau_c}$ τ_c = characteristic time

→

Solving for distance travelled

$$V_x = \frac{dx}{dt} = \frac{V_{ss}}{1 + \frac{t}{\tau_c}}$$

$$\int dx = \int \frac{V_{ss}}{1 + \frac{t}{\tau_c}} \, dt$$

$$x = V_{ss} \, \tau_c \ln(t + \tau_c) + C$$

At $t=0$, $x=0$ → use to solve for C

$$C = -\tau_c (\ln(\tau_c)) V_{ss}$$ plug back in to

$$\boxed{x = \tau_c V_{ss} \ln\left(\frac{t + \tau_c}{\tau_c}\right)}$$

Remember: $\tau = 8.7552\,s$ $V_{ss} = 700\,ft/s$

for $t = 78.8\,s$

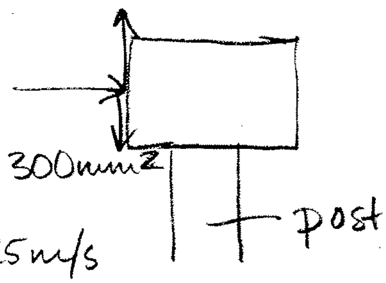
$$x = (8.7552\,\cancel{s})(700\,ft/\cancel{s}) \ln\left(\frac{78.8s + 8.7552s}{8.7552s}\right)$$

$$\boxed{x = 14.11 \times 10^3 \text{ ft}}$$

Known: Mailbox hit by stream of water

Find: (a) Shear force ~~[illegible]~~ of post on mailbox
(b) force of water acting on mailbox

Given:



$P_{atm} = 100\ kPa$

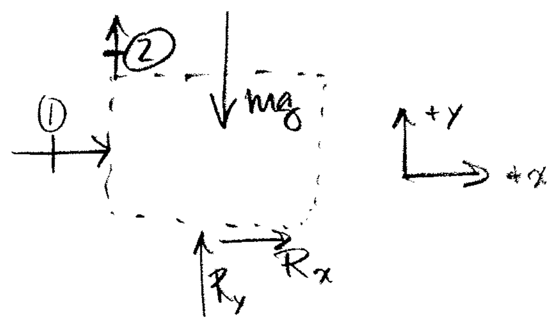
$A = 300\,mm^2$

$V_1 = 25\,m/s$

Analysis:

(a) system: mailbox & some water

Use Cons lin Mom



$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F}_{ext} + \sum \dot{m}_{in}\vec{V}_{in} - \sum \dot{m}_{out}\vec{V}_{out}$$

for x-direction

$\xrightarrow{+}$

$$\cancel{\frac{dP_{sys}}{dt}}_{ss} = \cancel{F_{atm}} + R_x + \dot{m}_1 V_1$$

uniform all around, so atm forces cancel

$$0 = R_x + \dot{m}_1 V_1$$

$$R_x = -\dot{m}_1 V_1$$

Remember: $\dot{m} = \rho A V$ so

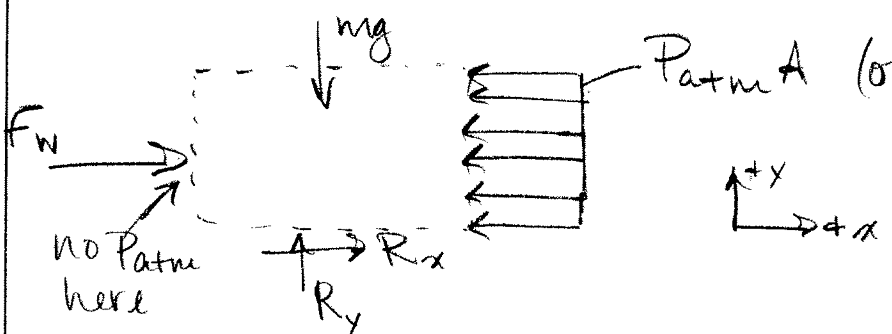
$$R_x = -\rho A V_1 \cdot V_1$$

$\longrightarrow$

$$R_x = -(1000\ \mathrm{kg}/\cancel{m^3})(300\ \cancel{mm^2})\left(\frac{1\ \cancel{m}}{1000\ \cancel{mm}}\right)^2(25\ \cancel{m}/s)(25\ m/s)$$

$$\boxed{R_x = -187.5\ \mathrm{kg \cdot m/s^2}}$$

(b) Force of water on mailbox



Apply Cons Lin Mom in x-direction

no mass flow this time

$$\xrightarrow{+} \underbrace{\frac{dP_{sys}}{dt}}_{0} = \sum F_{ext} + \sum \cancel{\dot{m}_{in}} V_{in} - \sum \cancel{\dot{m}_{out}} V_{out}$$

$$0 = F_W + R_x - P_{atm}A$$

$$F_W = P_{atm}A - R_x$$

$$F_W = (100\mathrm{E}3\ \mathrm{N}/\cancel{m^2})(300\ \cancel{mm^2})\left(\frac{1\ \cancel{m}}{1000\ \cancel{mm}}\right)^2 + 187.5\ \mathrm{N}$$

$$\boxed{F_W = 30\ \mathrm{kN}}$$

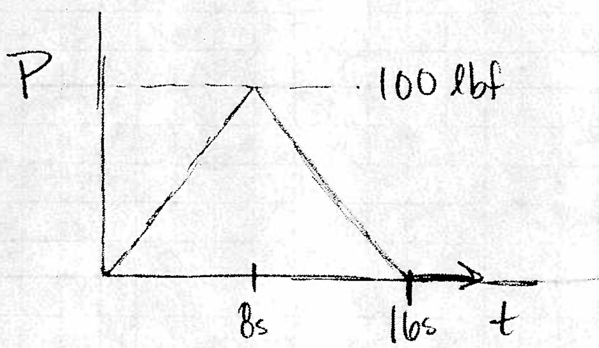
Known: block acted on by a time-varying force

Find: (a) Time t_1 when block first moves

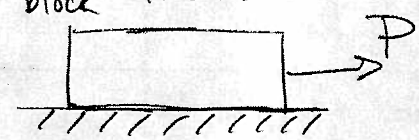
(b) Max velocity of block

(c) Time t_3 when block stops moving

Given:



$m_{block} = 125\ lbm$

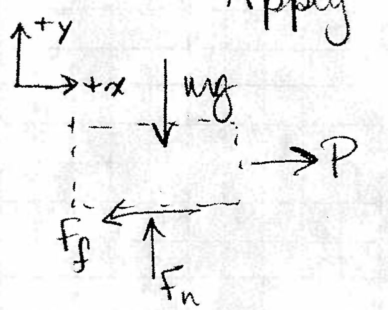


$\mu_s = 0.5$

$\mu_K = 0.4$

Analysis: System → block

Apply Cons Lin Mom



$$\frac{d\vec{P}_{sys}}{dt} = \sum \vec{F}_{ext} + \sum \dot{m}_{in}\vec{V}_{in} - \sum \dot{m}_{out}\vec{V}_{out}$$

y-direction ↑+

$$\underbrace{\cancel{\frac{dP_{sys,y}}{dt}}}_{0;\ V_y=0} = F_n - mg + \underbrace{\cancel{\ }\ - \cancel{\ }}_{\text{no mass entering/leaving}}$$

so $\boxed{F_n = mg}$

→

Cons Lin Mom x-direction

$$\xrightarrow{+} \quad \frac{d\mathbb{P}_{sys,x}}{dt} = P - F_{friction}$$

Remember: $F_{friction}$ (hereafter referred to as F_f) $= \mu F_n$

$$\frac{d\mathbb{P}_x}{dt} = P - \mu F_n$$

Also remember: $\mathbb{P} = mV$ and $F_n = mg$ (from above)

so

$$\frac{d\,mV_x}{dt} = P - \mu mg$$

$$\boxed{m\frac{dV_x}{dt} = P - \mu mg}$$

(a) at point of incipient motion $\frac{dV_x}{dt} = 0$, $V_x = 0$, and $\mu = \mu_s$

So

$$m\cancel{\frac{dV_x}{dt}}^{\,0} = P - \mu mg$$

$$0 = P - \mu mg$$

$$P = \mu mg = (0.5)(125\ \text{lbf})$$

$P = 62.5\ \text{lbf}$ at t_1

from "Given"

To find t_1, set up ratio: $\frac{62.5\ \text{lbf}}{t_1} = \frac{100\ \text{lbf}}{8\text{s}}$

$$\boxed{t_1 = 5\text{s}}$$

(b) at max velocity (V_{max}) $\longrightarrow$ $\frac{dV_x}{dt} = 0$ and $\mu = \mu_k$

$$\underbrace{m \cancel{\frac{dV_x}{dt}}}_{0} = P - \mu_k mg$$

$$P = \mu_k mg = (0.4)(125\, lbf)$$

$$P = 50\, lbf$$

$$t_2 = 8\, sec + \left(\frac{(100-50)\cancel{lbf}}{100\, \cancel{lbf}}\right) 8\, sec$$

$$\boxed{t_2 = 12\, sec}$$

FYI

$$\begin{bmatrix} 0 \le t \le 8 & P = \left(\frac{t}{8s}\right) 100\, lbf \\ 8 \le t \le 16 & P = 200\, lbf - 100\, lbf\left(\frac{t}{8s}\right) \\ t \ge 16 & P = 0 \end{bmatrix}$$

Calculate V_x vs t

$$m\frac{dV_x}{dt} = P - \mu_k mg$$ use μ_k b/c moving

$$\frac{dV_x}{dt} = \frac{P}{m} - \mu_k g = \frac{P}{W/g} - \mu_k g$$

$$\frac{dV_x}{dt} = \frac{P}{W} g - \mu_k g$$

$$\int dV_x = \int \frac{P}{W} g - \mu_k g \; dt$$

$$V_x = \underbrace{\int_5^t \frac{P}{W} g\, dt}_{} - \mu_k g (t-5)$$

$$= \int_5^8 \frac{P}{W} g\, dt + \int_8^t \frac{P}{W} g\, dt$$

$$\qquad * \qquad\qquad\qquad **$$

$$* = \int_5^8 \left(\frac{t}{8s}\right)\left(\frac{100}{125}\right)\left(32.174\ ft/s^2\right) = \left(3.2174\ ft/s^3\right) \frac{t^2}{2}\Bigg]_{5s}^{8s}$$

$$= 62.74\ ft/s$$

$$** = \int_8^t \left[\frac{200}{125} - \frac{100}{125}\left(\frac{t}{8s}\right)\right] g\, dt$$

$$= \int_8^t \left(51.48\ ft/s^2\right) dt - \int_8^t \left(3.2174\ ft/s^3\right) t\, dt$$

$$= 51.48\ ft/s^2 \,(t - 8s) - \left(3.2174\ ft/s^3\right) \frac{t^2}{2}\Bigg]_8^t$$

$$= \left(51.48\ ft/s^2\right)(t - 8s) - \left(3.2174\ ft/s^3\right)\left(\frac{t^2}{2} - 32s^2\right)$$

$$\int_5^t \frac{P}{W} g\, dt = \left(51.48 \frac{ft}{s^2}\right) t - \left(1.6087 \frac{ft}{s^3}\right) t^2 - 246.14\ ft/s$$

then: $\mu_k g(t-5s) = (0.4)(32.174\ ft/s^2)(t-5s)$

$= (12.87\ ft/s^2)t - 64.35\ ft/s$

plug everything back in to $V_x = \ldots$

$V_x = (38.61\ ft/s)t - (1.6087\ ft/s^3)t^2 - 181.79\ ft/s$

$\boxed{\text{At } t = 12s \quad V_x = 49.88\ ft/s \quad V_{max}}$

(c) Stop moving... check $t = 16s$

$V_{x,t=16s} = 24.14\ ft/s$, so still moving

for $t > 16s$ $P = 0$, so

$$\frac{dV_x}{dt} = -\mu_k g$$

$$V_x - V_{x,t=16s} = -\mu_k g(t - 16s)$$

$$t - 16s = \frac{0 - 24.14\ ft/s}{-(0.4)(32.174\ ft/s^2)}$$

$= 1.88s$

$t_3 = (16 + 1.88)s \boxed{= 17.88s}$

Known: 2 swimmers dive off a boat

Find: (a) V_{boat} if both swimmers dive together

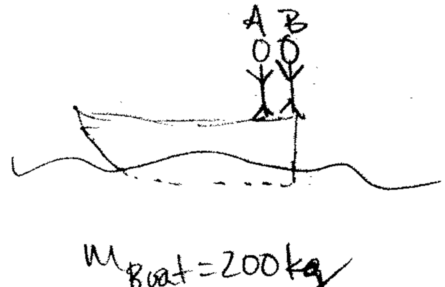
(b) " " " A dives 1st

Given: 2 swimmers, A & B, dive off the end of a 200 kg boat. Each has relative horizontal velocity 3 m/s when leaving.

Boat initially @ rest

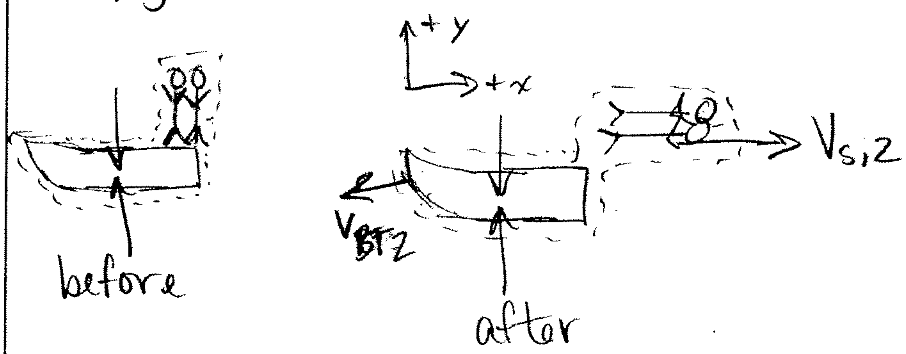
$m_A = 75$ kg

$m_B = 50$ kg



(a) System: both swimmers & boat before & after dive

Apply Cons Lin Mom Finite



Cons Lin Mom x-direction

$$\frac{d\mathbb{P}_{sys,x}}{dt} = \underbrace{\sum \cancel{F_x}}_{0} + \underbrace{\sum \cancel{\dot{m}_{in}} V_{in}}_{0} - \underbrace{\sum \cancel{\dot{m}_{out}} V_{out}}_{0}$$

closed

So integrate to get finite time

$$\int_{\mathbb{P}_{1x}}^{\mathbb{P}_{2x}} d\mathbb{P}_{sys\,x} = \int_{t}^{t_2} 0\, dt$$

$$\mathbb{P}_{x_2} - \mathbb{P}_{x_1} = 0$$

$$\underbrace{m_{Boat}(-V_{BT2}) + m_A V_{S2} + m_B V_{S2}}_{\mathbb{P}_{x2}} - \underbrace{0}_{\mathbb{P}_{x1}} = 0$$

$$V_{BT2} = \frac{(m_A + m_B)\, V_{S2}}{m_{Boat}} \qquad (1)$$

* Must be careful about relative velocities!! All velocities in (1) are given w/respect to GROUND. We are not given $V_{S,2}$, rather we have $V_{S,2/BT,2}$ in Vector form.

$$\vec{V}_{S2} = \vec{V}_{BT,2} + \vec{V}_{S2/BT,2}$$

The x-component of this is

$$V_{S2} = -V_{BT,2} + V_{S,2/BT,2} \qquad (2)$$

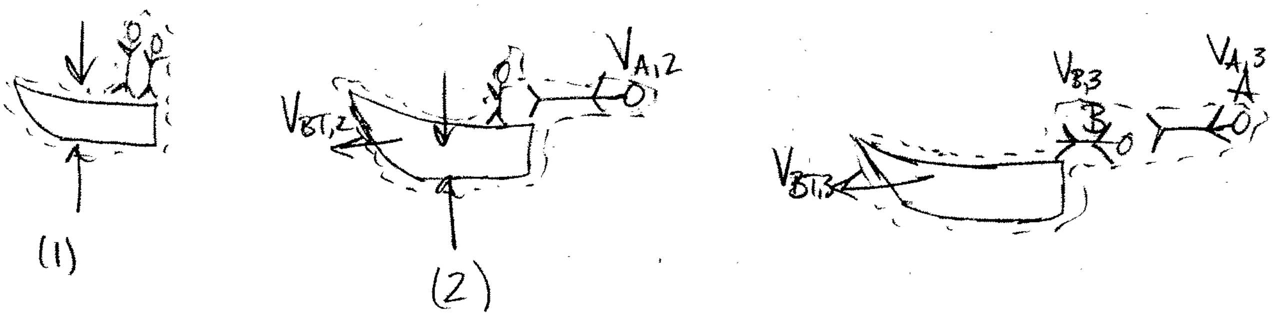
This is 3 m/s

(4) becomes

$$V_{BT,2} = \frac{(m_A + m_B)\left(-V_{BT,2} + V_{S2/BT,2}\right)}{m_{Boat}} \quad \boxed{= 1.15 \text{ m/s}}$$

(b) System: both swimmers & boat before & after dive

$+y$, $+x$



Cons lin Mom *x-dir only*
$1 \to 2$

$$\frac{d\mathbb{P}_{x\,sys}}{dt} = \underbrace{\cancel{\sum F_x}}_{0} + \underbrace{\cancel{\sum \dot m_{in} V_{in}}}_{0} - \underbrace{\cancel{\sum \dot m_{out} V_{out}}}_{0}$$

no mass flow

$$\frac{d\mathbb{P}_{sys,x}}{dt} = 0$$

$$\mathbb{P}_{x2} - \mathbb{P}_{x1} = 0$$

$$\underbrace{(m_{BT} + m_B)(-V_{BT,2}) + m_A V_{A,2}}_{\mathbb{P}_{x2}} - \underbrace{0}_{\mathbb{P}_{x1}} = 0 \quad (3)$$

Similar to Part(a) ↘

$$V_{A,2} = -V_{BT,2} + V_{A2/BT,2} = 3\,m/s$$

(3) Becomes ↘

$$(m_{BT} + m_B)(-V_{BT,2}) + m_A(-V_{BT,2} + V_{A2/BT,2}) = 0$$

$$\boxed{V_{BT/2} = 0.692\ m/s}$$

Now Cons Lin Mom in x-direction from 2 to 3

$$\xrightarrow{+} \frac{d\mathbb{P}_{sys,x}}{dt} = 0 + 0 - 0$$

$$\mathbb{P}_{x3} - \mathbb{P}_{x2} = 0$$

$$\underbrace{\left[m_{BT,3}(-V_{BT,3}) + \cancel{m_A V_{A3}} + m_B V_{B3}\right]}_{\mathbb{P}_{x3}} - \underbrace{\left[(m_{BT} + m_B)(-V_{BT,2}) + \cancel{m_A V_{A2}}\right]}_{\mathbb{P}_{x2}} = 0$$

$$m_{BT}(-V_{BT,3}) + m_B V_{B3} - (m_{BT} + m_B)(-V_{BT,2}) = 0$$

Again: ↘

$$V_{B,3} = -V_{BT,3} + V_{B3/BT3} = 3\,m/s$$

$$m_{BT}(-V_{BT3}) + m_B\left[-V_{BT3} + V_{B3/BT3}\right] - \left[m_{BT} + m_B\right] - V_{BT2} = 0$$

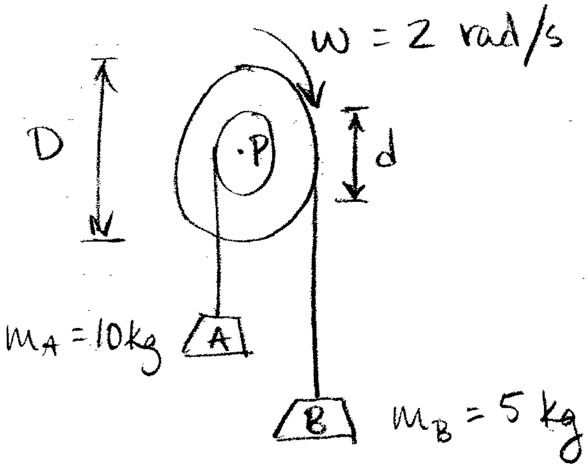
$$\boxed{V_{BT3} = 1.292\ m/s}$$

Known Pulley mass system

Find (a) Velocity of each mass, LM, & AM

(b) Angular momentum about P if pulleys are locked

Given



D = 0.5 m
d = 0.25 m

Analysis

(a) Velocities ... remember the relationship $\underline{V = r\omega}$ so

$$V_A = \frac{d}{2}\omega = \left(\frac{0.25\,m}{2}\right)\left(\frac{\cancel{2}\,rad}{s}\right) \boxed{= 0.25\ m/s \uparrow}$$

$$V_B = \frac{D}{2}\omega = \left(\frac{0.5\,m}{\cancel{2}}\right)\left(\frac{\cancel{2}\,rad}{s}\right) \boxed{= 0.5\ m/s \downarrow}$$

* Be sure to specify direction :)

Angular Momentum about "P"

Find magnitude & direction for each mass (A & B).
Remember $\vec{L} = m\vec{r} \times \vec{V}$

$$\vec{L}_{P,A} = m_A(\vec{r}_A \times \vec{V}_A) = (10\,kg)\left(\frac{0.25}{2}\,m\right)(0.25\,m/s) \downarrow$$

$$\boxed{\vec{L}_{P,A} = 0.3125\ \frac{kg\,m^2}{s}}$$

$$\vec{L}_{P,B} = m_B(\vec{r}_B \times \vec{V}_B) = (5\,kg)\left(\frac{0.5}{2}\,m\right)(0.5\,m/s) \downarrow$$

$$\boxed{\vec{L}_{P,B} = 0.625\ kg\,m^2/s}$$

Linear Momentum

Find Magnitude & direction for each mass (A & B)
Remember $\vec{P} = m\vec{V}$

$$\vec{P}_A = m_A \vec{V}_A$$

$$\vec{P}_A = (10\,kg)(0.25\,m/s \uparrow) \boxed{= 2.5\ \frac{kg\cdot m}{s} \uparrow}$$

$$\vec{P}_A = (5\,kg)(0.5\,m/s \downarrow) \boxed{= 0.5\ \frac{kg\cdot m}{s} \downarrow}$$

(b) ——— ~~now~~

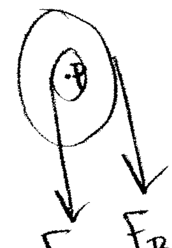
(b) If pulleys are locked:

$+\circlearrowright \sum M_P = M_{P,A} - M_{P,B}$

$+\circlearrowright M_{P,A} = \vec{r}_A \times {}^{*}\vec{F}_A$

$+\circlearrowleft M_{P,B} = \vec{r}_B \times \vec{F}_B$

*$\vec{F}$ is simply the force extorted on the pulley system by each block ... so FBD ↓



$m_A g = F_A \quad F_B = m_B g$

$$+\circlearrowright \sum M_P = \left[\left(\frac{0.25}{2}\,m\right)(10\,kg)(9.81\,m/s^2)\right] - \left[\left(\frac{0.5}{2}\,m\right)(5\,kg)(9.81\,m/s^2)\right]$$

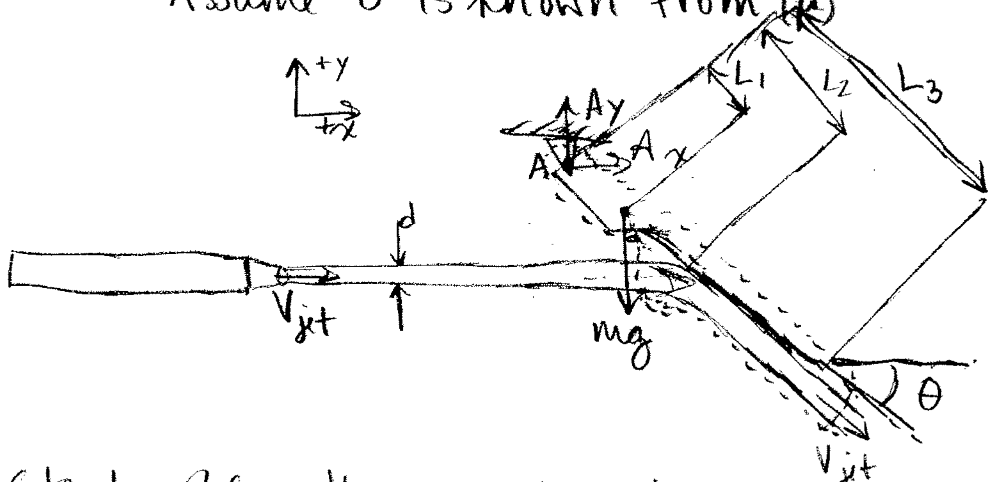
$$\boxed{+\circlearrowright \sum M_P = 0}$$

Known: Jet of water w/density ρ hits hinged flap w/mass m. Velocity of water = V_{jet}. Incoming jet is circular w/diameter d.

Find (a) θ that stationary flap makes w/horizontal

(b) horizontal & vertical reaction forces @ A.

Assume θ is known from (a)



(a) Start w/ Cons Mass on system above

$$\cancel{\frac{d m_{sys}}{dt}} = \Sigma \dot{m}_{in} - \Sigma \dot{m}_{out}$$

Assume steady state

$$0 = \dot{m}_{in} - \dot{m}_{out}$$

remember in general $\dot{m} = \rho A V$

$$\dot{m}_{in} = \dot{m}_{out} = \rho A V_{jet}$$

$$\boxed{\dot{m}_{in} = \dot{m}_{out} = \rho \frac{\pi d^2}{4} V_{jet}}$$

Cons AM about A

$$+\circlearrowright \quad \cancel{\frac{d L_{A,sys}}{dt}} = \Sigma \vec{M}_A + \Sigma(\vec{r} \times \vec{V})\dot{m}_{in} - \Sigma(\vec{r} \times \vec{V})\dot{m}_{out}$$

SS

$$0 = -L_1 \cos(\theta)(mg) + L_2 \sin(\theta) V_{jet} \dot{m} - 0$$

$\rightarrow 0$ b/c exiting V_{jet} is collinear w/A

$\rightarrow$ PTO

$$0 = -L_1\cos(\theta)\, mg + L_2 \sin(\theta)\, V_{jet}\, \rho \frac{\pi d^2}{4} V_{jet}$$ Solve for θ

$$\tan\theta = \frac{L_1 mg}{L_2 \dot{m} V_{jet}} = \frac{L_1 mg}{L_2 \rho \frac{\pi d^2}{4} V_{jet}^2}$$

$$\boxed{\theta = \tan^{-1}\left(\frac{L_1 mg}{L_2 \rho \frac{\pi d^2}{4} V_{jet}^2}\right)}$$

(b) Use Cons Lin Mom

$$\cancel{\frac{d\vec{P}_{sys}}{dt}}_{SS} = \Sigma F + \Sigma \dot{m}_{in} \vec{V} - \Sigma \dot{m}_{out} \vec{V}$$

x-direction $\xrightarrow{+}$

$$0 = A_x + \dot{m} V_{jet} - \dot{m}(V_{jet}\cos\theta)$$ Solve for A_x

$$A_x = \dot{m}(V_{jet}\cos(\theta)) - \dot{m} V_{jet}$$

$$A_x = \dot{m} V_{jet}(\cos(\theta) - 1)$$

$$\boxed{A_x = \rho \frac{\pi d^2}{4} V_{jet}^2 (\cos(\theta) - 1)}$$

y-direction

$\uparrow +$

negative b/c in negative y-direction

$0 = A_y - mg - \dot{m}\left(-V_{jet}\sin(\theta)\right)$ Solve for A_y

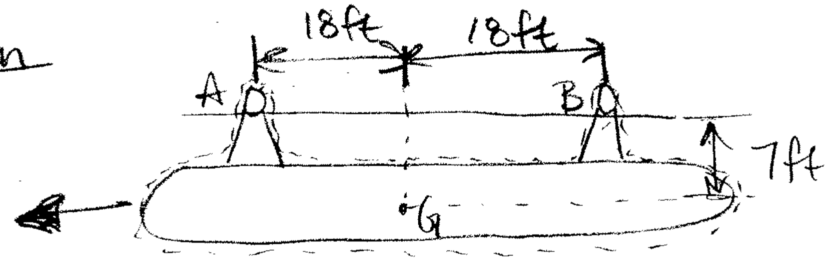
$A_y = mg - \dot{m}\,V_{jet}\sin(\theta)$

$$\boxed{A_y = mg - \rho\frac{\pi d^2}{4}V_{jet}^2\sin(\theta)}$$

Known Monorail moves on a track

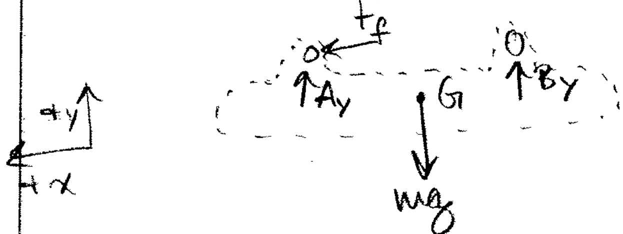
Find Maximum acceleration for car

Given



$\mu_s = 0.6$ wheel to track

Analysis Apply LM & AM to car (system defined above)



Car moves b/c of F_f between wheel + track... Assume wheels do not slip

$F_{f,max} = \mu_s A_y$

Cons Lin Mom

no mass flow

$$\frac{d\mathbb{P}_{sys}}{dt} = \Sigma f + \cancel{\Sigma \dot{m}_{in} V}^{\,0} - \cancel{\Sigma \dot{m}_{out} V}^{\,0}$$

$\leftarrow x+$ $\quad \dfrac{d\mathbb{P}_{sys,x}}{dt} = F_f$ remember $\mathbb{P} = mV$ and m isn't changing, so

$m\dfrac{dV_{max}}{dt} = F_{f,max}$ and $\dfrac{dV}{dt} = a$ (acceleration)

$$\boxed{m\frac{dV_{max}}{dt} = \mu_s A_y \quad (1)}$$

y-direction $\uparrow +$

$$\frac{d\cancel{P_{sys,y}}}{\cancel{dt}}^{0} = A_y + B_y - mg$$

$$\boxed{A_y + B_y = mg \quad (2)}$$

Solve for a_{max}

$$m\frac{dV_{max,a}}{dt} = \mu_s(mg - B_y) \Rightarrow 2 \text{ unknowns } (V_{max}, B_y)$$

Do Cons Ang Mom about G

$$+\circlearrowright \quad \frac{d\cancel{L_{sys,G}}}{\cancel{dt}}^{0} = \sum \vec{M}_G + \cancel{\sum(\vec{r}\times\vec{V})\dot{m}_{in}}^{0} - \cancel{\sum(\vec{r}\times\vec{V})\dot{m}_{out}}^{0}$$

$$\frac{d\cancel{L_{sys,G}}}{\cancel{dt}}^{0} = A_y(18ft) - B_y(18ft) - F_f(7ft)$$

$$0 = A_y - B_y - \frac{7}{18}F_f \qquad \text{plug in } F_f = \mu_s A_y$$

$$\boxed{0 = A_y - B_y - \frac{7}{18}\mu_s A_y} \quad (3)$$

Combine (3) w/ (2)

$$A_y + 0.7667 A_y = mg \Rightarrow A_y = 0.566\,mg$$

$$B_y = 0.434\,mg$$

so

$$\frac{dV_{max,a}}{dt_{a}} = \frac{\mu_s}{m}A_y = \frac{0.6}{\cancel{m}}(0.566\,\cancel{m}g) \boxed{= 0.3396g}$$

ES201-07 Date | HW# P7.1 | Name CM | 1/4

Known China has now joined the USA & Russia in putting a ~~man~~ in space
Person (you must always be P.C)

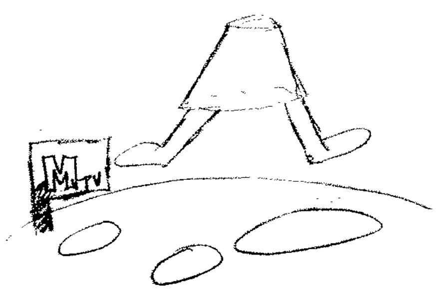
Find a) Max height "h" above lunar surface pilot can shut off lunar module if lunar module velocity relative to the moon is:

i) 0
ii) 3 m/s up
iii) 3 m/s down

Using the Work-Energy Principle

b) Repeat a) using Cons Lin Mom

Given



$g_{moon} = \frac{g_{earth}}{6}$

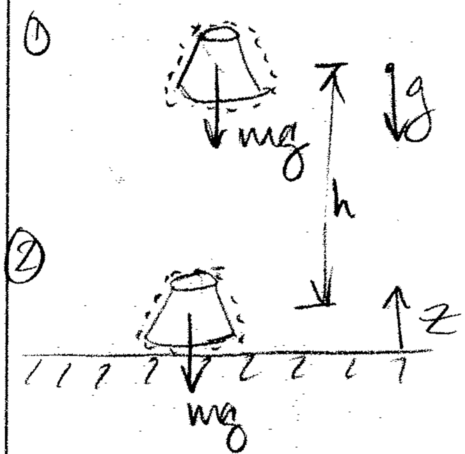
$V_{max} \leq 5\ m/s \downarrow$ (vertical max velocity for lunar module to make a safe landing)

Analysis:

<u>Analysis</u>

a) System: lunar module

<u>Use</u> Work-Energy
Time: Finite



$$W_{mech(1\to2)} = KE_2 - KE_1 + PE_2 - PE_1$$

<u>Remember</u>

$$KE = \frac{1}{2} m V^2$$

$$PE = mgz$$

So

$$\underset{\substack{0 \text{ b/c no} \\ \underline{\text{contact}} \text{ forces} \\ \text{doing work}}}{\cancel{W_{mech(1\to2)}}} = \frac{1}{2} m \underset{V_{max}}{\cancel{V_2}}^2 - \frac{1}{2} m V_1^2 + m g_{moon} \underset{0}{\cancel{z_2}} - m g_{moon} \underset{h}{\cancel{z_1}}$$

Make proper substitutions (shown) and get

$$0 = \frac{1}{2} m V_{max}^2 - \frac{1}{2} m V_1^2 - m g_{moon} h \quad \text{Solve for } h$$

$$m g_{moon} h = \frac{1}{2} m (V_{max}^2 - V_1^2)$$

$$h = \frac{\frac{1}{2} \cancel{m} (V_{max}^2 - V_1^2)}{\cancel{m} g_{moon}} = \frac{1}{2} \frac{V_{max}^2 - V_1^2}{g_{moon}} \quad \text{(EQN 1)}$$

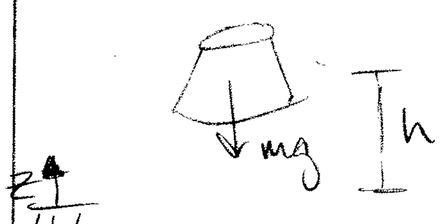
Solve EQN 1 for each situation listed in (a)

i) $h = \dfrac{\frac{1}{2}(5^2 - 0)\ \cancel{m^2}/\cancel{s^2}}{\frac{9.81}{6}\ \cancel{m}/\cancel{s^2}} \boxed{= 7.65\ m}$

ii) $h = \dfrac{\frac{1}{2}(5^2 - 3^2)\ m^2/s^2}{\frac{9.81}{6}\ \cancel{m}/\cancel{s^2}} \boxed{= 4.89\ m}$

iii) $h = \dfrac{\frac{1}{2}(5^2 - 3^2)\ m^2/\cancel{s^2}}{\frac{9.81}{6}\ \cancel{m}/\cancel{s^2}} \boxed{= 4.89\ m}$

(b) System: lunar module
Use Lin Mom
Time: Finite



$$\frac{d\mathbb{P}_{sys}}{dt} = \sum \vec{F} + \cancel{\sum \dot{m}_{in}\vec{V}} - \cancel{\sum \dot{m}_{out}\vec{V}}$$

closed system

Remember $\mathbb{P} = m\vec{V}$ so

$$\frac{d\,m\vec{V}}{dt} = m\frac{d\vec{V}}{dt} = \sum \vec{F}$$

z direction +↑

$$\cancel{m}\frac{dV_z}{dt} = -\cancel{m}g_{moon}$$

From Kinematics: $\underbrace{\frac{dV_z}{dt} = V_z\frac{dV_z}{dz}}_{\text{Plug in}}$

$$V_z\frac{dV_z}{dz} = -g_{moon}$$

$$\int_{V_{z1}}^{V_{z2}} V_z \, dV_z = \int_{z=h}^{z=0} -g_{moon} \, dz$$

$$\frac{V_2^2 - V_1^2}{2} = -g_{moon}(0-h)$$ Solve for h

$$h = \frac{\frac{1}{2}(V_{max}^2 - V_1^2)}{g_{moon}}$$ = Same as EQN, thus

$$\boxed{\begin{array}{l} \text{i) } 7.65 \text{ m} \\ \text{ii) } 4.89 \text{ m} \\ \text{iii) } 4.89 \text{ m} \end{array}}$$

ES201-0? Date | HW# P7.10 | Name CM | 1/2

Known Conditions of a closed system

Find Change in

a) KE of the system in ft·lbf

b) PE " " " " "

c) U " " " " " & BTU

Given Closed system undergoes a process

$m = 5\ lbm$

$Q_{out\,(1-2)} = 200\ ft\text{-}lbf$

$W_{1-2} = 0$

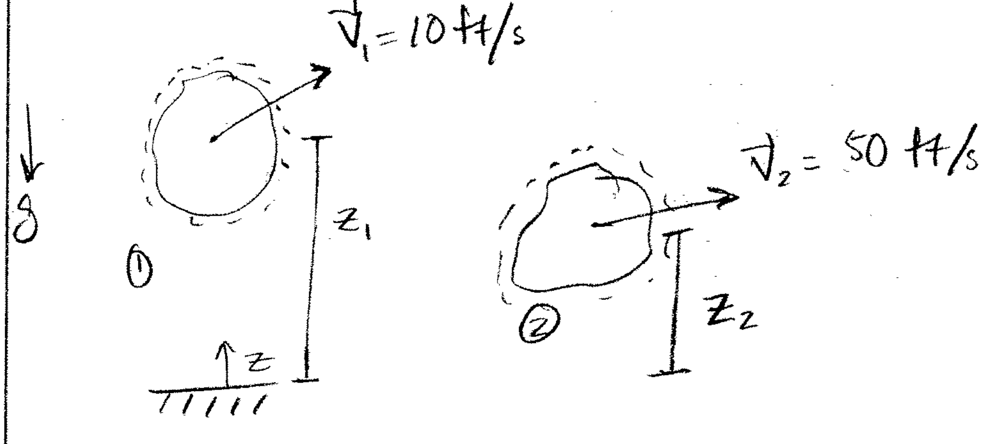
$g = 32\ ft/s^2$

① $V_{sys} = 10\ ft/s$

② $V_{sys} = 50\ ft/s$

$z_2 = z_1 - 150\ ft$

Analysis



Cons Energy

$$\frac{dE_{sys}}{dt} = \dot{Q}_{in} + \cancel{\dot{W}_{in}} + \cancel{\sum \dot{m}_{in}(e_{in} + P_{in} v_{in})} - \cancel{\sum \dot{m}_{out}(e_{out} + P_o v_o)}$$

0 Given

Closed System

→

$E_{sys2} - E_{sys1} = \dot{Q}_{in} = -\dot{Q}_{out}$ (1)

a) Apply (1) to KE

$$\Delta KE = KE_2 - KE_1 = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 = \frac{1}{2}\, 5 \cdot (50^2 - 10^2) \frac{lbm\text{-}ft^2}{s^2}$$

$$= 6000 \, \frac{lbm\, ft^2}{s^2} \cdot \frac{lbf\text{-}s^2}{32.2\, lbm\, ft} \; \boxed{= 186.3 \text{ ft-lbf}}$$

b) Apply (1) to PE

$$\Delta PE = PE_2 - PE_1 = mgz_2 - mgz_1 = (5)(32)(-150)$$

$$= -24{,}000 \, \frac{lbm\, ft^2}{s^2} \cdot \frac{lbf\text{-}s^2}{32.2\, lbm\, ft} \; \boxed{= -745.3 \text{ ft-lbf}}$$

(c) $E_{sys2} - E_{sys1} = (U_2 - U_1) + (KE_2 - KE_1) + (PE_2 - PE_1)$

(Assume all other modes of energy unimportant)

Now from (1):

$$U_2 - U_1 + \Delta KE + \Delta PE = -Q_{out\, 1-2}$$

$$U_2 - U_1 = -\Delta KE - \Delta PE - Q_{out\, 1-2}$$

$$= -186.3 - (-745.3) - 200 \; \boxed{= 359 \text{ ft} \cdot \text{lbf}}$$

Convert to BTU

$$359 \, \cancel{ft \cdot lbf} \cdot \frac{BTU}{778 \, \cancel{ft \cdot lbf}} \; \boxed{= 0.461 \text{ BTU}}$$

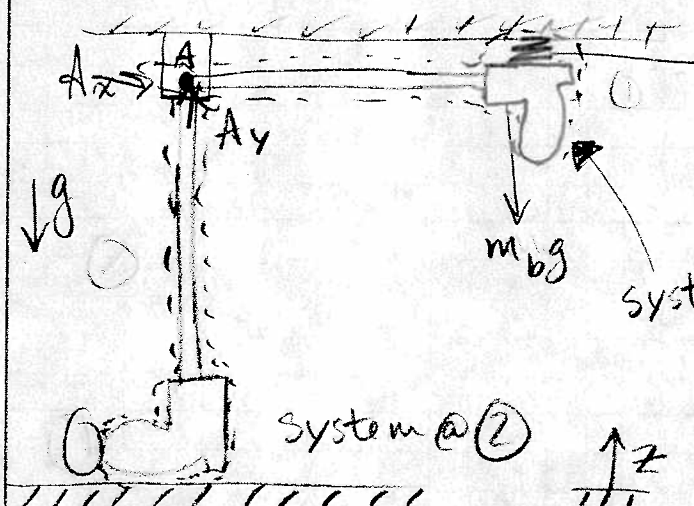
Known Spring loaded boot-on-a-stick kicks a marble

Find (a) an expression for V_{boot} right before it kicks the marble

(b) assuming the boot & the marble stick together, find an expression for V_{marble} right after it gets kicked

(c) if the spring was initially compressed @ d/3 before the device was loaded, would V_{Boot} increase, decrease, or remain the same? Why?

Given



Spring constant k
compressed distance d
system includes spring

Analysis

(a) Work-Energy Principle

$$\cancel{W_{12}} = KE_2 - \cancel{KE_1} + \cancel{PE_2} - PE_1 + \cancel{E_{s2}} - E_{s1}$$

W_{12}: 0, connection @ A does no work

KE_1: 0, not moving @ 1

PE_2: 0, $z_2 = 0$

E_{s2}: 0, no spring @ 2

Remember: $KE = \frac{1}{2} m V^2$; $PE = mgz$; $E_s = \frac{1}{2} k x^2$

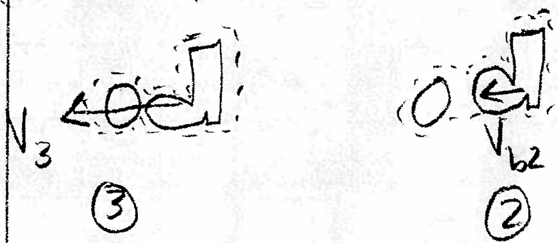
$$0 = \frac{1}{2} m_b V_{b2}^2 - m_b g L - \frac{1}{2} k d^2 \rightarrow \text{Solve for } V_{B2}$$

$$\frac{1}{2} m_b V_{b2}^2 = m_b g L + \frac{1}{2} k d^2$$

$$V_{b2}^2 = \frac{\cancel{m_b} g L + \cancel{\tfrac{1}{2}} k d^2}{\frac{1}{2} m_b} = 2gL + \frac{kd^2}{m_b}$$

(a) $$\boxed{V_{b2} = \sqrt{2gL + \frac{kd^2}{m_b}}}$$

(b) impact, so use Lin Mom finite



Cons Lin. Mom

closed system

$$\frac{d\mathbb{P}_{sys}}{dt} = \Sigma \vec{F} + \cancel{\Sigma \dot{m}_{in} \vec{V}_{in}}^{\,0} - \cancel{\Sigma \dot{m}_{out} \vec{V}_{out}}^{\,0}$$

$$\frac{d\mathbb{P}_{sys}}{dt} = \vec{F}$$ separate & integrate

$$\int d\mathbb{P}_{sys} = \int \vec{F}\, dt$$

$$\vec{\mathbb{P}}_{sys,3} - \mathbb{P}_{sys,2} = \vec{F}_{avg} \Delta t$$

x - component

$$\mathbb{P}_{x3} - \mathbb{P}_{x2} = \cancel{F_{avg\,x}}^{\,0} \Delta t$$

$$(m_m + m_b) V_3 - m_b V_{b2} = 0$$ Solve for V_3

$$V_3 (m_m + m_b) = m_b V_{b2}$$

$$V_3 = \frac{m_b V_{b2}}{m_m + m_b}$$

$$\boxed{V_3 = \frac{m_b}{m_m + m_b} \sqrt{2gL + \frac{kd^2}{m_b}}}$$

(c) V_{b2} will decrease. There is less energy w/ d/3 compression, so less energy can be converted to KE.

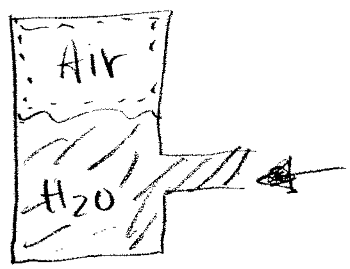
Known Air trapped above water in a storage tank

Find (a) Final P of air in tank

(b) W & Q for air as it is compressed

Given

Air is ideal gas!!



① $\forall_{air} = (0.4 - 0.3)\, m^3 = 0.10\, m^3$

$P_1 = 240$ kPa

$T_1 = 20°C = 293$ K

② $\forall_{air} = (0.4 - 0.35)\, m^3 = 0.05\, m^3$

$T_1 = T_2 \Rightarrow$ constant

$\forall_{tank} = 0.4\, m^3$

① → ② Isothermal compression of air

Analysis

(a) $m_{air} = \frac{P_1 V_1}{\cancel{R T_1}} = \frac{P_2 V_2}{\cancel{R T_2}}$ solve for P_2

$P_1 V_1 = P_2 V_2$

$P_2 = \frac{P_1 V_1}{V_2} \boxed{= 480 \text{ kPa}}$

(b) for Q & W use E.B. for air

$\frac{dE_{sys}}{dt} = \dot{Q}_{in} + \dot{W}_{in}$ closed system

$\Delta \cancel{E} = Q + W$ Finite time

ΔU b/c $\Delta KE = \Delta PE = 0$

Now $\Delta U = m \Delta u = m c_v (T_2 - T_1) = 0$ (b/c $T_2 = T_1$)

So $Q_{in} = -W_{in}$

$W_{in} = -\int_1^2 P d\forall$ and $P = \frac{mRT}{\forall}$

$$W_{in} = -\int_1^2 \frac{mRT}{\forall} d\forall$$

$$W_{in} = -mRT \int_1^2 \frac{1}{\forall} d\forall$$

$$W_{in} = -mRT \left(\ln \forall \Big|_1^2 \right)$$

$$W_{in} = -mRT (\ln \forall_2 - \ln \forall_1)$$

$$W_{in} = -mRT \ln\left(\frac{\forall_2}{\forall_1}\right)$$

$W_{in} = -P_1 \forall_1 \ln\left(\frac{\forall_2}{\forall_1}\right)$ ← Plug in & solve

$$\boxed{\begin{aligned} W_{in} &= 16.64 \text{ kJ} \\ Q_{in} &= -16.64 \text{ kJ} \end{aligned}}$$

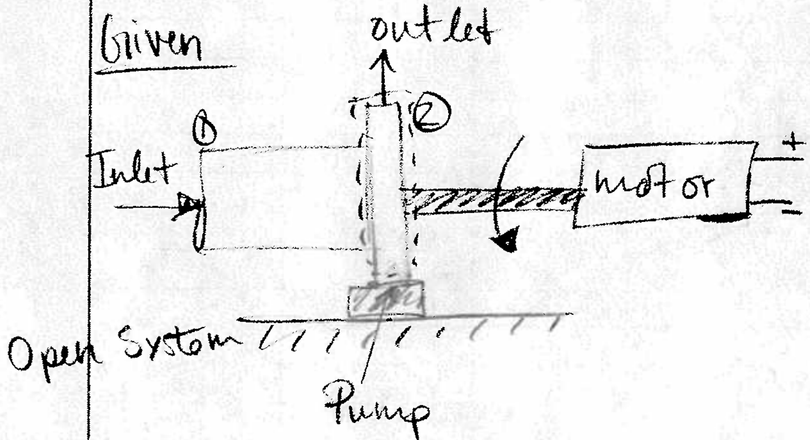
Known Centrifugal pump driven by a motor

Find (a) Heat transfer rate for the pump, kW

(b) Torque for the shaft, N·m

(c) electric current to motor, amps

Given



	①	②
T	20°C	20°C
P	100 kPa	500 kPa
A	180 cm²	125 cm²
V	5 m/s	?

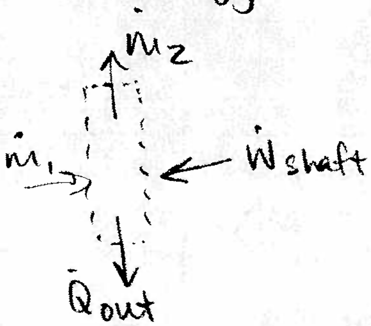
$N_{shaft} = 1750$ rpm

$\dot{W}_{shaft} = 40$ kW $\quad \dot{W}_{elec} = 42$ kW

440 Volt ac motor $\quad$ Power Factor = 1

Analysis

strategy → Cons of Mass & Energy



$$\underbrace{\cancel{\frac{dE_{sys}}{dt}}}_{ss} = \dot{Q}_{in} + \dot{W} + \dot{m}_1\left(h_1 + \frac{V_1^2}{2} + \cancel{gz_1}\right) - \dot{m}_2\left(h_2 + \frac{V_2^2}{2} + \cancel{gz_2}\right)$$

$z_1 = z_2$

$$\therefore \dot{Q}_{out} = \dot{W}_{in} + \underbrace{\dot{m}_1}_{\text{ss mass balance } \dot{m}_1 = \dot{m}_2}\left(h_1 - h_2 + \frac{V_1^2 - V_2^2}{2}\right)$$

$$\dot{m} = \rho A_1 V_1 = \left(997 \frac{kg}{m^3}\right)\left(180 \times 10^{-4} \, m^2\right)(5 \, m/s)$$

$$= 89.73 \, kg/s$$

$$\dot{m}_1 = \dot{m}_2 \xrightarrow[\rho \text{ constant}]{} \dot{V}_1 = \dot{V}_2 \longrightarrow A_1 V_1 = A_2 V_2$$

$$V_2 = \frac{A_1}{A_2} V_1$$

$$V_2 = 7.20 \, m/s$$

$$\Delta h = c \cancel{\Delta T} + v \Delta P$$

$T_1 = T_2$

$$\therefore \dot{Q}_{out} = \dot{W}_{in} + \dot{m}\left(v(P_1 - P_2) + \frac{V_1^2 - V_2^2}{2}\right)$$

plugin #'s & solve

$$\dot{Q}_{out} = 40 \, kW + 89.73 \, kg/s \left[\frac{100 - 500 \, kPa}{997 \, kg/m^3} + \frac{(5^2 - 7.2^2)}{2} \frac{m^2}{s^2}\right]$$

(a)

$$\boxed{\dot{Q}_{out} = 2.80 \, kW}$$

Find Torque

(b) $\tau = \frac{\dot{W}_{shaft}}{\omega}$ $\omega = \frac{1750 \, \cancel{rev}}{\cancel{min}} \cdot \frac{2\pi \, rad}{\cancel{rev}} \cdot \frac{1 \, \cancel{min}}{60 s} = 183.3 \, rad/s$

$$\tau = \frac{40{,}000 W}{183.3 \, rad/s} \boxed{= 218.3 \, N \cdot m}$$

(c) Finding Current

$$\dot{W}_{elec} = i \Delta V$$

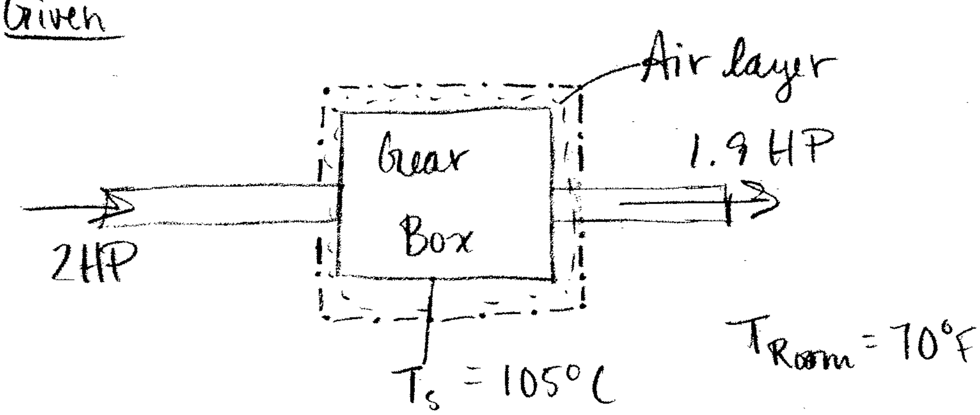
$$i = \frac{\dot{W}_{elec}}{\Delta V}$$

$$i = \frac{42{,}000 \text{ W}}{440 \text{ Volts}} \boxed{= 95.46 \text{ amps}}$$

Known: Gearbox operates at steady state

Find
(a) $\dot{Q}$ (in or out) in B/h
(b) $\dot{S}_{gen}$ for gearbox & shafts
(c) " " air layer
(d) " " " " & gearbox & shafts as a system
(e) comment on (b), (c), & (d)

Given



Analysis

(a) Gearbox is system; use Conservation of Energy

$$\underbrace{\cancel{\frac{dE_{sys}}{dt}}}_{0 \; SS} = \dot{Q}_{in} + \dot{W}_{in} + \underbrace{\cancel{\ldots}}_{\text{no mass flow}}$$

$$0 = \dot{Q}_{in} + \dot{W}_{in}$$

$$\dot{Q}_{in} = -\dot{W}_{in}$$

$$\dot{Q}_{in} = -(2HP - 1.9HP)$$

$\dot{Q}_{in} = -0.1\,HP$ so must be $\dot{Q}_{out}$ (b/c negative)

$\dot{Q}_{out} = 0.1\ HP$ convert to $\frac{B}{h}$

$$0.1\ \cancel{HP} \cdot 2546\ \frac{B/h}{\cancel{HP}} \boxed{= 254.6\ B/h}$$

(b) Entropy — System is gearbox & shafts

$$\frac{d\cancel{S_{sys}}}{dt} = \sum \frac{\dot{Q}_j}{T_j} + \cancel{\sum \ldots} - \cancel{\sum \ldots} + \dot{S}_{gen}$$

0 steady state

$$0 = \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen} \quad \text{solve for } \dot{S}_{gen}$$

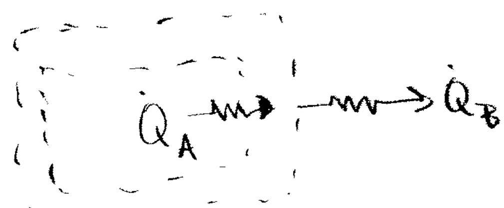
$$\dot{S}_{gen} = -\frac{\dot{Q}_j}{T_j}$$

$$\dot{S}_{gen} = \frac{-254.6\ B/h}{(105 + 460)} \boxed{= 0.451\ \frac{B}{h\,°R}}$$

(c) System is air layer

Conservation of Energy

$$\frac{d\cancel{E_{sys}}}{dt} = \dot{Q}_{in} + \dot{W}_{in} + \cancel{\sum \ldots} - \cancel{\sum \ldots}$$



0 (SS)

$$0 = \dot{Q}_A - \dot{Q}_B$$

$$\dot{Q}_B = \dot{Q}_A = 254.6\ B/h \quad \text{(makes sense ☺)}$$

Entropy Accounting

$$\underbrace{\frac{d \cancel{S_{sys}}}{dt}}_{0} = \sum \frac{\dot{Q}_j}{T_j} + \underbrace{\cancel{\sum \dots}}_{0} - \underbrace{\cancel{\sum \dots}}_{0} + \dot{S}_{gen}$$

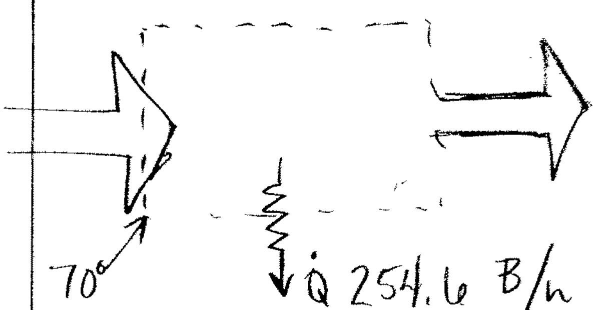
$$0 = \sum \frac{\dot{Q}_j}{T_j} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = -\sum \frac{\dot{Q}_j}{T_j}$$

$$\dot{S}_{gen} = -\left[\frac{\dot{Q}_A}{T_A} - \frac{\dot{Q}_B}{T_B}\right] = -\left[\frac{254.6}{(105+460)} - \frac{254.6}{(70+460)}\right]$$

$$\boxed{\dot{S}_{gen} = 0.0297 \ \frac{B}{h \cdot {}^\circ R}}$$

(d) System is now gearbox w/air layer



Entropy Accounting

$$\underbrace{\frac{d \cancel{S_{sys}}}{dt}}_{0 (ss)} = \sum \frac{\dot{Q}_j}{T_j} + \underbrace{\cancel{\sum \dots}}_{0} - \underbrace{\cancel{\sum \dots}}_{0} + \dot{S}_{gen}$$

$$\dot{S}_{gen} = -\sum \frac{\dot{Q}_j}{T_j} = -\frac{254.6 \ B/h}{(70+460)} \boxed{= 0.48 \ \frac{B}{h \cdot {}^\circ R}}$$

(e) From (b) + (d),

$$\dot{S}_{gen,\ gearbox\ \&\ air\ layer} = \dot{S}_{gen,\ gearbox} + \dot{S}_{gen,\ air\ layer}$$

<u>Known</u> Jet engine nozzle; adiabatic

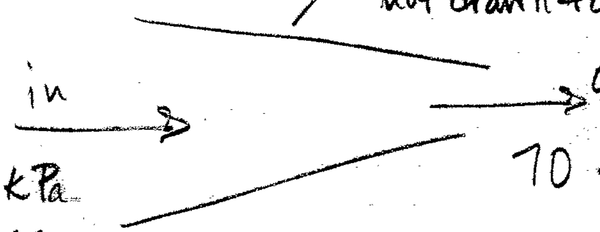
<u>Find</u> (a) T_{out}, V_{out} if internally reversible

(b) $\dot{S}_{gen}$, V_{out} if $T_{out} = 527°C$

(c) compare T_2, V_2, & $\dot{S}_{gen}$

<u>Given</u>

should be symmetric... not drawn-to-scale ☺



180 kPa
707°C
70 m/s

70 kPa

$\dot{m} = 3.0$ kg/s

$C_p = 1.00 \ \frac{kJ}{kg \cdot K}$

<u>Assume</u>

Steady state; ideal gas; negligible change in potential energy

<u>Strategy</u>

Cons Energy, 2nd law, open system

$$\underbrace{\frac{d\cancel{E_{sys}}}{dt}}_{0(ss)} = \underbrace{\cancel{\dot{Q}_{in}}}_{\text{adiabatic}} + \underbrace{\cancel{\dot{W}_{in}}}_{0 \text{ no work done}} + \dot{m}_{in}\left(h + \frac{V^2}{2} + \underbrace{\cancel{gz}}_{0 \text{ negl.}}\right)_{in} - \dot{m}_{out}\left(h + \frac{V^2}{2} + \underbrace{\cancel{gz}}_{0 \text{ negl}}\right)_{out}$$

$$h_{in} + \frac{V_{in}^2}{2} = h_{out} + \frac{V_{out}^2}{2} \qquad (1)$$

$$\frac{d\cancelto{0(ss)}{S_{sys}}}{\cancel{dt}} = \Sigma \cancelto{\text{adiabatic}}{\frac{\dot{Q}_j}{T_j}} + \dot{m}(s_{in} - s_{out}) + \dot{S}_{gen}$$

$$0 = \dot{m}(s_{in} - s_{out}) + \dot{S}_{gen}$$

$$\dot{S}_{gen} = \dot{m}\, s_{out} - \dot{m}\, s_{in}$$

(a) when system is reversible $\dot{S}_{gen} = 0$ so

$$0 = \dot{m}\, s_{out} - \dot{m}\, s_{in}$$

$$\cancel{\dot{m}}\, s_{in} = \cancel{\dot{m}}\, s_{out}$$

$$s_{in} = s_{out} \quad \text{or} \quad \underline{s_2 - s_1 = 0}$$

$$s_2 - s_1 = c_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right) = 0$$

$$c_p \ln\left(\frac{T_2}{T_1}\right) = R \ln \frac{P_2}{P_1}$$

$$\ln \frac{T_2}{T_1} = \frac{R}{c_p} \ln \frac{P_2}{P_1}$$

$$\left(\frac{T_2}{T_1}\right) = \left(\frac{P_2}{P_1}\right)^{R/c_p} = \left(\frac{70}{180}\right)^{(0.287/1)} = 0.76257$$

$$T_2 = (707 + 273)\, 0.76257 = 743.3\ K = 474\ ^\circ C$$

$$\boxed{T_2 = 474^\circ C}$$

Energy (1): $h_{in} - h_{out} = \frac{V_{out}^2 - V_{in}^2}{2}$ Solve for V_{out}

$$V_{out}^2 = V_{in}^2 + 2(h_{in} - h_{out})$$

$$V_{out}^2 = V_{in}^2 + 2(C_p(T_{in} - T_{out}))$$

$$V_{out} = \sqrt{(70\,m/s)^2 + 2\left[\left(1.00\,\frac{kJ}{kg}\right)(707-474)\right] \times 1000}$$

$$\boxed{V_{out} = 686\ m/s}$$ * if the system is reversible *

(b) $\dot{S}_{gen} = \dot{m}(s_{out} - s_{in})$

$$s_{out} - s_{in} = C_p \ln\left(\frac{T_2}{T_1}\right) - R \ln\left(\frac{P_2}{P_1}\right)$$

$$= 1.0 \ln\left(\frac{527+273}{707+273}\right) - 0.287 \ln\left(\frac{70}{180}\right)$$

$\dot{m} = 3$ kg/s so

$$\boxed{\dot{S}_{gen} = 0.204\ \frac{kW}{K}}$$

$$V_{out} = \sqrt{70^2 + 2[1.00(707-527)] \times 1000}$$

$$\boxed{V_{out} = 604\ m/s}$$

(c) Reversible Irreversible

Reversible		Irreversible
474°C	T_2	527°C
686 m/s	V_2	604 m/s

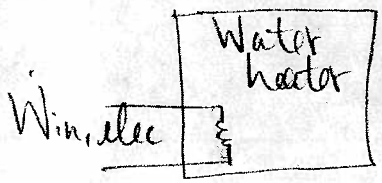
The irreversible case has lower V_2 & higer T_2.
The purpose of a nozzle is to convert internal energy (u) and flow work (pv) into kinetic energy ($\frac{V^2}{2}$). The reversible nozzle does a better job of this than the irreversible one.

ES201-0? Date | HW # P8.17 | Name CM | 1/2

Known Electric water heater

Find (a) $W_{in,elec}$
(b) S_{gen} water only
(c) S_{gen} water & resistor
(d) why do results differ?

Given



① $T_1 = 18°C$

$T_{resistor} = 97°C$

② $T_2 = 60°C$

$\rho = 1000\ kg/m^3$

$c = 4.18\ kJ/kg \cdot K$

$V = 100\ L = 0.1\ m^3$

Assume incompressible; no $\dot{Q}_{out}$ of heater; no storage in resistor or water

Strategy Cons Energy & 2nd law - finite time; closed system

$$E_{sys,2} - E_{sys,1} = Q_{1-2} + W_{1-2}$$

$$S_{sys,2} - S_{sys,1} = \frac{Q_{in}}{T_{in}} + S_{gen}$$

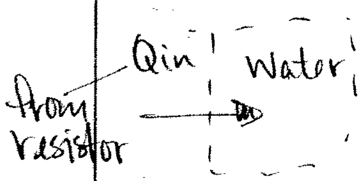
Solve: (a) $E_{sys,2} - E_{sys,1} = U_2 - U_1 = W_{1-2}$
System: water & resistor

$$U_2 - U_1 = m(u_2 - u_1) = m(c(T_2 - T_1)) = \rho \forall c (T_2 - T_1)$$

$$U_2 - U_1 = 1000\,(0.1)(4.18)(60 - 18)$$

$$U_2 - U_1 \boxed{= W_{1-2,elec} = 17560 \text{ kJ}}$$

(b) system: water only *not including the resistor*



$$\underbrace{Q_{in} = W_{1-2,elec} = 17560 \text{ kJ}}_{\text{from cons energy applied to resistor only}}$$

$$S_{gen} = S_1 - S_2 - \frac{Q_{in}}{T_{surf,in}} = m(s_2 - s_1) - \frac{Q_{in}}{T_{surf,in}}$$

$$s_2 - s_1 = c_p \ln\frac{T_2}{T_1} = 4.18 \ln\left(\frac{273+60}{273+18}\right)$$

$$T_{surf,in} = 97°C = 370 \text{ K}$$

$$S_{gen} = \rho \forall c \ln\left(\frac{T_2}{T_1}\right) - \frac{Q_{in}}{T_{surf,in}} \boxed{= 8.91 \frac{\text{kJ}}{\text{K}}}$$ *water only*

(c) system: water & resistor $\therefore Q = 0$

$$S_{gen} = m(s_2 - s_1) = \rho \forall c \ln\left(\frac{T_2}{T_1}\right)$$

$$\boxed{S_{gen} = 56.4 \frac{\text{kJ}}{\text{K}}}$$

(d) (c) includes resistor, (b) does not

There is heat transfer from the hot resistor to the cool water (which accounts for the S_{gen} discrepancy between the two).