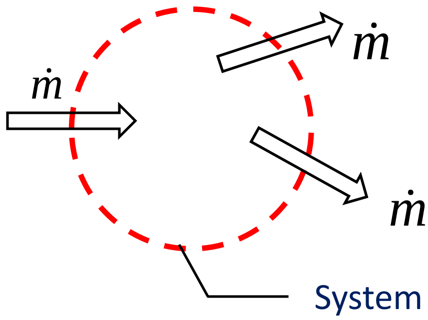
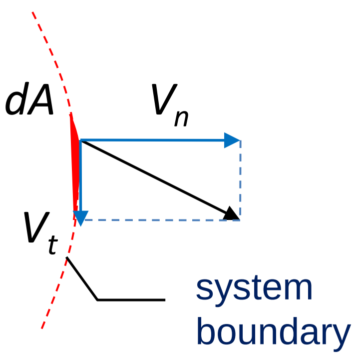
Expressions for mass flow rate

Remember



$$\frac{d}{dt}(m_{sys}) = \sum \dot{m}_{in} - \sum \dot{m}_{out}$$



How far does a mass particle travel away from the boundary in time dt?

$$V_n dt$$

What volume has crossed the boundary?

$$(V_n dt)(dA)$$

What mass has crossed the boundary?

$$\rho(V_n dt)(dA)$$

What is the mass flow rate across dA?

$$\rho(V_n)(dA)$$

Only V_n contributes to flow rate

Total mass flow rate across all A is, then…

$$\dot{m} = \int_A \rho V_n dA = \int_A \rho(\vec{V} \cdot \vec{n}) dA$$

Note: V_n is **relative** to the boundary!

Other expressions

If ρ = constant (**incompressible**)

$$\dot{m} = \rho \int_A V_n dA$$

Volumetric flow rate

What do you think $\int_A V_n dA$ is ?

$$\int_A V_n dA = \dot{\forall}$$

If V_n is also constant and/or uniform

$$\dot{\forall} = V_n A \quad \text{and} \quad \dot{m} = \rho\dot{\forall} = \rho V_n A$$

If ρ = constant but V_n is *not* uniform

$$\dot{m} = \rho V_{n,avg} A$$

where

$$V_{n,avg} \equiv \frac{\int_A V_n dA}{A}$$

Average velocity