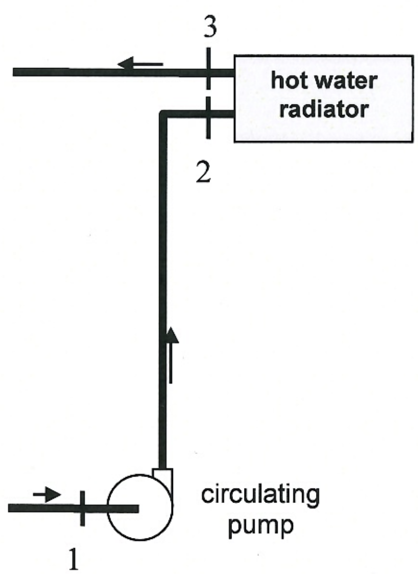
ROSE-HULMAN Institute of Technology
Sophomore Engineering Curriculum

Example

A hot-water heating system is shown in the figure below. The circulating pump is located in the basement of the building and the hot-water radiator is located on an upper floor. Under steady-state conditions, the radiator delivers 3.0 kW by heat transfer to the surroundings.

Pertinent operating conditions are shown in the figure. Assume that water can be treated as an incompressible substance with room-temperature specific heats. (Liquid Water Properties: c = 4.18 kJ/kg-K, ρ = 997 kg/m^3)

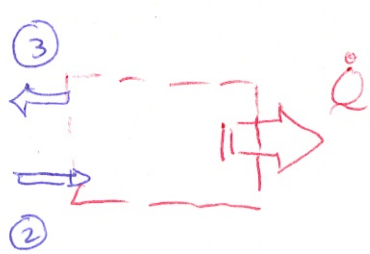


Operating Conditions				
State	T (°C)	P (kPa)	z (m)	A (m^2)
1	60	100	10.0	0.0020
2	60	125	30.0	0.0020
3	40	125	30.0	0.0020

Heat transfer from the pipes and the pump is negligible.
The only significant heat transfer occurs from the radiator.

(a) Determine the mass flow rate through the radiator, in kg/s.
(b) Determine the shaft power supplied to the pump, in kW, to move the water up to the radiator.
(c) Estimate the surface area of the radiator. Assume that convection heat transfer is the primary mechanism, the convection heat transfer coefficient h = 50 W/(m^2-°C), the room temperature is 22°C and the average radiator temperature is 50°C.

(a)



$$\frac{d}{dt}(\cancel{E_{sys}})^{\to 0} = \dot{Q}_{IN} + \cancel{\dot{W}_{IN}}^{\to 0} + \sum_{IN} \dot{m}(h+\dots) - \sum_{OUT} \dot{m}(h+\dots)$$

$$0 = -\dot{Q}_{OUT} + \dot{m}(h_2) - \dot{m}(h_3)$$

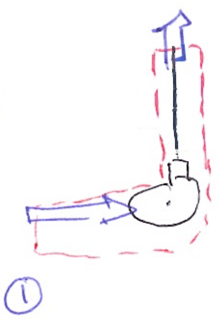
$$\dot{Q}_{OUT} = \dot{m}(h_2 - h_3)$$

$$\dot{m} = \frac{\dot{Q}_{OUT}}{h_2 - h_3} = \frac{\dot{Q}_{OUT}}{c(T_2 - T_3) + \cancel{\frac{P_2 - P_3}{\rho}}^{\to 0}}$$

$$= \frac{3\ \cancel{kW}}{4.18\ \frac{\cancel{kJ}}{kg\cdot \cancel{K}}(60^\circ - 40^\circ)\ \cancel{^\circ C}} \left\langle \frac{\cancel{kJ/s}}{\cancel{kW}} \right\rangle$$

$$= \boxed{0.036\ kg/s}$$

(b)



$$\frac{d}{dt}(E_{sys})^{\to 0} = \dot{Q}_{IN}^{\to 0} + \dot{W}_{IN} + \dot{m}(h_1 + gz_1) - \dot{m}(h_2 + gz_2)$$

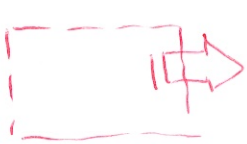
$$\dot{W}_{IN} = \dot{m}(h_2 - h_1) + \dot{m}g(z_2 - z_1)$$

$$= \dot{m}\left(c(T_2 - T_1)^{\to 0} + \frac{P_2 - P_1}{\rho} \right) + g(z_2 - z_1)$$

$$= 0.036\ \frac{kg}{s}\left[\frac{125 - 100}{997}\ \frac{k\cancel{Pa}}{\frac{kg}{\cancel{m^3}}} \left\langle \frac{kJ}{kPa\cdot \cancel{m^3}} \right\rangle + 9.81\ \frac{m}{s^2}(30 - 10)\ m \times \left\langle \frac{\cancel{J}/kg}{m^2/s^2} \right\rangle \left\langle \frac{kJ}{1000\ \cancel{J}} \right\rangle \right]$$

$$= 0.00797\ \frac{kJ}{s} = \boxed{7.97\ W}$$

(c)



$$\dot{Q} = h_{conv} A (T_s - T_{air})$$

$$A = \frac{\dot{Q}}{h_{conv}(T_s - T_{air})} = \frac{3\ \cancel{kW} \left\langle \frac{1000\ \cancel{W}}{\cancel{kW}} \right\rangle}{50\ \frac{\cancel{W}}{m^2 \cdot \cancel{^\circ C}}(50^\circ - 22)\ \cancel{^\circ C}}$$

$$= \boxed{2.14\ m^2}$$