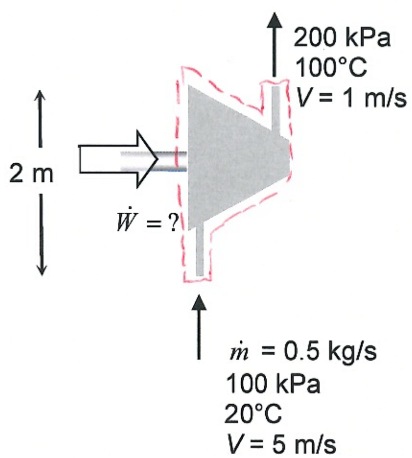
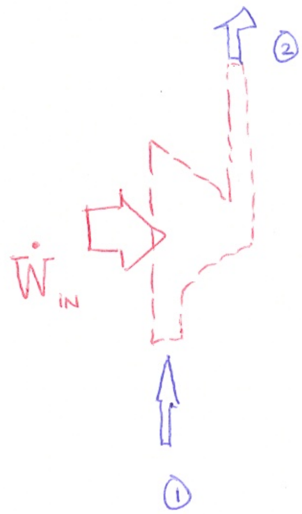
Example

0.5 kg/s of air flows steadily through a compressor. The air enters and exits the compressor at the states shown in the figure. If the compression is **adiabatic** (buzza buzza buzz) calculate the power input to the compressor. (R_{air} = 0.287 kJ/kg-K = 0.287 kPa-m^3/kg-K, c_v = 0.713 kJ/kg-K, c_p = 1.000 kJ/kg-K)





Cons of mass

$$\frac{d}{dt}(\cancel{m_{sys}}) = \dot{m}_1 - \dot{m}_2$$

$\to 0$ steady state

$$\dot{m}_1 = \dot{m}_2 = \dot{m}_a$$

Cons. of energy

$$\frac{d}{dt}(\cancel{E_{sys}}) = \cancel{\dot{Q}}_{IN,NET} + \dot{W}_{IN,NET} + \sum_{IN} \dot{m}\left(h + \frac{V^2}{2} + gz\right) - \sum_{OUT} \dot{m}\left(h + \frac{V^2}{2} + gz\right)$$

$\to 0$ S-S $\qquad \to$ Adiabatic

$$\dot{W}_{IN} = \dot{m}_a\left(h_2 + \frac{V_2^2}{2} + gz\right) - \dot{m}_a\left(h_1 + \frac{V_1^2}{2} + gz_2\right)$$

$$= \dot{m}_a\left[\underbrace{h_2 - h_1}_{\text{Ideal gas, constant specific heats}} + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$= \dot{m}_a\left[c_p(T_2 - T_1) + \frac{V_2^2 - V_1^2}{2} + g(z_2 - z_1)\right]$$

$$= 0.5\,\frac{kg}{s}\left[1.00\,\frac{kJ}{kg\text{-}\cancel{K}}\cdot(100-20)\cancel{K} + \frac{1^2 - 5^2}{2}\,\frac{m^2}{s^2}\left\langle\frac{J/kg}{m^2/s^2}\right\rangle\left\langle\frac{kJ}{1000\,J}\right\rangle + \right.$$

$$+ 9.81 \frac{m}{s^2} \cdot (2m) \left\langle \frac{J/kg}{m^2/s^2} \right\rangle \left\langle \frac{kJ}{1000 J} \right\rangle \Bigg]$$

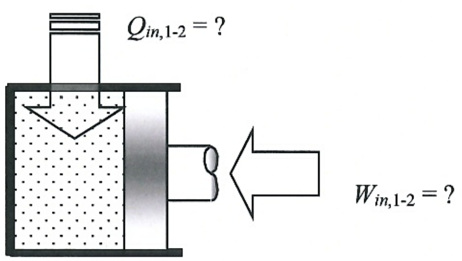
$$= \boxed{40.0 \text{ kW}}$$

Yes, °C cancels with K! The K on the bottom of c_p is a change in temperature, & changes in °C & K are the same.

Example

0.3 kg of air is contained in a piston-cylinder assembly. Initially, the air is at 200 kPa and 20°C. The air is then compressed in a process for which pV^2= constant until the pressure is 500 kPa. (R_{air} = 0.287 kJ/kg-K = 0.287 kPa-m³/kg-K, c_v = 0.713 kJ/kg-K, c_p = 1.000 kJ/kg-K)

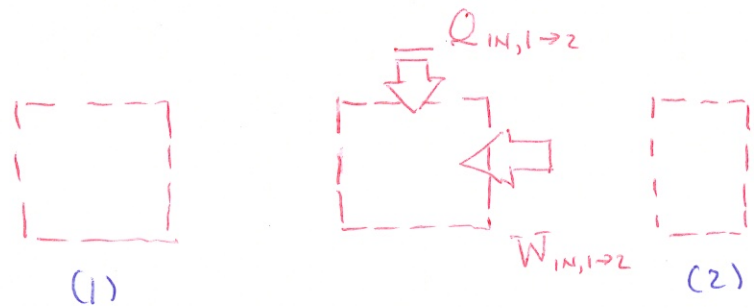
a) Sketch the p-V diagram and calculate the work (in kJ) into the piston cylinder.
b) Calculate the heat transfer (in kJ) into the piston cylinder during the process.



(a) See previous example for finding the work.

$$\therefore W_{IN,1\to 2} = 14.6 \text{ kJ}$$

(b)



Conservation of energy, finite time, closed system.

$$(E_2 - E_1)_{sys} = Q_{IN,1\to 2} + W_{IN,1\to 2}$$

Only internal energy is important.

$$(U_2 - U_1) = Q_{IN,1\to 2} + W_{IN,1\to 2}$$

$$\underbrace{m(u_2 - u_1)}_{} = \text{"} \quad \text{"}$$

Ideal gas with constant specific heats

$$m(c_v(\overset{?}{T_2} - \overset{\checkmark}{T_1})) = \overset{?}{Q}_{IN,1\to 2} + \overset{\checkmark}{W}_{IN,1\to 2} \qquad (1)$$

Need to find T_2.

Ideal gas:

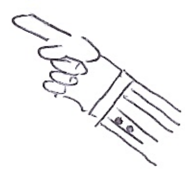
$$P_1\forall_1 = mRT_1 \quad \& \quad P_2\forall_2 = mRT_2$$

$$\therefore \quad T_2 = \frac{P_2\forall_2}{P_1\forall_1} T_1$$

Since $P_2\forall_2^2 = P_1\forall_1^2$, $\quad \forall_2/\forall_1 = (P_1/P_2)^{1/2}$

$$\therefore \quad T_2 = \frac{P_2}{P_1}\left(\frac{P_1}{P_2}\right)^{1/2} T_1 = \left(\frac{P_2}{P_1}\right)^{1/2} T_1$$

$$= \left(\frac{500 \ k\cancel{Pa}}{200 \ k\cancel{Pa}}\right)^{1/2} (20°C + 273)\,K$$



Don't forget!

$$= 463\,K$$

Returning to (1)

$$Q_{IN,1\to2} = mc_v(T_2 - T_1) - W_{IN,1\to2}$$

$$= (0.3\,kg)\left(0.713 \frac{kJ}{kg\cdot\cancel{K}}\right)(463\,K - [20°C + 273]\cancel{K})$$

$$- 14.6 \frac{kJ}{kg}$$

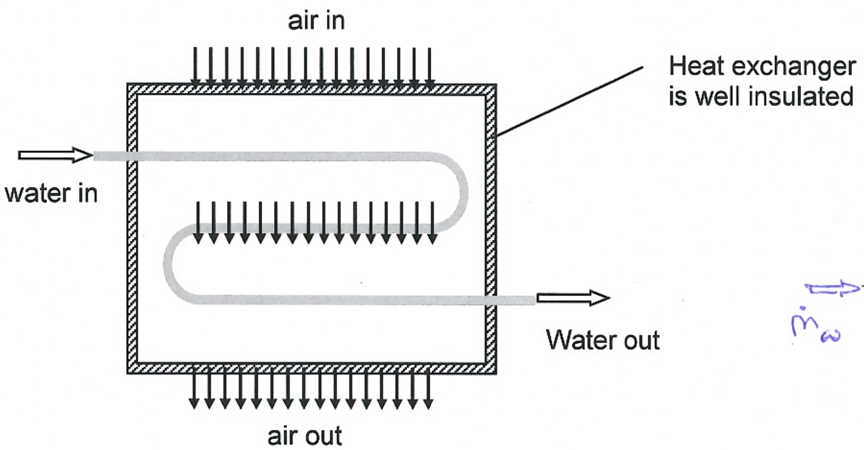
$$= \boxed{21.6 \ kJ}$$

Example

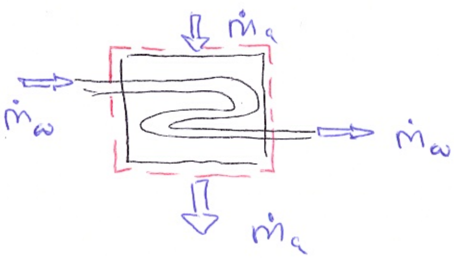
A heat exchanger operates at steady-state. 50 kg/min of air enters the device at 35°C and leaves at 45°C. Water flows through a coiled tube in the heat exchanger, entering at 250 kPa and 200°C and leaving at 240 kPa and 195°C. The kinetic and potential energies of the fluid streams are negligible. Property data are given below.

a) Find the mass flow rate of water through the coiled tube.
b) Find the rate of heat transfer from the water to the air.

Air: $c_v = 0.713$ kJ/(kg-K), $c_p = 1.000$ kJ/(kg-K), $R_{air} = 0.287$ kJ/(kg-K)
Water: $\rho = 865$ kg/m³, $c = 4.47$ kJ/(kg-K)



Pick entire HXR as system so that no $\dot{Q}$ crosses boundary:



Cons. of Energy

$$\underbrace{\frac{d}{dt}(E_{sys})}_{\to 0 \text{ steady}} = \cancelto{0}{\dot{Q}_{IN,NET}} + \cancelto{0}{\dot{W}_{IN,NET}} + \sum \dot{m}_{IN}(h + \dots) - \sum \dot{m}_{OUT}(h + \dots)$$

$$0 = \dot{m}_w h_{w,IN} + \dot{m}_a h_{a,IN} - \dot{m}_w h_{w,OUT} - \dot{m}_a h_{a,OUT}$$

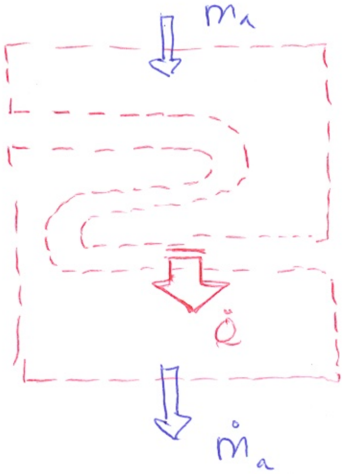
$$\dot{m}_w = \dot{m}_a \frac{(h_{a,OUT} - h_{a,IN})}{(h_{w,IN} - h_{w,OUT})} = \dot{m}_a \frac{c_p (T_{a,OUT} - T_{a,IN})}{c(T_{w,IN} - T_{w,OUT}) + \dfrac{P_{w,IN} - P_{w,OUT}}{\rho}}$$

$$= 50 \frac{kg}{s} \frac{1.00\ kJ/kg\text{-}K\ (45 - 35)K}{4.47 \frac{kJ}{kg\text{-}K}(200 - 195)K + \dfrac{(250 - 240)\ kPa}{865\ \frac{kg}{m^3}} \left(\frac{kJ}{kPa \cdot m^3}\right)}$$

$$= \boxed{22.4\ kg/s}$$

To find $\dot{Q}$, must pick either water or air by itself as system:

Just air



Cons. of energy

$$0 = \dot{Q}_{IN} + 0 + \dot{m}_a\left(h_{a,in} + \dots\right) - \dot{m}_a\left(h_{a,OUT} + \dots\right)$$

$$\dot{Q}_{IN} = \dot{m}_a\left(h_{a,OUT} - h_{a,in}\right)$$

$$= \dot{m}_a C_p\left(T_{a,OUT} - T_{a,in}\right)$$

$$= 50\,\frac{\cancel{kg}}{s} \cdot 1.00\,\frac{kJ}{\cancel{kg}\cdot \cancel{K}}\left(45\cancel{^\circ C} - 35\cancel{^\circ C}\right)\left\langle \frac{kW}{\cancel{kJ/s}} \right\rangle$$

$$= \boxed{8.34\ kW}$$

If you use just water as system, you get same number, except that it is <u>out</u> of system.