# Example C

As shown in the figure, an open tank with rigid walls and a base area $A_{base} = 0.5\ m^2$ contains a composite layer of liquid resting on solid. By cooling the base of the tank, solidification occurs at the liquid-solid interface. The liquid layer has density $\rho_L = 800\ kg/m^3$ and the solid layer has density $\rho_S = 1000\ kg/m^3$. Initially, the liquid layer and solid layer have identical thicknesses, i.e. $H = h = 150$ mm (see the figure).

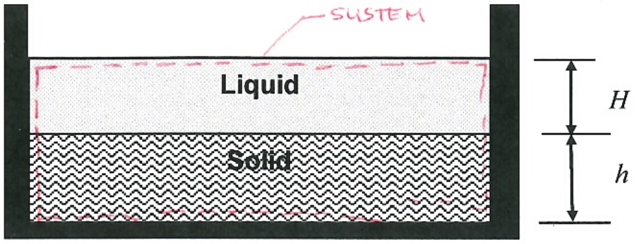
You have been asked to examine how the thicknesses of the individual layers and the composite layer are related during this process. In operation, the thickness $h$ of the solid layer increases at a constant rate $dh/dt = 5$ mm/min.

(a) Write a symbolic expression for the total mass inside the system.

(b) Apply Conservation of Mass to this closed system (the material in the tank) and answer the following questions:

- How is $dH/dt$, the rate of change of the liquid-layer thickness $H$, related to $dh/dt$?
- Does the top surface of the composite layer (liquid plus solid) layer rise or fall during this process? How fast?

(c) How would your answers to Part (b) change if the density values were reversed, i.e. $\rho_S < \rho_L$? If the density values were equal i.e. $\rho_S = \rho_L$?

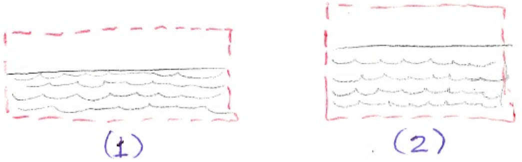


(a) $m_{sys} = m_{LIQ} + m_{SOLID}$

$= \rho_{LIQ} \forall_{LIQ} + \rho_{SOLID} \forall_{SOLID} = \rho_{LIQ} A_{base} H + \rho_{SOLID} A_{base} h$

$= A_{base} [ \rho_{LIQ} H + \rho_{SOLID} h ]$

(b) $\frac{d}{dt}(m_{sys}) = \sum \cancelto{0}{\dot{m}_{in}} - \sum \cancelto{0}{\dot{m}_{out}}$



$$\frac{d}{dt}\left[ A_{base} ( \rho_{LIQ} H + \rho_{SOLID} h ) \right] = 0$$

$$A_{base} \left[ \rho_{LIQ} \frac{dH}{dt} + \rho_{SOLID} \frac{dh}{dt} \right] = 0$$

$$\therefore \frac{dH}{dt} = - \frac{\rho_{SOLID}}{\rho_{LIQ}} \frac{dh}{dt} \quad (1)$$

$$= - \frac{1000\ kg/\cancel{m^3}}{800\ \cancel{kg/m^3}} \cdot 5 \frac{mm}{min} = -6.25 \frac{mm}{min}$$

Total height of material in tank is changing @ rate:

$$\frac{d}{dt}(H + h) = \frac{dH}{dt} + \frac{dh}{dt} = -6.25 + 500\ mm/min = -1.25\ mm/min$$

∴ The total height is falling @ 1.25 mm/min

(c) From equation (1)

$$\frac{dH}{dt} = -\frac{\rho_{SOLID}}{\rho_{LIQ}} \cdot \frac{dh}{dt}$$

If $\rho_s < \rho_L$, then $\left|\frac{dH}{dt}\right| < \left|\frac{dh}{dt}\right|$ and the level of the of the composite layer will rise; i.e., $\frac{dH}{dt} + \frac{dh}{dt} > 0$.

If $\rho_s = \rho_L$, then $\frac{dH}{dt} = -\frac{dh}{dt}$ and the height of the composite will remain constant.