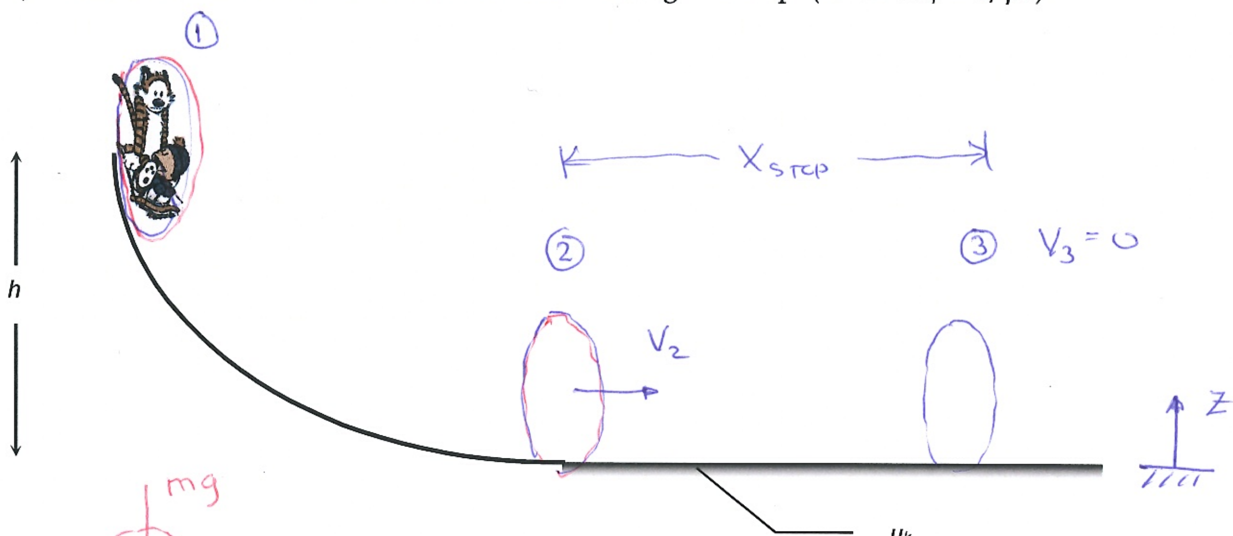
Example

Calvin and Hobbes are sledding down a smooth slide onto a rough surface with a coefficient of kinetic friction μ_k. Calvin, Hobbes and their sled have a combined mass of m_A and start at rest from the top of the slide.

a) Find the sled velocity as it comes onto the rough surface.
b) Find distance that the sled travels before coming to a stop. (ANS: $x_{stop} = h/\mu_K$)



(a)

Work-energy:

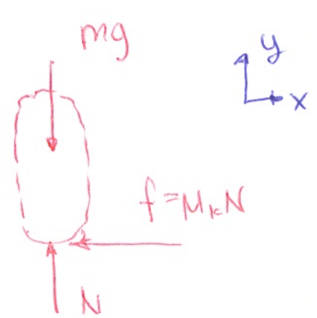
$$PE_2 - PE_1 + KE_2 - KE_1 = \underbrace{W_{1\to 2}}_{\to 0}$$

N Does no work, since always ⊥ to V.

$$mg(\underbrace{z_2}_{\to 0}) - mg(\underbrace{z_1}_{= h}) + \frac{1}{2} m V_2^2 - \frac{1}{2} m \underbrace{V_1^2}_{\to 0} = 0$$

$$\therefore \boxed{V_2 = \sqrt{2gh}}$$

(b)



CoLM, y-dir:

$$\underbrace{\frac{d}{dt}(P_y)}_{\to 0} = \sum F_y + \underbrace{0 - 0}_{\text{CLOSED}} \qquad 0 = N - mg$$

No V_y $\qquad N = mg$

Work-energy:

$$\cancel{PE_3} - \cancel{PE_2} + \underbrace{\cancel{KE_3}}_{0} - KE_2 = W_{23} = -f\, x_{STOP}$$

$$-\frac{1}{2}mV_2^2 = -f\,x_{stop}$$

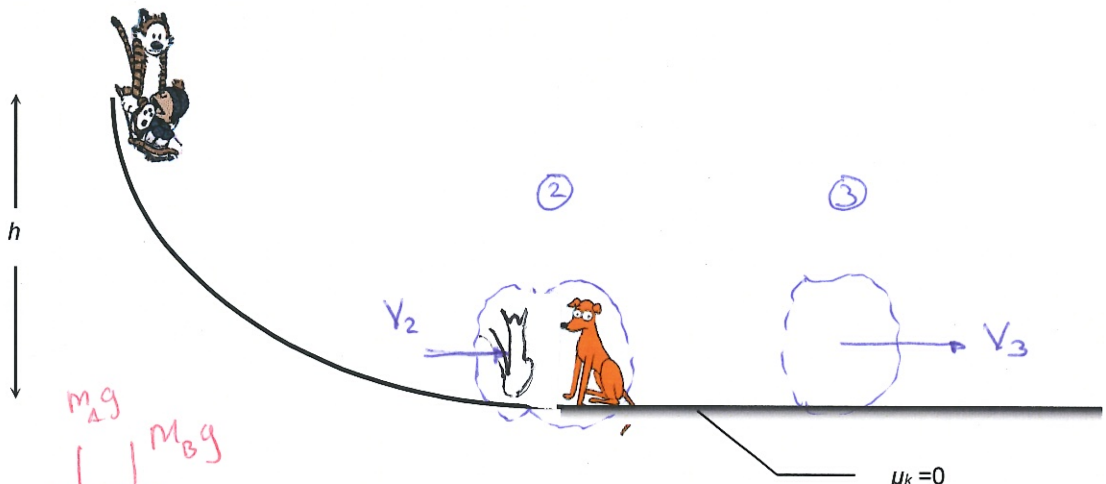
$$-\frac{1}{2}m\left[\sqrt{2gh}\right]^2 = -\left[\mu_k mg\right]x_{stop}$$

$$\boxed{x_{stop} = h/\mu_k}$$

Example

Reconsider the sled in the last problem. This time, make the flat surface frictionless, but place Santa's Little Helper in the sled path. Santa's Little Helper has mass m_B.

Find the velocity of Calvin, Hobbes, Santa's Little Helper and the sled after impact using the **conservation of energy**. Be sure to start with the most general form of conservation of energy.



<u>Work-energy:</u>

$$\cancel{PE_3} - \cancel{PE_2} + KE_3 - KE_2 = \cancel{W_{23}} \rightarrow 0$$

No forces in direction of displacement.

$$\frac{1}{2}(m_A + m_B)V_3^2 - \frac{1}{2}m_A V_2^2 = 0$$

Only Calvin/Hobbes have KE @ ②.

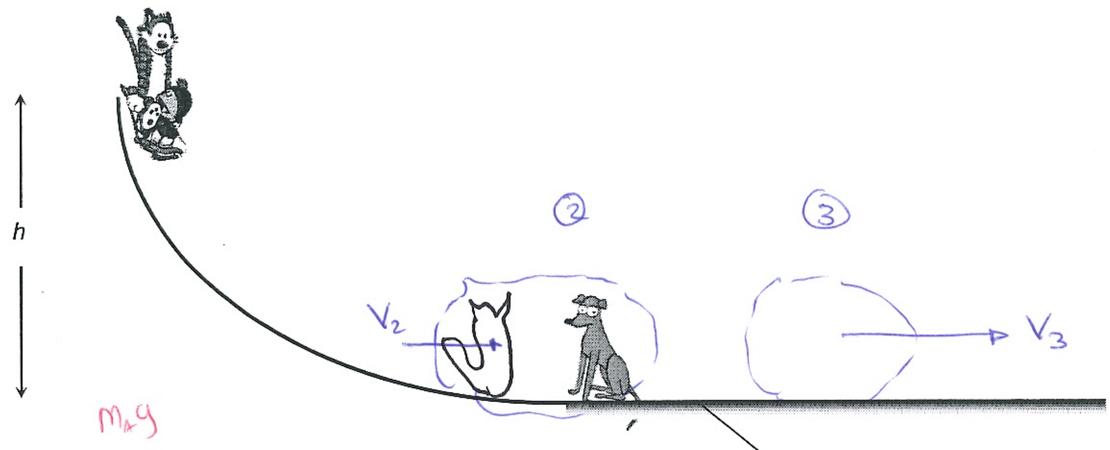
$$V_3 = \sqrt{\frac{m_A}{m_A + m_B}} \cdot V_2$$

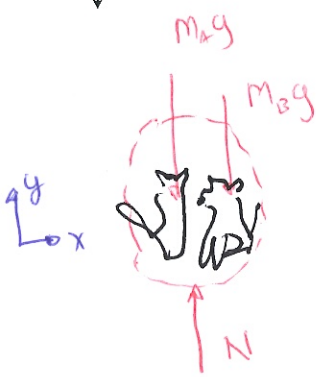
Actually <u>wrong</u> result!

Example

Reconsider the sled in the last problem. This time, make the flat surface frictionless, but place Santa's Little Helper in the sled path. Santa's Little Helper has mass m_B.

Find the velocity of Calvin, Hobbes, Santa's Little Helper and the sled after impact using the **conservation of linear momentum.**





Time frame: Just before impact to just after.

<u>CoLM in x-direction</u>

$$\frac{d}{dt}(\mathbb{P}_x) = \cancel{\sum F_x} + 0 + 0$$

closed.

$$\int_{P_{x2}}^{P_{x3}} d\mathbb{P}_x = \int_0^t 0\, dt$$

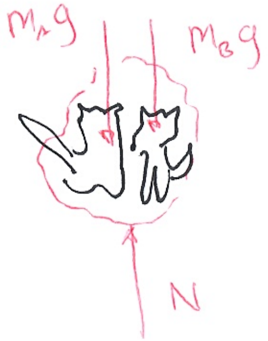
$$\mathbb{P}_{x,3} - \mathbb{P}_{x,2} = 0$$

$$(m_A + m_B) V_3 - m_A V_2 = 0$$

$$\boxed{V_3 = \frac{m_A}{m_A + m_B} \cdot V_2}$$

Note this is <u>not</u> same result as from Work-energy! This one is right...

You can not neglect internal energy during impact. The full-blown conservation of energy is needed:



Cons. of energy, closed finite time:

$$\cancel{PE_3} - \cancel{PE_2} + KE_3 - KE_2 + \underbrace{U_3 - U_2}_{\neq 0!} = \underbrace{\cancel{Q_{23}}}_{0} + \underbrace{\cancel{W_{23}}}_{0}$$

$$U_3 - U_2 = KE_2 - KE_3$$

$$= \frac{1}{2}(m_A V_2^2) - \frac{1}{2}(m_A + m_B) V_3^2$$

$$= \frac{1}{2} m_A V_2^2 - \frac{1}{2}(m_A + m_B) \underbrace{\left[\frac{m_A}{m_A + m_B}\right]^2 V_2^2}_{\text{Result from CoLM.}}$$

$$\underbrace{U_3 - U_2 = \frac{1}{2}\left[\frac{m_A m_B}{m_A + m_B}\right] V_2^2}$$

During collison, the difference between the kinetic energies is converted to internal energy.