ROSE-HULMAN Institute of Technology

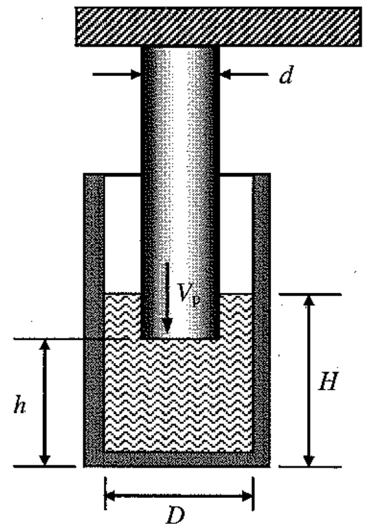
Sophomore Engineering Curriculum

Example

To help isolate a large and heavy optics table from building vibrations, the table is floated on four isolation pads like the one shown in the figure. The isolation pad consists of a cylindrical cavity of diameter D filled with a dense liquid ($S.G.$ = 13.6) and a piston with diameter d attached to the table.

Initial Conditions:

- $H = 20$ cm
- $h = 15$ cm
- $D = 18$ cm
- $d = 9$ cm
- $V_p = 1$ cm/s



You have been asked to determine how the vertical motion of the piston affects the motion of the liquid in the cavity.

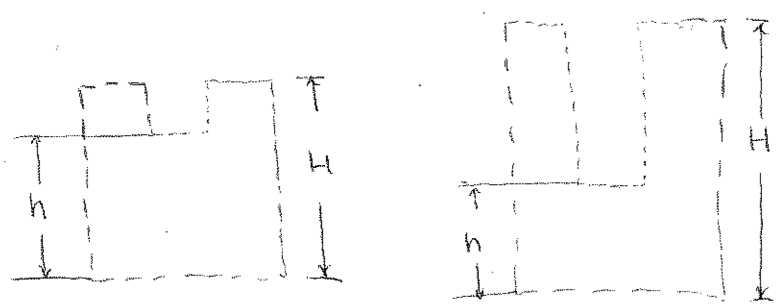
(a) Determine the mass of liquid in the cavity, in kg. **Use symbols first!**
(b) If the piston moves downward with a constant velocity V_p, what is the direction and magnitude of the motion of the free surface of the liquid: i.e. what is dH/dt in terms of V_p? **Use symbols first!**
(c) In another design, liquid can be added or removed from the base of the cavity, to maintain H at a constant value. Determine the direction and magnitude of the required mass flow rate.

$$m_{sys} = \rho \forall_{sys}$$

$$m_{sys} = \rho \left\{ \frac{\pi D^2}{4} H - \frac{\pi d^2}{4} [H - h] \right\}$$

$$= \rho_w S.G. \cdot \left\{ \frac{\pi}{4} \left[H(D^2 - d^2) + d^2 h \right] \right\}$$

(a)



(1) (2)

$$m_{sys} = 1000\frac{kg}{m^3} \cdot 13.6 \cdot \frac{\pi}{4} \left[20cm \cdot (18^2 cm^2 - 9^2 cm^2) + 9^2 cm^2 \cdot 15 cm \right] \cdot \frac{m^3}{1\times10^6 cm^3}$$

$$= 64.9 \text{ kg}$$

(b) Cons. of mass

$$\frac{d}{dt}(m_{sys}) = \sum \cancel{\dot{m}_{in}}^{\,0} - \sum \cancel{\dot{m}_{out}}^{\,0}$$

Closed system

$$\frac{d}{dt}(m_{sys}) = 0$$

Using expression for m_{sys}

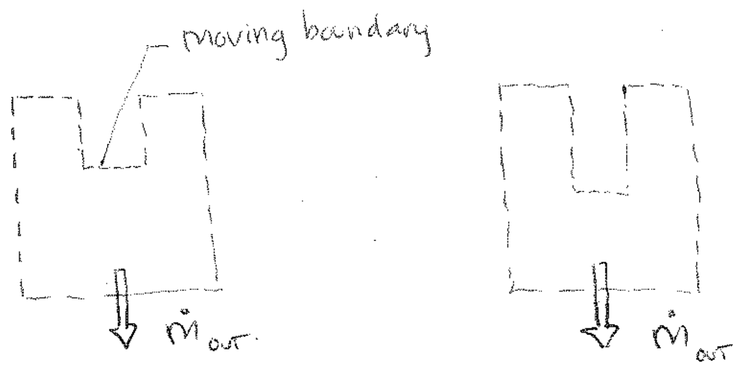
$$\frac{d}{dt}\left(\rho_w \cdot SG \cdot \frac{\pi}{4}\left[H(D^2-d^2)+d^2h\right]\right)=0$$

$$\rho_w \cdot SG \cdot \frac{\pi}{4}\cdot\left[\frac{dH}{dt}\cdot(D^2-d^2)+d^2\underbrace{\frac{dh}{dt}}_{=-V_p}\right]=0$$

$$\frac{dH}{dt}=\frac{d^2 V_p}{D^2-d^2}=\frac{9^2\,cm^2}{18^2\,cm^2-9^2\,cm^2}\cdot 1\ cm/s$$

$$=\boxed{0.333\,cm/s}$$

(c)



Cons. of mass

$$\frac{d}{dt}(m_{sys})=\sum \cancel{\dot{m}_{IN}}^{\,0}-\sum \dot{m}_{out}$$

$$\frac{d}{dt}\left(\rho_w SG\cdot\frac{\pi}{4}\cdot\left[H(D^2-d^2)+d^2h\right]\right)=-\dot{m}_{out}$$

$$\rho_w \cdot SG\cdot\frac{\pi}{4}\cdot d^2\underbrace{\frac{dh}{dt}}_{=-V_p}=-\dot{m}_{OUT}$$

$$\dot{m}_{OUT}=\rho_w\cdot SG\cdot\frac{\pi}{4}\cdot d^2\cdot V_p=1000\frac{kg}{m^3}\cdot 13.6\cdot\frac{\pi}{4}\cdot 9^2\,cm^2\cdot 1\frac{cm}{s}$$

$$=865{,}300\ \frac{kg}{\cancel{m^3}}\cdot\frac{\cancel{cm^3}}{s}\left\langle\frac{\cancel{m^3}}{10^6\,\cancel{cm^3}}\right\rangle=\boxed{0.865\ \frac{kg}{s}}$$