ROSE-HULMAN Institute of Technology

Foundation Coalition Sophomore Engineering Curriculum

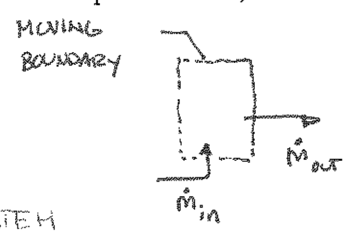
ES201 – Conservation & Accounting Principles

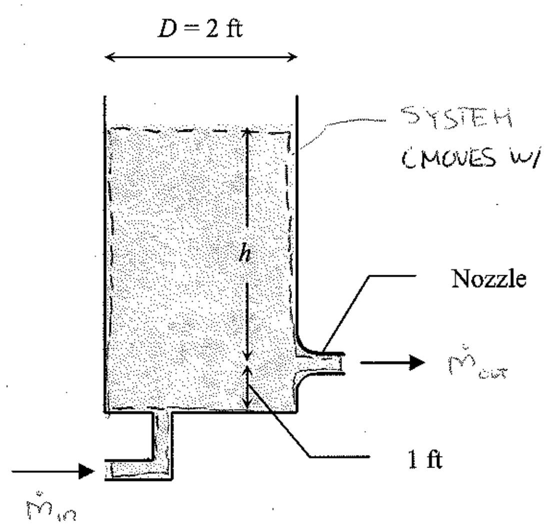
EXAMPLE

Water enters a tank through a pipe at a volumetric flow rate of 0.35 ft^3/s. The tank has a diameter of 2 ft and the axis of the tank is vertical as shown in the figure. During the initial filling process all tank outlets are closed.

When the depth of the water in the tank reaches 10 ft, a nozzle is unplugged. The nozzle has a diameter of 1 in and is smoothly rounded. Therefore, the velocity of the water exiting the nozzle is $V_{exit} = \sqrt{(2gh)}$.

a) Determine how long it takes to fill the tank in seconds.
b) Develop an equation that could be used to solve for $h(t)$ for the time period after the nozzle is opened. (Assume that $t = 0$ corresponds to a water depth of 10 ft, and that the nozzle is unplugged simultaneously.)





a) Cons of mass →

$$\underbrace{\frac{d}{dt}(m_{sys})}_{?} = \sum_{in} \dot{m} - \underbrace{\sum_{out} \dot{m}}_{0}$$

$$m_{sys} = \rho \forall_{sys} = \rho A_{TANK}(h + 1\,ft)$$

$$= \rho \frac{\pi D^2}{4}(h + 1\,ft)$$

(Note we have ignored water in inlet & exit pipes.)

SO:

$$\frac{d}{dt}\left(\rho \frac{\pi D^2}{4}(h + 1\,ft)\right) = \dot{m}_{in}$$

$$= \rho \dot{\forall}_{in}$$

(Note $\dot{m}_{in} \neq 0.35\ ft^3/s$!!!)

$$\cancel{\rho} \frac{\pi D^2}{4} \frac{dh}{dt} = \cancel{\rho} \dot{\forall}_{in}$$

Separate variables →

$$dh = \frac{4\dot{\forall}_{in}}{\pi D^2} dt$$

~~Not~~ $h = 0$ to 10 ft!

Integrate →

$$\int_{h=-1\,ft}^{9\,ft} dh = \int_{t=0}^{t_{fill}} \frac{4\dot{\forall}_{in}}{\pi D^2} dt$$

$$h\Big|_{-1ft}^{9ft} = \frac{4\dot{\forall}}{\pi D^2}\, t\Big|_{0}^{t_{fill}} \Rightarrow 9ft - (-1ft) = \frac{4\dot{\forall}}{\pi D^2}\, t_{fill}$$

$$t_{fill} = \frac{(9-(-1))\,ft\ \pi D^2}{4\dot{\forall}} = \frac{10\,ft\ \pi\ (2)^2\ ft^2}{4 \cdot 0.35\ \frac{ft^3}{s}}$$

$$= \boxed{89.8\ s}$$

b) Same system, but now $\dot{m}_{OUT} \neq 0$

Cons of mass $\rightarrow$

$$\frac{d}{dt}(m_{sys}) = \Sigma\, \dot{m}_{IN} - \Sigma\, \dot{m}_{OUT}$$

AS BEFORE

$$\frac{d}{dt}\left[\rho\, \frac{\pi D^2}{4}(h + 1ft)\right] = \dot{m}_{IN} - \overset{?}{\overline{\dot{m}_{OUT}}}$$

$$'' \qquad = \rho\, \dot{\forall}_{IN} - \rho\, V A_{OUT}$$

$$'' \qquad '' \qquad - \rho \sqrt{2gh} \cdot \frac{\pi d^2}{4}$$

$$\boxed{\cancel{\rho}\, \frac{\pi D^2}{4} \frac{dh}{dt} = \cancel{\rho}\, \dot{\forall}_{IN} - \rho \sqrt{2gh}\ \frac{\pi d^2}{4}}$$