Homework, Lesson 4

Problem 1

An incompressible liquid with density ρ is contained in a tank with rigid, vertical walls. (Fig. 1.) The tank is open to the atmosphere on the top and divided into two sections, Side A and Side B.

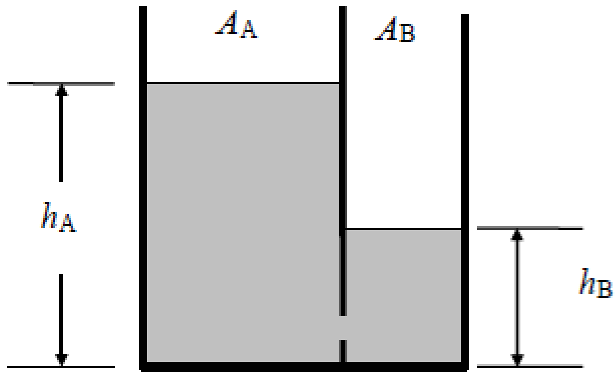

Figure 1: Two-sectioned tank with incompressible liquid

The height h of the liquid and the cross-sectional or foot-print area A on each side of the tank (area perpendicular to the coordinate h) are shown in the figure. There is a hole in the partition and the liquid can flow freely between the two sides.

Assume that the system is the mass of liquid in both tanks.

(a) Write an equation for the mass of the system, m_{sys}, in terms of the given information. One approach is to find $m_{sys,A}$ and $m_{sys,B}$ separately and then add them together to find m_{sys} Your answer should only contain symbols.

(b) Using your result from Part (a), calculate the first derivative of the system mass with respect to time: dm_{sys}/dt. Your answer should include two derivatives dh_A/dt and dh_B/dt. What is the physical meaning of these two terms and why aren't there more derivatives? [FYI –The derivative dm_{sys}/dt is called the time rate of change of the mass inside the system.]

(c) If the system mass is a constant, determine dh_A/dt, the time rate of change of the liquid level in Tank A, , when h_B, the liquid level in Tank B, is rising at the rate of 5 cm/s. Assume A_A = 0.5 m^2 and A_B = 0.2 m^2.

Problem 2

You have been asked to predict how the pressure inside a basketball changes as a player dribbles the ball. For a first-order model (Fig. 2), assume

- the basketball can be modeled as a sphere of constant radius R containing pressurized air and air can be modeled as an ideal gas.
- during a dribble, a section of the sphere is compressed a distance h by the floor and deforms without a wrinkling or changing R into a spherical shape that is missing a segment (see figure). This missing segment is called a *spherical cap*. The spherical cap has a volume $\forall_{cap} = (\pi/3)[h^2 (3R - h)]$.

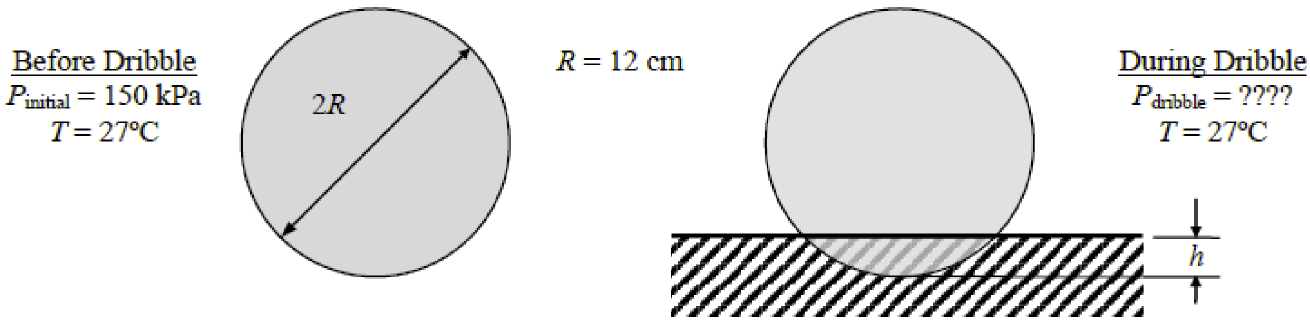

Figure 2: First-order model of a basketball

(a) Calculate the mass of air, in kg, inside the basketball before the dribble.

(b) If the sphere is compressed a distance h = 3 cm during the dribble, determine the ratio $P_{dribble}/P_{initial}$. If necessary, assume the temperature of the air does not change and the mass inside the basketball is a constant.

(c) If the mass of air inside the basketball is a constant, show that $d\rho/dt = -(\rho/V\!\!\!-)(dV\!\!\!-/dt)$ where ρ is the uniform density of the air inside the basketball and $V\!\!\!-$ is the volume of the air. Note ρ and $V\!\!\!-$ both depend on time.

(d) Write an expression for the volume $V\!\!\!-$ inside the basketball in terms of R and h that would be valid at any time during the dribble. (Your answer will only contain symbols and pure numbers.)

(e) Using your result from Part (d), calculate $dV\!\!\!-/dt$ in terms of dh/dt, h, and R, then determine the numerical value for K in the equation, $dV\!\!\!-/dt = K\,(dh/dt)$ when h = 3 cm.