Problem 1 (35 points)

A desalination plant operates at steady-state conditions and produces pure water vapor and brine from seawater.

$\dot{m}_2$ $\dot{m}_1$ ρ_1

Sea water enters the plant with a mass flow rate of 100,000 kg/hr and a density of 1025 kg/m^3. Brine exits the plant with a mass flow rate of 14,630 kg/hr and a density of 1200 kg/m^3. Water vapor leaves the plant at a pressure of 120 kPa and temperature of 105°C. The water vapor can be modeled as an ideal gas with

P_3 T_3 ρ_2

$$R_{water} = 0.4614 \text{ kJ/(kg-K)}; \quad R_u = 8.314 \text{ kJ/(kmol-K)}; \quad M_{water} = 18.02 \text{ kg/kmol.}$$

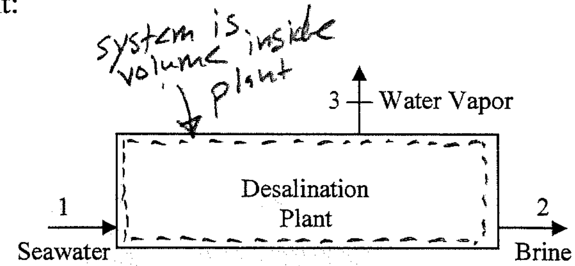
Determine the following information for the desalination plant:

(a) the mass flow rate of **water vapor**, in kg/hr.

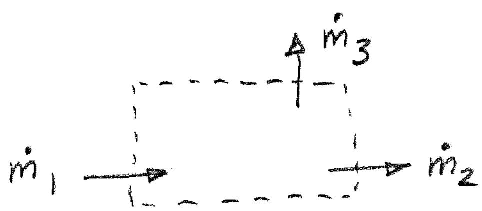
(b) the volumetric flow rate of **water vapor** leaving the plant, in m^3/hr.

(c) the volumetric flow rate of **brine**, in m^3/hr.

(d) the average velocity of the **brine**, in *m/s*, if the cross-sectional area of the brine pipe is 0.5 m^2. [*Note the units for velocity.*]



(a)



COM $\quad \cancel{\dfrac{dm_{sys}}{dt}}^{\,0\ (s.s.)} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$

$$0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_3$$

$$\dot{m}_3 = \dot{m}_1 - \dot{m}_2 = 100{,}000 \text{ kg/hr} - 14{,}630 \text{ kg/hr}$$

$$\boxed{\dot{m}_3 = 85{,}370 \text{ kg/hr}}$$

(b) $\dot{m}_3 = \rho_3 \dot{\forall}_3 \Longrightarrow \dot{\forall}_3 = \dot{m}_3 / \rho_3$

assumes that density is constant across outlet area

$$\dot{m} = \underbrace{\int_A \rho}_{\text{const.}} (\vec{V}_{rel} \cdot \hat{n})\, dA \Rightarrow \rho \underbrace{\int_A (\vec{V}_{rel} \cdot \hat{n})\, dA}_{\triangleq\ \dot{\forall}}$$

Ideal Gas Model

$$P\forall = mRT \Rightarrow P = \frac{m}{\forall} RT \Rightarrow P_3 = \rho_3 \underbrace{R}_{=R_{water}} T_3$$

(Cont'd →)

$$\rho_3 = \frac{P_3}{R_{water}\, T_3} = \frac{120\ \cancel{kPa}}{\left(0.4614\ \frac{\cancel{kJ}}{kg\cdot\cancel{K}}\right)(378\ \cancel{K})} \underbrace{\left(\frac{10^5\ \frac{\cancel{N}}{m^2}}{100\ \cancel{kPa}}\right)\left(\frac{1\ \cancel{kJ}}{10^3\ \cancel{N}\cdot m}\right)}_{\text{unit conversions}}$$

$$\rho_3 = 0.688\ kg/m^3$$

$$\dot{V}_3 = \dot{m}_3/\rho_3 = \frac{85{,}370\ \cancel{kg}/hr}{0.688\ \cancel{kg}/m^3} \rightarrow \boxed{\dot{V}_3 = 124{,}084\ \frac{m^3}{hr}}$$

(c) $\dot{m}_2 = \rho_2 \dot{V}_2$ (<u>assumes</u> ρ = const/uniform i.e. <u>incompressible</u>)

$$\dot{V}_2 = \dot{m}_2/\rho_2 = \frac{14{,}630\ \cancel{kg}/hr}{1200\ \cancel{kg}/m^3} \Rightarrow \boxed{\dot{V}_2 = 12.2\ m^3/hr}$$

(d) $\dot{V}_2 = V_{avg,2}\, A_2$

$$V_{avg,2} = \frac{\dot{V}_2}{A_2} = \frac{12.2\ m^3/hr}{0.5\ m^2} \underbrace{\left(\frac{1\ hr}{3600\ s}\right)}_{\text{unit conversion}}$$

$$\boxed{V_{avg,2} = 0.0068\ m/s}$$

Problem 2 (30 points)

To shape the head of a steel bolt, a cylindrical steel rod is placed into a die. A punch then moves downward at constant velocity V_P to forge the head. The bolt head remains cylindrical during the entire process, as shown below. The density of the steel is constant and uniform during the entire process.

Determine the following:

(a) the time rate-of-change of the bolt head diameter (dD/dt) in terms of the punch velocity (V_P), the instantaneous bolt-head diameter (D), and the instantaneous bolt-head thickness (h).

(b) the time rate-of-change of the bolt-head thickness (dh/dt) if the velocity of the punch is $V_P = 1$ m/s.

(c) the final bolt-head thickness (h_f) in terms of the punch diameter (D_P), the steel-rod diameter (D_0), and the initial height of the rod above the lower die cavity (h_0).

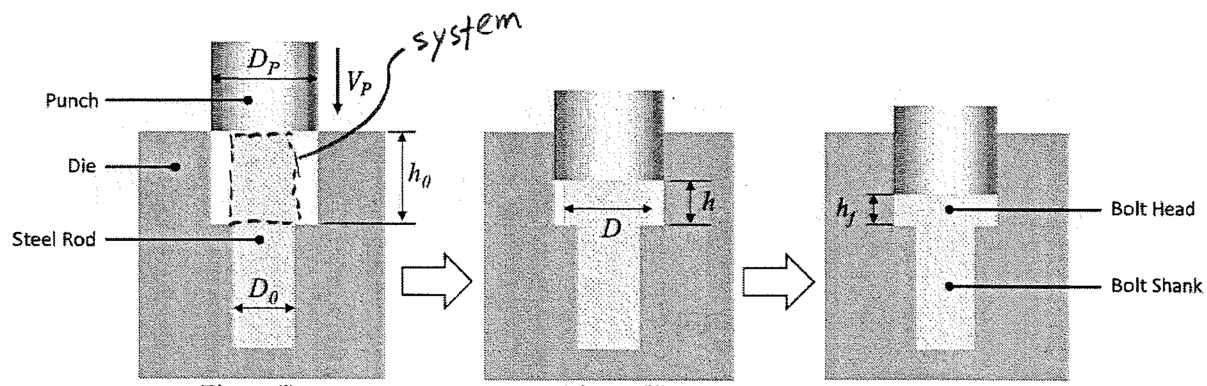
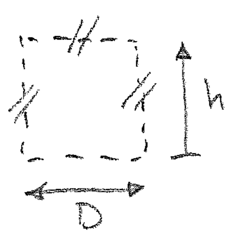


Figure (i) Figure (ii) Figure (iii)

(a)



closed system

$$\frac{dm_{sys}}{dt} = \cancel{\sum \dot{m}_{in}}^{\,0} - \cancel{\sum \dot{m}_{out}}^{\,0}$$

$$\frac{d}{dt}\left(\rho \pi \left(\frac{D}{2}\right)^2 h\right) = 0$$

assume $\rho =$ const (incompressible)

$$\cancel{\frac{\rho \pi}{4}}\left(2D\frac{dD}{dt}h + D^2\frac{dh}{dt}\right) = 0$$

$$\frac{dh}{dt} = -V_p$$

$$2\cancel{D}\frac{dD}{dt}h - D^{\cancel{2}}V_p = 0$$

$$\boxed{\frac{dD}{dt} = \frac{D V_p}{2h}}$$

(b) $\quad \dfrac{dh}{dt} = -V_p \quad \Longrightarrow \quad \boxed{\dfrac{dh}{dt} = -1\,\text{m/s}}$

(c) $\quad \dfrac{dm_{sys}}{dt} = 0 \quad \Longrightarrow \quad m_{sys,i} - m_{sys,f} = 0$

$$m_{sys,i} = m_{sys,f}$$

$$\cancel{\rho}\,\forall_i = \cancel{\rho}\,\forall_f$$

$$\cancel{\pi}\,\frac{D_o^2}{\cancel{4}}\,h_o = \cancel{\pi}\,\frac{D_p^2}{\cancel{4}}\,h_f$$

$$\boxed{h_f = \frac{D_o^2}{D_p^2}\,h_o}$$

Problem 3 (35 points)

A tank with a concrete basin is shown in the figure and is filled with water to a height h. Openings to fill and drain the tank are located in the concrete basin (see figure).

The volume of water in the tank $\forall_{water}$ depends on the height h of water in the tank:

$$\forall_{water} = \forall_{base} + wh^2 \qquad \text{where} \qquad \forall_{base} = 144\ \text{ft}^3 \text{ and } w = 100\ \text{ft.}$$

The volumetric flow rate of the water draining from the tank also depends on the height h:

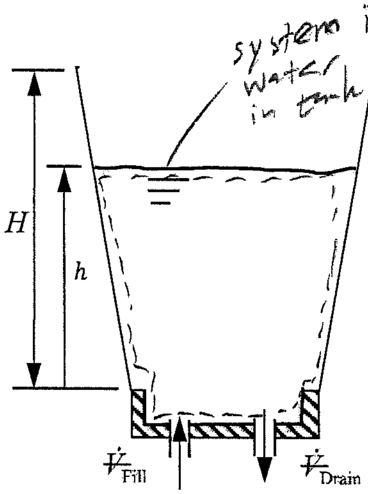
$$\dot{\forall}_{Drain} = K_{Drain}\sqrt{h} \qquad \text{where} \qquad K_{Drain} = 20.0\ \text{ft}^{5/2}/\text{s}.$$

Assume water is incompressible with a density of 62.4 lbm/ft^3.

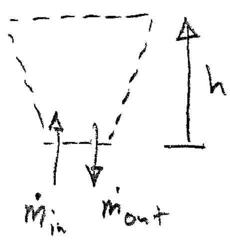
(a) Develop a symbolic equation for dh/dt, the time rate-of-change of the water level in the tank when $\dot{\forall}_{Fill}$, $\dot{\forall}_{Drain}$ and h are all known.

(b) If the tank is initially empty but $\dot{\forall}_{Fill} = 50$ ft^3/s and the drain is open, determine the steady-state height h of the liquid in the tank, in ft.

(c) If the height of the water is $h = 14$ ft and only the drain is open, i.e. $\dot{\forall}_{Fill} = 0$, determine how long it will take, in seconds, for the tank to drain to $h = 3$ ft.



(a)



$$\frac{dm_{sys}}{dt} = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$

$$\frac{d}{dt}(\cancel{\rho} \forall_{water}) = \cancel{\rho}\dot{\forall}_{fill} - \cancel{\rho}\dot{\forall}_{Drain}$$

assume ρ = incompressible (= const)

$$\frac{d}{dt}(\forall_{base} + wh^2) = \dot{\forall}_{fill} - \dot{\forall}_{Drain}$$

$$2h\frac{dh}{dt}w = \dot{\forall}_{fill} - \dot{\forall}_{drain}$$

$$\boxed{\frac{dh}{dt} = \frac{\dot{\forall}_{fill} - \dot{\forall}_{drain}}{2hw} = \frac{\dot{\forall}_{fill} - K_{drain}\sqrt{h}}{2hw}}$$

(b) at steady state..

$$\frac{\cancel{dm}}{\cancel{dt}}_{0\,(s.s.)} = \rho \dot{V}_{fill} - \rho \dot{V}_{drain} \Rightarrow \dot{V}_{fill} = \dot{V}_{drain}$$

$$\dot{V}_{fill} = K_d \sqrt{h} \Rightarrow h = \left(\frac{\dot{V}_{fill}}{K_D}\right)^2$$

$$h = \left(\frac{50 \text{ ft}^3/\cancel{s}}{20 \text{ ft}^{5/2}/\cancel{s}}\right)^2$$

$$\boxed{h = 6.25 \text{ ft}}$$

(c) from (a) w/ $\dot{V}_{fill} = 0 \Rightarrow \frac{dh}{dt} = \frac{-K_D\sqrt{h}}{2hw}$

$$\int_{14\text{ft}}^{3\text{ft}} h^{1/2}\, dh = \frac{-K_D}{2w}\int_0^{t_f} dt$$

$$\frac{2}{3} h^{3/2} \Big|_{14\text{ ft}}^{3\text{ ft}} = \frac{-20 \text{ ft}^{5/2}/s}{2(100 \text{ ft})} t_f$$

$$t_f = \frac{\frac{2}{3}(3\text{ft})^{3/2} - \frac{2}{3}(14\text{ ft})^{3/2}}{\left(-\frac{20 \text{ ft}^{5/2}/s}{2(100 \text{ ft})}\right)}$$

$$\boxed{t_f = 314.6 \text{ s}}$$