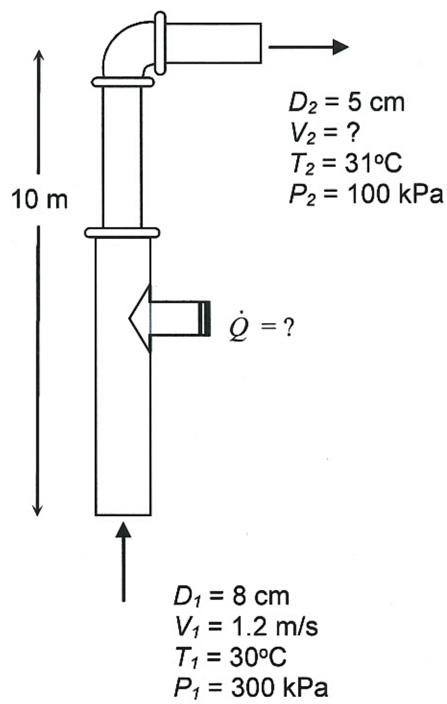
ROSE-HULMAN Institute of Technology
Sophomore Engineering Curriculum

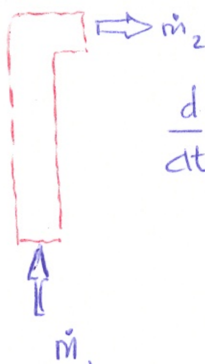
Example

Water flows through a piping system as shown in the figure. The properties at the inlet and the exit of the pipe are known. Modeling water as an incompressible substance with $\rho = 996\ \mathrm{kg/m^3}$ and $c = 4.47\ \mathrm{kJ/kg\text{-}K}$

a) find the exit velocity of the water, and
b) find the rate of heat transfer added to the water.
c) How does the enthalpy change compare with the kinetic and potential energy terms?



(a) Use cons. of mass



$$\frac{d}{dt}\cancel{(m_{sys})}^{\,0} = \sum \dot m_{IN} - \sum \dot m_{OUT}$$

$$= \dot m_1 - \dot m_2$$

$$\dot m_1 = \dot m_2$$

$$= \rho A_1 V_1 = \cancel{996\,k}$$

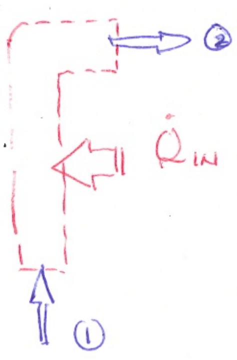
$$= \rho \frac{\pi D_1^2}{4} V_1 = \left(996 \frac{kg}{m^3}\right)\left(\frac{\pi 0.05^2}{4} m^2\right) \times \frac{1.2 m}{s}$$

$$= 6\ kg/s$$

$$\dot m_1 = \dot m_2 \Rightarrow \rho \frac{\pi D_1^2}{4} V_1 = \rho \frac{\pi D_2^2}{4} V_2$$

$$V_2 = \left(\frac{D_1}{D_2}\right)^2 V_1 = \left(\frac{8}{5}\right)^2 1.2 \frac{m}{s} = 3.07\ m/s$$

(b) Use conservation of energy



$$\frac{d}{dt}\cancel{(E_{sys})}^{\,0} = \dot Q_{IN} + \cancel{\dot W_{IN}}^{\,0} + \sum \dot m_{in}\left(h + \frac{V^2}{2} + gz\right) - \sum \dot m_{OUT}\left(h + \frac{V^2}{2} + gz\right)$$

$$\dot Q_{IN} = \dot m\left[h_2 - h_1 + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$\dot{Q}_{IN} = \dot{m}\Big[\underbrace{c(T_2 - T_1) + \frac{P_2 - P_1}{\rho}}_{\text{Since incompressible.}} + \frac{V_2^2 - V_1^2}{2} + g(Z_2 - Z_1)\Big]$$

$$= 6\,\frac{kg}{s}\Big[4.47\,\frac{kJ}{kg\text{-}K}(31 - 30)^{\circ}C + \frac{(300 - 100)\,kPa}{996\,kg/\cancel{m^3}}\left\langle\frac{kJ}{\cancel{kPa}\cdot\cancel{m^3}}\right\rangle$$

$$+ \frac{3.07^2 - 1.2^2}{2}\,m^2/s^2\left\langle\frac{J/kg}{m^2/s^2}\right\rangle\left\langle\frac{kJ}{1000\,J}\right\rangle + 9.81\,\frac{\cancel{m}}{\cancel{s^2}}(10\,\cancel{m})\left\langle\frac{J/kg}{\cancel{m^2/s^2}}\right\rangle\left\langle\frac{kJ}{1000\,J}\right\rangle\Big]$$

$$= 6\,\frac{kg}{s}\Big[4.67\,\frac{kJ}{kg} + \underbrace{0.0040\,\frac{kJ}{kg} + 0.0098\,\frac{kJ}{kg}}\Big]$$

Note that these are negligible, & this is compared to Δh caused by on 1°C temperature change!

$$= 28.0\,\frac{kJ}{s} = \boxed{28.0\,kW}$$