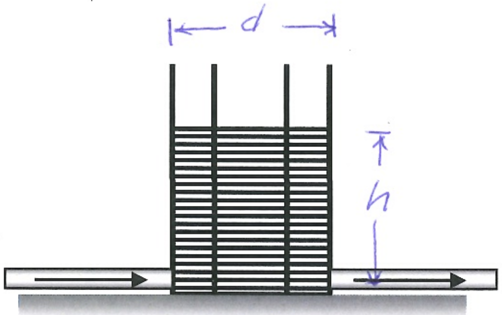
Example

On the east coast, natural gas is frequently stored in large flexible tanks that can expand and contract while maintaining a relatively constant pressure. The walls of the tank are made of a flexible accordion membrane, and the roof and floor of the tank are rigid. As the tank fills up the roof of the tank "floats" up. The vertical tank has diameter d = 50 ft and a height h which change from 5 ft to 30 ft. Under normal operating conditions, the temperature and pressure inside the tank is uniform and constant. T he density of the natural gas in the tank under these conditions is ρ = 0.12 lbm/ft^3.

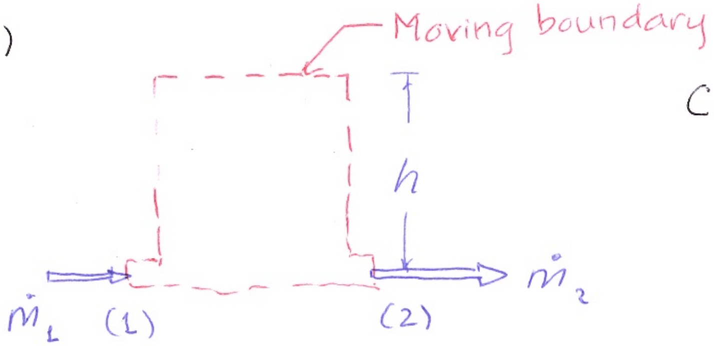
(a) Under one set of operating conditions, h = 10 ft and natural gas is flowing into the tank at 470 lbm/min and flowing out of the tank at 235 lbm/min, and natural gas has a density ρ = 0.12 lbm/ft^3.
- Is the roof of the tank moving up or down? How fast?
- How far will it move in 5 minutes?

(b) On another day, the tank operators observe a drop in the roof height h even though all tank valves are shut and they are worried there could be a leak. Or it might just be caused be the change in temperature due to the cold snap. Initially, the roof height h_1 = 10 ft when the air (and natural gas) temperature T_1 = 40°F (500°R). And after the cold snap, the temperature T_2 = 10°F (470°R).



If the tank pressure is uniform and constant and natural gas (methane) can be modeled as an ideal gas with $R_{methane}$ =96.3 ft-lbf/lbm-°R, determine the new roof height after the cold snap.

(a)



Cons. of mass →

$$\frac{d}{dt}(m_{sys}) = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$$

$$m_{sys} = \rho V_{sys} = \rho \frac{\pi d^2}{4} \cdot h$$

$$\therefore \frac{d}{dt}\left(\rho \frac{\pi d^2}{4} h\right) = \dot{m}_1 - \dot{m}_2$$

$$\rho \frac{\pi d^2}{4} \frac{dh}{dt} = \dot{m}_1 - \dot{m}_2$$

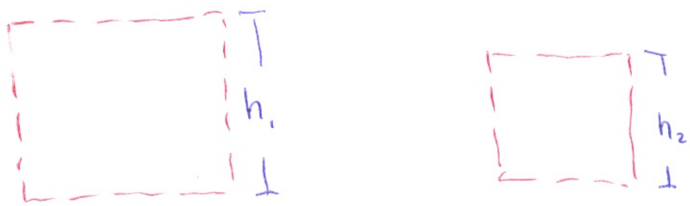
$$\frac{dh}{dt} = \frac{4(\dot{m}_1 - \dot{m}_2)}{\rho \pi d^2} = \frac{4\left(470 \frac{\cancel{lbm}}{min} - 235 \frac{lbm}{min}\right)}{0.12 \frac{\cancel{lbm}}{ft^{\cancel{3}}} \cdot \pi \cdot 50^2 \cancel{ft^2}} = 0.997 \frac{ft}{min}$$

It is moving ~ 1 ft/min up.

In 5 mins (since dh/dt = CONST)

$$\text{dist} = \frac{dh}{dt} \cdot \text{time} = (1\ \text{ft/min}) \cdot (5\ \text{min}) = \boxed{5.0\ \text{ft}}$$

(b)



(1) (2)

Cons. of mass

$$\frac{d}{dt}(m_{sys}) = \Sigma \cancel{\dot{m}_{in}}^{0} - \Sigma \cancel{\dot{m}_{out}}^{0}$$

closed.

$$m_{sys} = \rho \forall = \rho \cdot \frac{\pi d^2}{4} h = \left(\frac{P}{RT}\right) \frac{\pi d^2}{4} h \qquad (1)$$

(For an ideal gas $P = \rho R T$)

$$\frac{d}{dt}(m_{sys}) = 0$$

$$\int_{m_1}^{m_2} d\,m_{sys} = \int_{t_1}^{t_2} 0 \cdot dt$$

$$m_{sys,2} - m_{sys,1} = 0$$

$$m_{sys,2} = m_{sys,1}$$

Using (1):

$$\frac{\cancel{P_2}}{\cancel{R_{METHANE}} \cdot T_2} \frac{\cancel{\pi d^2}}{\cancel{4}} h_2 = \frac{\cancel{P_1}}{\cancel{R_{METH}}\, T_1} \frac{\cancel{\pi d^2}}{\cancel{4}} h_1$$

$$h_2 = \frac{T_2}{T_1} \cdot h_1 = \frac{470°R}{500°R} (10\ \text{ft}) = \boxed{9.4\ \text{ft}}$$