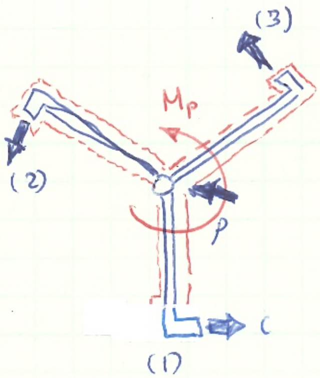

SYSTEM: SPRINKLER

Cons of Mass →

$$\frac{dM_{sys}}{dt} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$$

$\frac{dM_{sys}}{dt} \to 0$ (S-S)

$$0 = \dot{m}_{IN,P} - [\dot{m}_1 + \dot{m}_2 + \dot{m}_3]$$

$$\dot{m}_p = \rho \dot{\forall}_{IN}$$

$$= (1000)(2.7)\ \frac{kg}{m^3}\cdot\frac{m^3}{h}\left\langle\frac{h}{3600\ s}\right\rangle$$

$$= \underline{0.75\ kg/s}$$

$$\dot{m}_1 = \rho A_1 V_1$$

$$A_1 = A_2 = A_3 = A = \frac{\pi d^2}{4} \qquad \dot{m}_1 = \dot{m}_2 = \dot{m}_3 = \dot{m}_{OUT}$$

$$V_1 = V_2 = V_2 = \underline{V}$$

SO:

$$0 = \underline{0.75} - 3\dot{m}_{OUT}$$

$$\dot{m}_{OUT} = \underline{0.25\ \frac{kg}{s}} = \rho A V$$

$$V = \frac{\dot{m}_{OUT}}{\rho \frac{\pi}{4} d^2} = \frac{0.25\ kg/s}{(1000)\frac{kg}{m^3}\ \frac{\pi}{4}(0.007)^2} = 6.50\ m/s$$

Cons of Angular Momentum about P:

$$\circlearrowleft + \quad \frac{dL_p}{dt} = \sum M_p + \sum_{IN} (\vec{r} \times \vec{V})\dot{m} - \sum_{OUT} (\vec{r} \times \vec{V})\dot{m}$$

$\frac{dL_p}{dt} \to 0$, $\sum_{IN} (\vec{r} \times \vec{V})\dot{m} \to 0$

$$0 = M_p - [R \cdot V \cdot \dot{m}_{OUT} + RV\dot{m}_{OUT} + RV\dot{m}_{OUT}]$$

$$M_p = 3 \cdot R \cdot V \cdot \dot{m}_{OUT}$$

$$= 3 \cdot (0.15\ m)(\underline{6.50}\ \frac{m}{s})(0.25\ \frac{kg}{s})$$

$$= \boxed{0.70\ N \cdot m} \qquad N \cdot m$$

b) Cons. of Mass → Same.

$\dot{m}_{OUT} = 0.75$ kg/s

$V = 6.50$ m/s

Cons of Angular Momentum about P.

$$\frac{dL_p}{dt} = \sum M_p + \sum_{IN} (\bar{r} \times \bar{V})\dot{m} - \sum (\bar{r} \times \bar{V})\dot{m}_{OUT}$$

$\to 0$ $\to 0$ $\to 0$

$$0 = -[R \cdot V \cdot \dot{m}_{OUT} + RV\dot{m}_{OUT} + RV\dot{m}_{OUT}]$$

$$= -3RV\dot{m}_{OUT}$$

? DOES $\dot{m}_{OUT} = 0$? NO! (Where else does m go?)

" $R = 0$? "

" $3 = 0$?

$\therefore$ $V = 0$! HOW CAN THIS BE?

BACK TO MASS: $\dot{m} = \rho A V_{REL}$

$V_{REL} = 6.50$ m/s

REL TO BOUNDARY

THUS:

$$V = V_{BOUND} + V_{rel}$$

$$0 = V_{BOUND} + 6.5 \text{ m/s}$$

$$V_{BOUND} = -6.5 \text{ m/s}$$

$$= R\omega$$

$$\omega = \frac{-6.5 \text{ m/s}}{0.15 \text{ m}} = 43.3 \frac{r}{s} = \boxed{414 \frac{\text{rev}}{\text{min}}}$$

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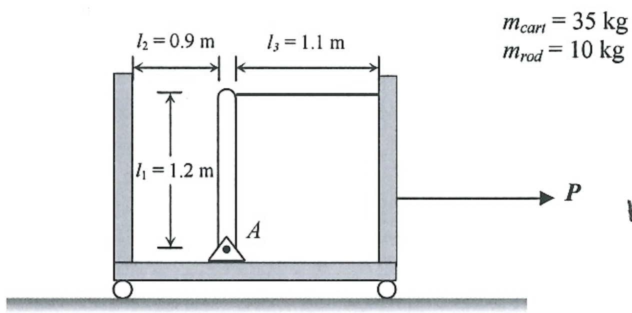
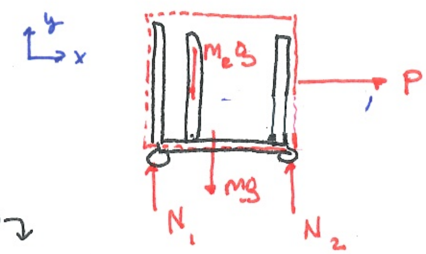
EXAMPLE B

A cord holds a 10 kg, 1.2-m tall rod vertically in a cart as shown in the figure. The rod is pinned to the cart at *A*. The cart itself has a mass of 35 kg and rolls without friction on the horizontal surface.

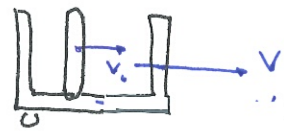
If the cord will break when the tension in it reaches 12 N, find the maximum force *P* that can be exerted on the cart without breaking the cord. You may assume that the center of mass of the rod is at the geometric center.



SYSTEM:



K.D



CoLM y DIR ↑

$$\frac{d}{dt}(P_y) = \sum F_y + \not{\Delta} - \not{\Delta}$$

$$\frac{d}{dt}(m \cdot \underbrace{V_y}_{0}) = N_1 + N_2 - m_c g - m_r g$$

$$0 = N_1 + N_2 - (m_c + m_R) g \quad (1)$$

CoLM X DIR →

$$\frac{d}{dt}(P_x) = \sum F_x + \not{\Delta} - \not{\Delta}$$

$$\frac{d}{dt}(m_c V_c + m_R V_R) = P$$

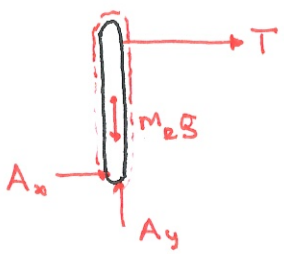
$$V_c = V_R = V \quad \text{SO}$$

$$\frac{d}{dt}\left[(m_c + m_R) V\right] = P$$

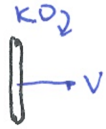
$$(m_c + m_R)\frac{dV}{dt} = P$$

$$(m_c + m_R)\overset{?}{a_x} = \overset{?}{P} \quad (2)$$

NEW SYSTEM: ROD



KD



↺ CoAM @ A

$$\frac{d}{dt}(L_A) = \sum M_A + \not{\Delta}_0 \; \not{\Delta}_0$$

$$\frac{d}{dt}\left(-\frac{l_1}{2} m_R V\right) = -1_1 \cdot T$$

$$-\frac{l_1}{2} m_R \frac{dV}{dt} = -1_1 T$$

$$-\frac{l_1}{2} m_R \overset{?}{a_x} = -1_1 T \quad (3)$$

EQN'S 3 & 2 ARE TWO EQN'S W/ 2 UNKNOWNS (a_x & P). EQN 1 DIDN'T HELP. OH WELL!...

FROM 3:

$$a_x = \frac{2T}{m_R} = \frac{2 \cdot 12\,\cancel{N}}{10\,\cancel{kg}} \left\langle \frac{\cancel{kg} \cdot m}{\cancel{N} \cdot s^2} \right\rangle = 2.4\ m/s^2$$

FROM 2:

$$P = (m_c + m_R)\, a_x = (35+10)\,kg \cdot 2.4\,\frac{m}{s^2} = \boxed{108\ N}$$

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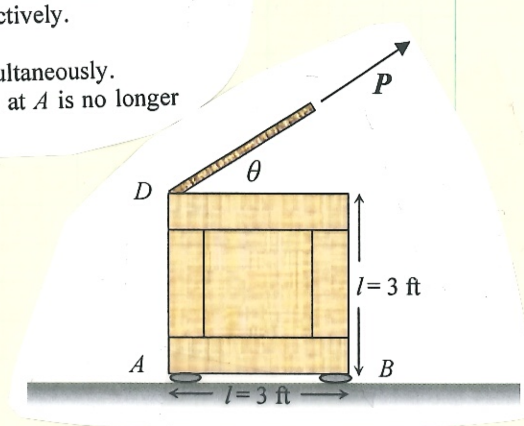
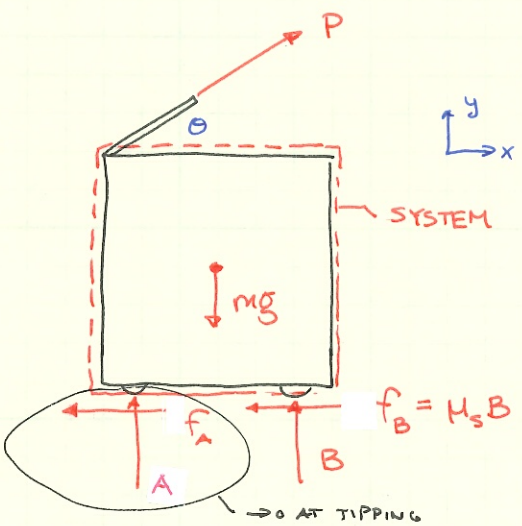
EXAMPLE C

A 100 lb crate is pulled by a rope attached to the corner D at an angle of θ as shown. The coefficients of static and kinetic friction between the crate and the surface are $\mu_S = 0.4$ and $\mu_K = 0.3$, respectively.

a) Find the angle θ and the force P for which tipping and impending sliding happen simultaneously.
b) Find the acceleration of the crate just after it starts sliding. (Hint: The normal force at A is no longer zero when sliding ensues.)





(a)

AT IMPENDING SLIPPING & TIPPING

$A \to 0 \quad f_A \to 0 \quad f_B = \mu_s B$

COLM x +→

$$\frac{d}{dt}(P_x) = \sum F_x + / - /$$

$$\frac{d}{dt}(m V_x) = \sum F_x$$

$$\frac{d}{dt}(\cancel{m} \cdot 0)_{\to 0} = -f_B + P\cos\theta$$

$$0 = -\mu_s \overset{?}{B} + \overset{?}{P}\cos\overset{?}{\theta} \quad (1)$$

COLM y DIR ↑

$$\frac{d}{dy}(P_y) = \sum F_y + / - /$$

$$\frac{d}{dt}(\cancel{m V_y})_{\to 0} = B + P\sin\theta - mg \quad (2)$$

COAM @ B (↺+)

$$\frac{d}{dt}(L_B) = \sum M_B + / - /$$

CLOSED

$$\frac{d}{dt}\left(\tfrac{1}{2} \cancel{m l}\right)_{\to 0} = -\cancel{l} P\sin\theta - \cancel{l} P\cos\theta + \frac{\cancel{l}}{2} mg \quad (3)$$

3 EQNS W/ 3 UNKNOWNS ... (P, θ, B)

DON'T BE A WIMP! SOLVE 'EM BY HAND! !!

- SUB. 1 & 2 INTO 3:

$$0 = (B - mg) - \mu_s B + \frac{1}{2} mg$$

$$B = \frac{\frac{1}{2} mg}{1 - \mu_s} = \frac{\frac{1}{2}\, 100 \text{ lbf}}{(1 - 0.4)} = 83.3 \text{ lbf}$$

- DIVIDING 2 BY 1:

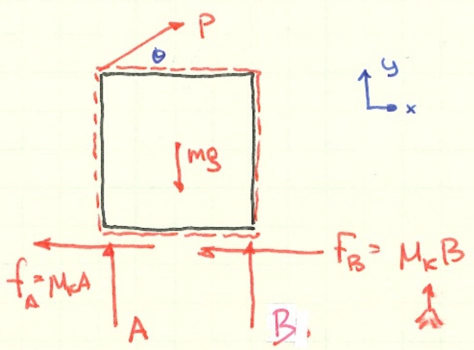
$$\tan\theta = \frac{mg - B}{\mu_s B} = \frac{100 - 83.3}{(0.4)(83.3)} = 0.5 \quad \therefore \boxed{\theta = 26.6^\circ}$$

- NOW FROM 1:

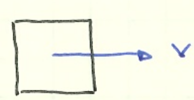
$$P = \frac{\mu_s B}{\cos\theta} = \frac{(0.4)(83.3)}{\cos(26.6)} = \boxed{37.3 \text{ lbF}}$$

(b) MOTION ENSUES!

NEW SYSTEM DIAGRAM(S)



KINETIC DIAGRAM



COLM x+→

$$\frac{d}{dt}(P_x) = \sum F_x \quad + \leftarrow \; - \angle$$

$$\frac{d}{dt}(m v_x) = P\cos\theta - f_B - f_A$$

$$m \frac{dv_x}{dt} = P\cos\theta - \mu_k B - \mu_k A$$

$$m a_x = P\cos\theta - \mu_k B - \mu_k A$$

$$a_x = \frac{P\cos\theta - \mu_k B - \mu_k A}{m} = \frac{P\cos\theta - \mu_k (A + B)}{m} \quad (1)$$

CoLM y-DIR ↑

$$\frac{d}{dt}(P_y) = \sum F_y + \cancel{} - - \cancel{}$$

$$\frac{d}{dt}(\cancel{m \cdot 0}) = A + B - mg + P\sin\theta \quad (2)$$

CoAM @ B ↻

$$\frac{d}{dt}(L_B) = \sum M_B + \cancel{} - - \cancel{}$$

$$\frac{d}{dt}\left(-\frac{\cancel{\ell}}{2} mv\right) = -\cancel{\ell}P\sin\theta - \cancel{\ell}P\cos\theta + \frac{\cancel{\ell}}{2}mg - \cancel{\ell}A$$

$$-\frac{m}{2}a_x = -P\sin\theta - P\cos\theta + \frac{mg}{2} - A \quad (3)$$

3 EQNS 3 UNKNOWNS → OK, FINE, SOLVE USING MAPLE or WHATEVER...

$$\boxed{\begin{aligned} A &= 4.13 \text{ lbf} \\ B &= 79.2 \text{ lbf} \\ a_x &= 2.69 \text{ ft/s}^2 \end{aligned}}$$