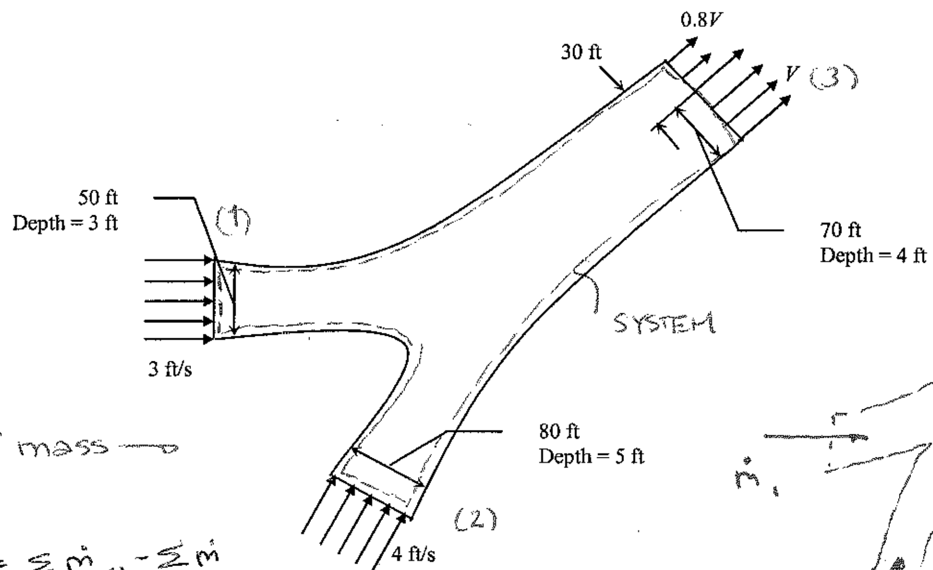
ROSE-HULMAN Institute of Technology

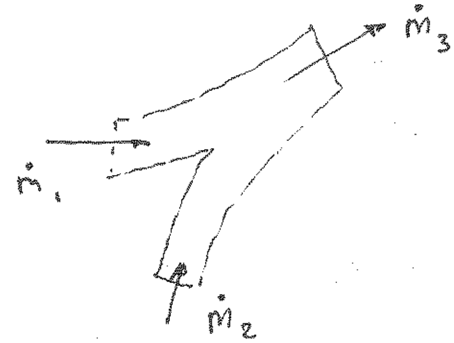
Foundation Coalition Sophomore Engineering Curriculum

ES201 – Conservation & Accounting Principles

EXAMPLE

Two rivers merge to form one river ("And sorry I could not travel both…") as shown. At a ~~junction~~ location downstream from the junction, the velocity profile is as shown in the figure. Determine the value of V.





Cons. of mass →

$$\cancel{\frac{dm_{sys}}{dt}} = \Sigma \dot m_{IN} - \Sigma \dot m_{OUT}$$

→0 SS

$$\dot m = \int \rho V dA \qquad @(1) \quad \dot m_1 = \rho A_1 V_1$$

$$@(2) \quad \dot m_2 = \rho A_2 V_2$$

$$\dot m_3 = \rho A V + \rho A (0.8V)$$

$$0 = \dot m_1 + \dot m_2 - \dot m_3$$

$$0 = \cancel{\rho}(50\times3)(3)\ ft^2 \frac{ft}{s}$$

$$+ \cancel{\rho}(80\times5)(4)\ ft^2 \frac{ft}{s}$$

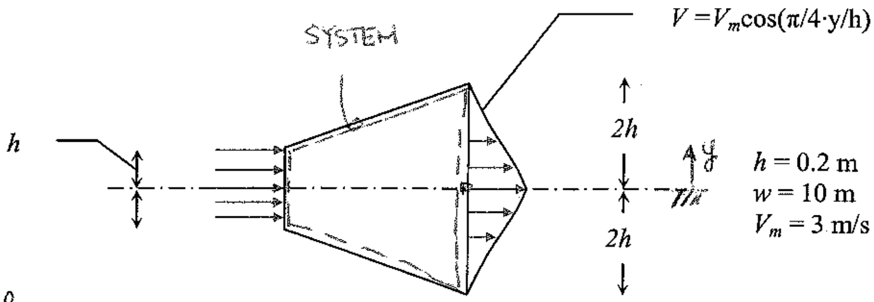
$$- [\cancel{\rho}(30\times4)(0.8V) + \cancel{\rho}(70\times4)V]$$

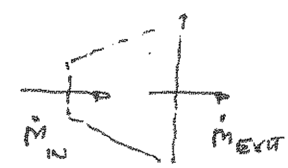
$$\boxed{V = 5.45\ ft/s}$$

EXAMPLE

Liquid mercury (ρ = 13,500 kg/m^3) enters a diverging channel as shown below. It enters the channel with a one-dimensional velocity profile and exits with the profile shown in the figure. The flow is steady, and the channel has a constant width (into the page) of w. Other known information is shown.

a) Find the mass flow rate of the mercury at the channel exit in kg/s.
b) Find the average velocity at the inlet of the channel in m/s.





a) $\dot{m}_{exit} = \int \rho V \, dA$

$$= \int_{y=-2h}^{2h} \rho \left[V_m \cos\left(\frac{\pi}{4}\cdot\frac{y}{h}\right)\right]\left[w\,dy\right] = \rho V_m w \int \cos\left(\frac{\pi}{4}\cdot\frac{y}{h}\right) dy$$

$$= \rho V_m w \frac{4h}{\pi} \sin\frac{\pi}{4}\frac{y}{h}\Bigg|_{y=-2h}^{2h} = \rho V_m w \frac{4h}{\pi}\left[\sin\frac{\pi}{2} - \sin\left(-\frac{\pi}{2}\right)\right]$$

$$= \frac{\rho V_m w 8h}{\pi} = \frac{13{,}500 \frac{kg}{m^3}\cdot 3\frac{m}{s}\cdot 10m \cdot 8 \cdot 0.2m}{\pi} = \boxed{206{,}264 \text{ kg/s}}$$

b) SYSTEM: CHANNEL

PROP: MASS

TIME: RATE

$$\cancel{\frac{dm_{sys}}{dt}} = \sum_{IN}\dot{m} - \sum_{OUT}\dot{m}$$

$$0 = \dot{m}_1 - \dot{m}_2$$

$$\dot{m}_1 = \dot{m}_2 = \underline{206{,}264 \text{ kg/s}}$$

$$= \rho A_1 V_1$$

$$V_1 = \frac{\dot{m}}{\rho A_1} = \frac{206{,}264 \text{ kg/s}}{13{,}500 \frac{kg}{m^3}\cdot(2\cdot h\cdot w)} = \boxed{3.82 \text{ m/s}}$$