Conservation of Angular Momentum

Four Questions

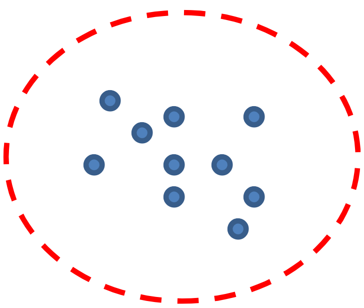
1. What is it?
   - Moment of LM?
   - For a particle, it's

It's a vector

$$\vec{L}_{point} = \vec{r} \times \vec{P} = \vec{r} \times m\vec{V}$$

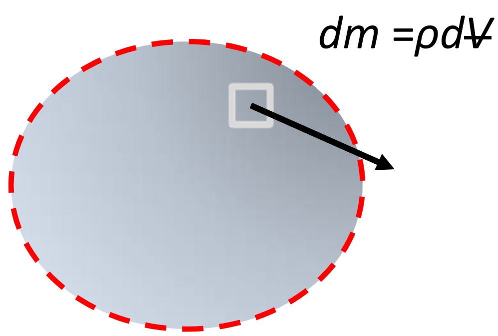
2. How can it be stored (What is $\vec{L}_{point,sys}$?)

System of particles



$$\vec{L}_{point,sys} = \sum_i \vec{r}_{point} \times m_i \vec{V}_i$$

Continuum



$$\vec{L}_{point,sys} = \int_{\forall_{sys}} \vec{r}_{point} \times \vec{V} \rho d\forall$$

If translating only $\vec{r}_{point} \times m\vec{V}_G$

If translating and rotating, expressions like

$$\bar{\bar{I}}\vec{\omega} + \vec{r}_{point} \times m\vec{V}_G$$

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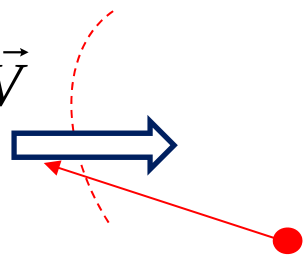
3. How can it be transported? (How does it cross system boundaries?)

Flow boundaries

$$\vec{r}_{point} \times \dot{m}\vec{V}$$

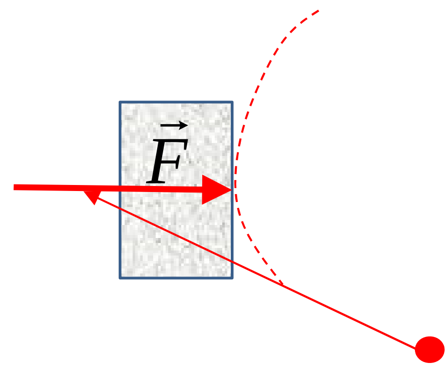


~~Rate of LM of mass *m* at boundary is~~

$$\vec{r}_{point} \times \dot{m}\vec{V}$$

Rate of AM transport due to mass flow is

Non-flow boundaries



~~Rate of LM transport at a non-flow boundary is~~

$$\vec{M}_{point} = \vec{r} \times \vec{F}$$

A moment is a rate of AM transport at a non-flow boundary!

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4. How is generated and/or consumed?

It's not! It's **conserved**.

$$\frac{d}{dt}\left(B_{sys}\right) = \dot{B}_{in} - \dot{B}_{out} + \dot{B}_{gen} - \dot{B}_{con}$$

$$\frac{d}{dt}\left(\vec{L}_{point,sys}\right) = \sum \vec{M}_{point,net} + \sum \dot{m}\left(\vec{r}_{point} \times \vec{V}_{in}\right) - \sum \dot{m}\left(\vec{r}_{point} \times \vec{V}_{out}\right)$$

Net *rate* of AM transport at non-flow boundaries

Net *rate* of AM transport at flow boundaries