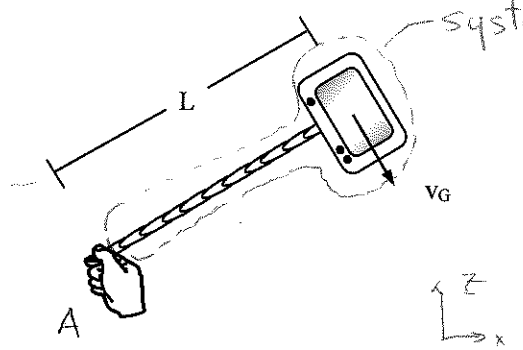
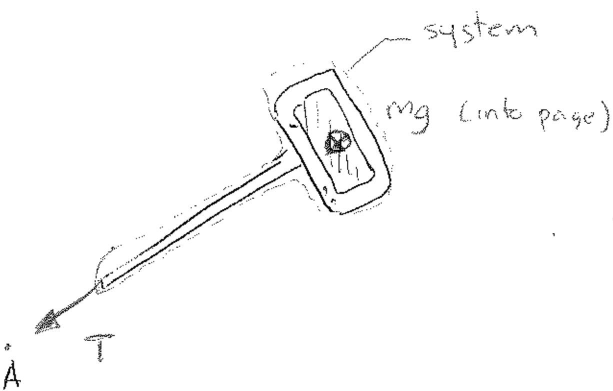
ROSE-HULMAN Institute of Technology
Sophomore Engineering Curriculum

Example

An over-worked Rose-Hulman student has decided to release some frustration by swinging an old 50-lb computer monitor around his head by a rope. Suddenly the rope unravels and its length increases from 3 feet to 10 feet. If the monitor was originally traveling at a (tangential) velocity of 18 ft/s, calculate the velocity after the rope has unraveled.





Cons. of AM
(About point A)

$$\frac{d}{dt}(\vec{\mathbb{L}}_{sys,A}) = \sum \vec{M}_A + \sum(\vec{r}\times\vec{v})\cancelto{0}{\dot{m}_{in}} - \sum(\vec{r}\times\vec{v})\cancelto{0}{\dot{m}_{out}}$$

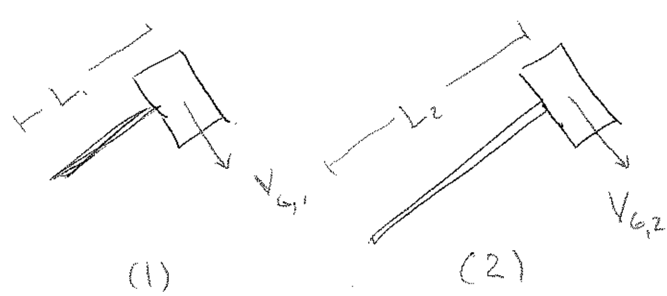
Z-COMPONENT

Closed system.

$$\frac{d}{dt}(\mathbb{L}_{sys,A,z}) = \sum \cancelto{0}{M_{A,z}}$$ (Since T goes through A)

$$\frac{d}{dt}(\mathbb{L}_{sys,A,z}) = 0$$

$$\mathbb{L}_{sys,A,z,2} - \mathbb{L}_{sys,A,z,2} = 0$$



$$L_2 \cdot m V_{G,2} - L_1 \cdot m V_{G1} = 0$$

↑
Length, not AM.

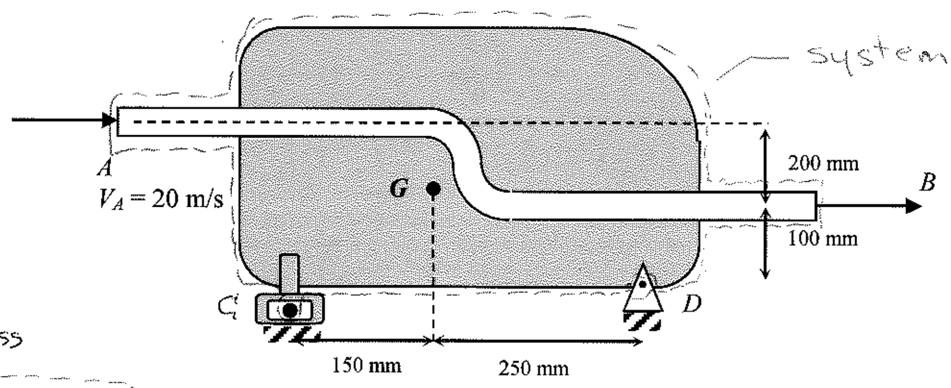
$$V_{G,2} = \frac{L_1}{L_2} \frac{\cancel{m} V_{G1}}{\cancel{m}}$$

$$= \frac{L_1}{L_2} V_{G1} = \frac{3\,\cancel{ft}}{10\,\cancel{ft}} \cdot 18\ ft/s$$

$$= \boxed{5.4\ ft/s}$$

Example

A stream of water (ρ = 1000 kg/m³) enters a constant cross-sectional area flow channel as shown in the figure. The channel area is 600 mm² and the water enters at A at a velocity of 20 m/s. The flow channel is welded to a vertical plate. The combined mass of the channel and the plate is 5 kg. Find the reactions at the pin-in-slot support C and the pin support D.



System for mass

Cons. of mass

$$\underbrace{\frac{d}{dt}(\cancel{m_{sys}})}_{\underset{s\text{-}s}{=0}} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$$

$$0 = \dot{m}_1 - \dot{m}_2$$

$$\dot{m}_2 = \dot{m}_1 = \dot{m}$$

$$\dot{m}_1 = \rho A_1 V_1$$

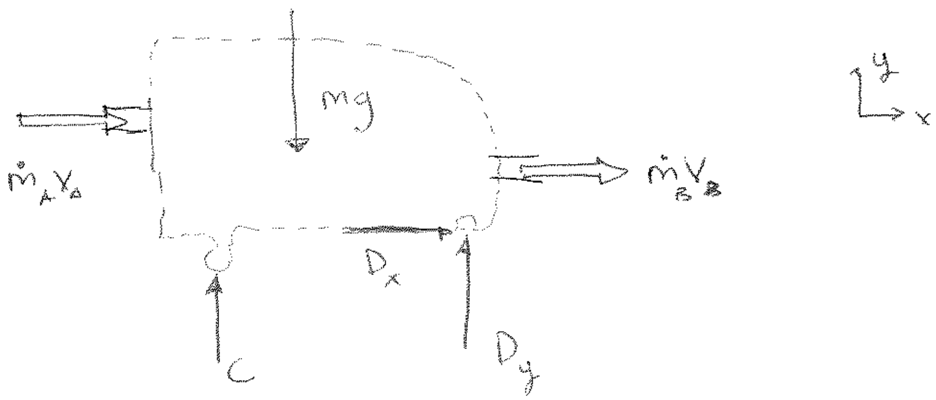
$$= 1000 \frac{kg}{m^3} \cdot 600\,mm^2 \left(\frac{1\,m^2}{10^6\,mm^2}\right) 20\frac{m}{s}$$

$$= 12.\ kg/s$$

$$\dot{m}_2 = \rho A_2 V_2$$

$$V_2 = \frac{\dot{m}_2}{\rho A_2} = \frac{\dot{m}_1}{\rho A_2} = \frac{\rho \cancel{A_1} V_1}{\rho \cancel{A_2}} = V_1 = 20\ m/s$$

System for linear momentum / angular momentum



Cons of L.M.

$$\frac{d}{dt}\cancel{(\vec{P}_{sys})} = \sum \vec{F} + \sum_{IN} \dot{m}\vec{V} - \sum_{OUT} \dot{m}\vec{V}$$

$\to 0$ S.S

x- DIR

$$0 = D_x + \cancel{\dot{m} V_A - \dot{m} V_B}$$

Because $\dot{m}_A = \dot{m}_B$ & $V_A = V_B$ NOT because it is steady-state!

$$\boxed{D_x = 0}$$

y- DIR

$$0 = \overset{?}{C} + \overset{?}{D_y} - mg \qquad (1)$$

One eqn. & two unknowns. Need another equation $\longrightarrow$ Cons. of AM.

Cons. of AM (about point D)

$$\frac{d}{dt}\cancel{(\vec{H}_{sys,o})} = \sum \vec{M}_D + \sum_{IN} \dot{m}(\vec{r}\times\vec{V}) - \sum_{OUT} \dot{m}(\vec{r}\times\vec{V})$$

$\to 0$

Z-COMPONENT

$$0 = \sum M_{D,z} + \dot{m}(\vec{r}\times\vec{V}_A)_z - \dot{m}(\vec{r}\times\vec{V}_B)_z$$

$$0 = -[(0.150 + 0.250)\,m]\,C + (0.250\,m)\,mg$$

$$+ \dot{m}\left[-(0.300\,m)(V_A)\right] - \dot{m}\left[-(0.100\,m)(V_B)\right]$$

$$C = \frac{(-0.200\,m)\,V_A \cdot \dot{m} + (0.250\,m)(mg)}{0.400\,m}$$

$$= \frac{(-0.200\,\cancel{m})\left(20\,\frac{m}{s}\right)\left(12\,\frac{kg}{s}\right) + (0.250\,\cancel{m})(5\,kg)\left(9.81\,\frac{m}{s^2}\right)}{0.400\,\cancel{m}}$$

$$= -89.3\,\frac{kg\cdot m}{s^2} = \boxed{-89.3\ N}$$

It is pointing <u>down</u>.

Eqn. (1) gives

$$D_x = mg - C = \ldots = \boxed{138\ N}$$