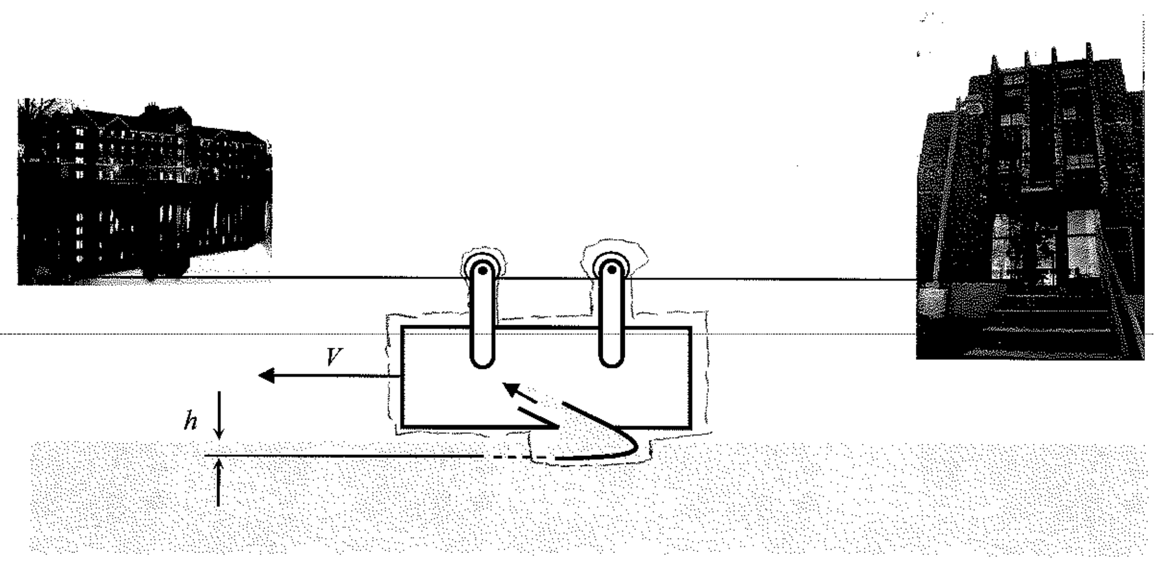
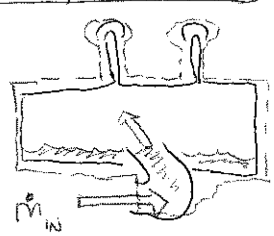
ROSE-HULMAN Institute of Technology
Sophomore Engineering Curriculum

Example

A new cable car connecting the Triplets with Percopo Hall is moving at a constant velocity $V_0 = 30$ m/s when suddenly the cable drops and the cable car starts taking on water from Speed Lake. If the car can be modeled as a car with a scoop as shown in the figure below, calculate the deceleration of the cart when it has taken on 100 kg of water. The car has a mass of 100 kg when empty, and the scoop has a width of w = 1 m, extending a depth of h = 2 cm into Speed Lake. Assume there is no friction between the cable and the rollers.



Cons. of mass:



$$\frac{d}{dt}(m_{sys}) = \sum \dot{m}_{IN} - \cancel{\sum \dot{m}_{OUT}} \rightarrow 0$$

$$m_{sys} = m_{CART} + m_{WAT}$$

$$\frac{d}{dt}(m_{CART} + m_{water}) = \dot{m}_{IN}$$

$$\frac{d}{dt}(m_{WAT}) = \dot{m}_{IN} \qquad \dot{m}_{IN} = \rho_{WAT} A_{IN} V_{IN}$$

This is *relative* to system boundary

$$\therefore V_{IN} = V$$

$$\frac{d}{dt}(m_{WAT}) = \rho_{WAT} A_{IN} V$$

$$\frac{dm_{WAT}}{dt} = (\rho_{WAT})(w \cdot h)(V) \qquad (1)$$

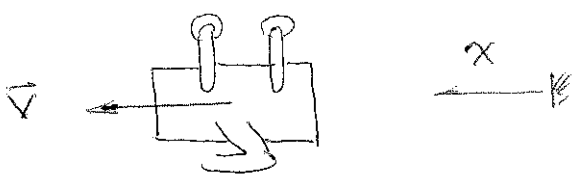
$$\vec{V}_{WAT} = \vec{V}_{CART} + \underbrace{\vec{V}_{WAT/CART}}_{\text{Needed for } \dot{m}}$$

<u>In the x direction</u>

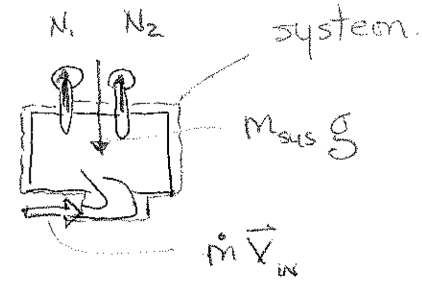
$\leftarrow + x$

$$\underbrace{\cancel{V_{WAT,x}}}_{\to 0} = -V + V_{WAT/CART} \quad \therefore \quad \vec{V}_{WAT/CART} = V$$

<u>Cons. of linear momentum</u>



↑ Diagram for L.H.S.



$$\frac{d}{dt}(\vec{P}_{sys}) = \sum \vec{F} + \sum \dot{m}_{IN} \vec{V}_{IN} - \sum \dot{m}_{OUT} \vec{V}_{OUT}$$

$$\vec{P}_{sys} = m_{sys} \vec{V}_{sys}$$

<u>Both</u> m_{sys} & $\vec{V}_{sys}$ are changing!

<u>In the x direction</u>

$$\frac{d}{dt}(m_{sys} V) = \underbrace{\cancel{\sum F_x}}_{\to 0} + \dot{m}_{IN} V_{x,IN} - \underbrace{\cancel{\dot{m}_{OUT} V_{x,OUT}}}_{\to 0}$$

$$\underbrace{V \frac{dm_{sys}}{dt}}_{= \rho_{WAT} w h V} + m_{sys} \underbrace{\frac{dV}{dt}}_{= a_x} = \dot{m}_{IN} V_{x,IN}$$

$$(V)(\rho_{WAT} w h V) + m_{sys} a_x = \dot{m}_{IN} \underbrace{V_{x,IN}}_{\text{What is } V_{x,IN}?}$$

For $\dot{m} = \rho A V$, V must be referenced to system boundary.

In consv. of linear momentum, however,

$$\frac{d}{dt}\left(m_{sys}\underbrace{\vec{V}_{sys}}\right) = \sum \vec{F} + \sum_{in} \dot{m} \underbrace{\vec{V}_{IN}} - \sum_{out} \dot{m} \underbrace{\vec{V}_{OUT}}$$

These $\vec{V}$s must all be referenced to the same inertial reference frame. (I.e, a non-accelerating frame)

Our system is accelerating! We cannot reference $V_{x,in}$ to system boundary. We have already referenced $\vec{V}_{sys}$ to the earth. We must therefore reference $V_{x,in}$ to the earth as well. What is it?

$$(V)(\rho_{WAT}\, w h V) + m_{sys}\, a_x = \dot{m}_{IN} \cancel{V_{x,IN}} \rightarrow 0!!$$

$$\therefore \; a_x = -\frac{\rho_{WAT}\, w h V^2}{m_{sys}}$$

When $m_{sys} = m_{CART} + m_{WAT} = 100\text{ kg} + 100\text{ kg}$,

$$a_x = -\frac{\left(1000\,\frac{kg}{m^3}\right)(1\,m)(0.02\,m)\left(15^2\,\frac{m^2}{s^2}\right)}{(100+100)\,kg} = \boxed{-22.5\,\frac{m}{s^2}}$$

Why did we use $V = 15$ m/s ?

Revisit x-diretion linear momentum:

$$\frac{d}{dt}(m_{sys} V_{sys,x}) = \cancelto{0}{\sum F_x} + \cancelto{0}{\sum \dot{m}_{in} V_{x,in}} - \cancelto{0}{\sum \dot{m}_{out}} V_{x,out}$$

x- momentum is NOT changing!

$$\frac{d}{dt}(m_{sys} V_{sys,x}) = 0$$

$$\int d(m_{sys} V_{sys,x}) = \int 0 \, dt$$

$$m_{sys} V_{sys,x} = CONST$$

$$\therefore \quad m_{sys,1} V_{sys,x,1} = m_{sys,2} V_{sys,x,2}$$

$$(m_{CART})(V_0) = (m_{CART} + m_{WAT}) V_2$$

$$(100 \text{ kg})(30 \tfrac{m}{s}) = (100 + 100) \text{ kg} \cdot V_2$$

$$V_2 = 15 \text{ m/s}$$