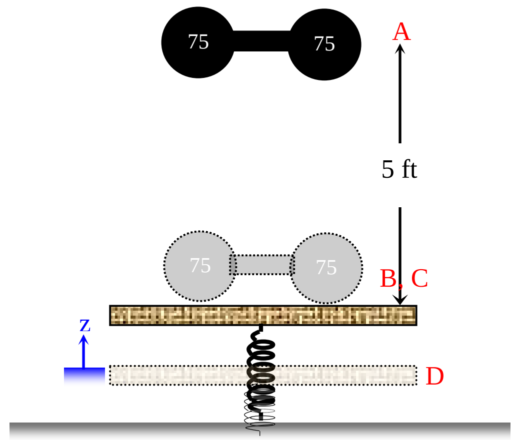
Example

Dr. Thom drops a 75 lb dumbbell from a height of 5 ft onto a springboard. The spring board has a mass of 10lbm with springs of stiffness 450 lb/in. The springs are initially compressed 0.022 in. Calculate the maximum deflection of the springboard in inches.



Divide the problem into three stages:

A→ B: Weight drops from 5 ft to instant *just before impact.*

B→ C: Just before impact to just after impact.

C→ D: Just after impact to point of maximum spring deflection

A→ B

System: dumbbell

Conservation of energy

$$\frac{d}{dt}(E_{sys}) = \dot{Q}_{in,net} + \dot{W}_{in,net} + \ldots + \ldots$$

No heat transfer   Closed system

Integrate to get finite time:

$$E_{sys,B} - E_{sys,A} = W_{A\text{-}B}$$

Only KE and PE for this system:

$$(KE_B - KE_A) + (PE_B - PE_A) = W_{A\text{-}B}$$

No surface (contact) forces do work. Plug and chug…

$$0 = (\tfrac{1}{2} \cdot m_{db} \cdot V_B^2 - 0) + m_{db} \cdot g \cdot (5 \text{ ft})$$

$$V_B = (2 \cdot g \cdot 5 \text{ ft})^{1/2} = (2*32.2*5)^{1/2} \text{ ft/s} = \boxed{17.9 \text{ ft/s}}$$

B→ C

System: dumbbell & board

z-dirction Linear momentum, finite time, closed system (You should be able to derive this on your own!)

$$P_{sys,C} - P_{sys,B} = F_{avg,impulsive}\,\Delta t$$

$$[(m_{db} + m_{board})V_C] - [m_{db}V_B + m_{board}\,V_{B,board}] = 0$$

(No *impulsive* forces acting on system. Spring forces are negligible compared to impulsive forces.)

$$[(m_{db} + m_{board})V_C] - [m_{db}V_B + m_{board}\,0] = 0$$

$$V_C = [m_{db}/(m_{db} + m_{board})]\ V_B = \ldots = \boxed{15.83\text{ ft/s}}$$

C→ D

System: dumbbell, board and spring

Conservation of energy reduces as before:

$$\frac{d}{dt}(E_{sys}) = \dot{Q}_{in,net} + \dot{W}_{in,net} + \ldots + \ldots$$

No heat transfer　　Closed system

Integrate to get finite time:

$$E_{sys,D} - E_{sys,C} = W_{C\text{-}D}$$

$$W_{C\text{-}D} = (KE_D - KE_C) + (PE_D - PE_C) + (E_{S,D} - E_{S,C})$$

$$0 = [\tfrac{1}{2}\cdot(m_{db} + m_{board})\cdot V_D^2 - \tfrac{1}{2}\cdot(m_{db} + m_{board})\cdot V_C^2] + [m_{db}\cdot g\cdot(z_D - z_C)] + [(\tfrac{1}{2}\,k\,x_D^2 - \tfrac{1}{2}\,k\,x_C^2)]$$

$$0 = [\tfrac{1}{2}\cdot(m_{db} + m_{board})\cdot 0^2 - \tfrac{1}{2}\cdot(m_{db} + m_{board})\cdot V_C^2] + m_{db}\cdot g\cdot(0 - z_C) + [\tfrac{1}{2}\,k\,(z_C + 0.022\text{ in})^2 - \tfrac{1}{2}\,k\,(0.022\text{ in})^2]$$

$$0 = -\tfrac{1}{2}\cdot(m_{db} + m_{board})\cdot V_C^2 - m_{db}\cdot g\cdot z_C + [\tfrac{1}{2}\,k\,(z_C + 0.022\text{ in})^2 - \tfrac{1}{2}\,k\,(0.022\text{ in})^2]$$

Note that x_D is the initial compression in the spring plus the additional compression from C→D, which is z_C.

The only unknown here is z_C. Solving anyway you want (and paying close attention to **units**)

$z_C = 4.34$ in

max deflection $= z_C + 0.022$ in $= 4.36$ in