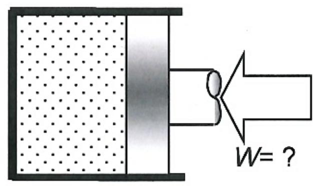
Example

A gas contained in a piston-cylinder at an initial state of $T_1 = 298$ K, $p_1 = 500$ kPa and $\forall_1 = 0.1$ m³ is placed in a furnace with $T_{furnace} = 1500$ K. This causes the gas to expand to a final state where $p_2 = 100$ kPa. During the expansion of the gas, the pressure and volume are related by $p\forall^{0.5}$ = constant.



a) Calculate the compression/expansion work into the gas.

$$W_{IN,12} = -\int_1^2 p\, d\forall$$

$$p = \frac{CONST}{\forall^{0.5}} = \frac{p_1\forall_1^{0.5}}{\forall^{0.5}}$$

$$W_{IN,1\to2} = -\int_{\forall_1}^{\forall_2} \frac{p_1\forall_1^{0.5}}{\forall^{0.5}} d\forall = -p_1\forall_1^{0.5}\int_{\forall_1}^{\forall_2} \frac{1}{\forall^{0.5}} d\forall = -p_1\forall_1\left[2\forall^{0.5}\right]_{\forall_1}^{\forall_2}$$

Conceptually we're done. Algebra...

$$-2p_1\forall_1^{0.5}\left[\forall_2^{0.5} - \forall_1^{0.5}\right] = -2p_1\forall_1\left[\left(\frac{\forall_2}{\forall_1}\right)^{0.5} - 1\right] = -2p_1\forall_1\left[\left(\frac{p_1}{p_2}\right) - 1\right]$$

$$= -2(500\text{ kPa})(0.1\text{ m}^3)\left[\left(\frac{500}{100}\right) - 1\right] = \boxed{-400\text{ kJ}}$$

(−) means work is out.

b) If the gas is air, calculate the initial and final temperatures. Assume air is an ideal gas with $R_{air} = 0.287$ kJ/kg-K

$$p_1\forall_1 = mRT_1 \quad \& \quad p_2\forall_2 = mR_2T_2$$

$$\therefore T_2 = \frac{p_2\forall_2}{p_1\forall_1} T_1 = \left(\frac{p_2}{p_1}\right)\cdot\left(\frac{p_1}{p_2}\right)^2 T_1$$

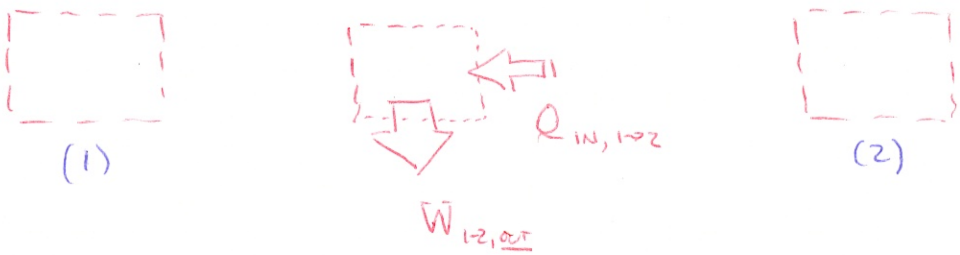
$$= \left(\frac{p_1}{p_2}\right)T_1 = \left(\frac{500}{100}\right)(298\text{ K}) = \boxed{1490\text{ K}}$$

Must be K, not °C.

c) Based on your answer to b), do you think there is heat transfer into or out of the system? Which way is it going?

T_{sys} is always less than $T_{furnace}$ ($T_{surroundings}$), and so yes, there should be heat transfer & it should be <u>into</u> the system.

d) If the heat transfer into the system is 900 kJ, what is the change of energy of the system?



Co Energy, Finite time, closed system.

$$E_2 - E_1 = Q_{IN,1\to2} + W_{IN,12} = 900\text{ kJ} - 400\text{ kJ} = \boxed{500\text{ kJ}}$$

e) Is there significant system kinetic energy in this system? Gravitational potential energy? So where is the energy?

All of the system energy is internal energy; i.e.,

$$(E_2 - E_1)_{sys} = (U_2 - U_1)_{SYSTEM}$$