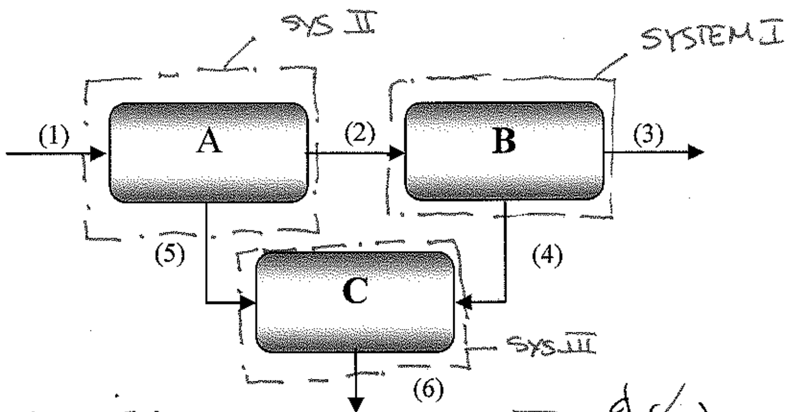
EXAMPLE

A system of three tanks is connected as shown. The flow network formed by the tanks and their piping operates at **steady-state.** Known flow rates are $\dot{m}_1 = 10$ kg/s, $\dot{m}_3 = 30$ kg/s and $\dot{m}_4 = 20$ kg/s. Find the unknown flow rates.



I. $\cancel{\frac{d}{dt}(m)_{sys}}_{\to 0} = \sum_{IN} \dot{m} - \sum_{OUT} \dot{m}$

$0 = \dot{m}_2 - \dot{m}_3 - \dot{m}_4$

$\dot{m}_2 = \dot{m}_3 + \dot{m}_4$

$= \boxed{50 \text{ kg/s}}$

II. $\cancel{\frac{d}{dt}(m)_{sys}}_{\to 0} = \sum \dot{m}_{IN} - \sum \dot{m}_{OUT}$

$0 = \dot{m}_1 - \dot{m}_2 - \dot{m}_5$

$\dot{m}_5 = \dot{m}_1 - \dot{m}_2$

$= (10 - 50)$ kg/s

$= \boxed{-40 \ \cancel{\text{kg}} \text{ kg/s}}$

$\cancel{\frac{d}{dt}(m_{sys})}_{\to 0} = \sum_{IN} \dot{m} - \sum_{OUT} \dot{m}$

$0 = \dot{m}_5 + \dot{m}_4 - \dot{m}_6$

$\dot{m}_6 = \dot{m}_4 + \dot{m}_5$

$= (\cancel{40 + 50) = 10 \text{ kg}}$/s

$20 \ -40)$ kg/s $= \boxed{-20 \text{ kg/s}}$

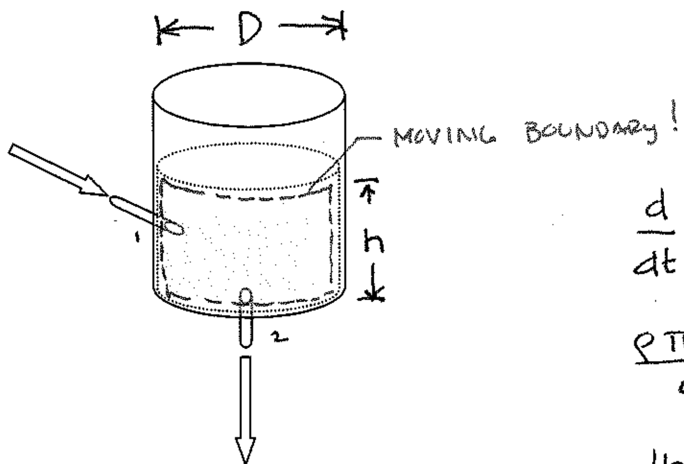
EXAMPLE

Water is being added to a storage tank at the rate of 4200 lbm/min. At the same time water flows out the bottom through a small diameter pipe at a flow rate of 4900 lbm/min. The storage tank has an inside diameter of 10 ft.

a) Find the rate at which the water level rises or falls. (ρ_{water}= 62.4 lbm/ft^3)
b) If the exit flowrate is actually 1.23 lbm/ft $\sqrt{2gh}$ where h is measured in feet, find the **steady state** height.
c) If the inlet flow is turned off for an initial water height of h_{int} = 10 ft, how long will it take to drain the tank? The exit flow is the same as given in b).

(a) $$\frac{d}{dt}(m)_{sys} = \sum_{IN} \dot{m} - \sum_{OUT} \dot{m}$$

$$= \dot{m}_1 - \dot{m}_2$$



$$m_{sys} = ? = \rho A h = \rho \frac{\pi D^2}{4} h$$

$$\frac{d}{dt}\left[\rho \frac{\pi D^2}{4} h\right] = \dot{m}_1 - \dot{m}_2$$

$$\frac{\rho \pi D^2}{4} \frac{dh}{dt} = \text{"}$$

$$\frac{dh}{dt} = \frac{(\dot{m}_1 - \dot{m}_2)4}{\rho \pi D^2}$$

$$= \frac{(4200 - 4900)\frac{lbm}{min} \cdot 4}{62.4 \frac{lbm}{ft^3} \cdot \pi \cdot 10^2 \, ft^2} = \boxed{-0.143 \frac{ft}{min}}$$

(b) SYSTEM IS NOW S-S

$$\cancel{\frac{d}{dt}(m)_{sys}}_{0} = \dot{m}_1 - \dot{m}_2 = \dot{m}_1 - C\sqrt{2gh}$$

$$\dot{m}_1 = C\sqrt{2gh}$$

$$h = \left(\frac{\dot{m}_1}{C}\right)^2 \frac{1}{2g} = \frac{4200^2 \frac{lbm^2}{min^2}}{1.23^2 \frac{lbm^2}{ft^2}} \cdot \frac{\left(1 \frac{min^2}{60^2 \, s^2}\right)}{2 \cdot 32.2 \frac{ft}{s^2}}$$

$$= \boxed{50.3 \text{ ft}}$$

$$\frac{d}{dt}(m)_{sys} = 0 - \dot{m}_2$$

$$\frac{d}{dt}\left(\rho \frac{\pi D^2}{4} h\right) = -c\sqrt{2gh}$$

$$\frac{\rho \pi D^2}{4}\frac{dh}{dt} = -c\sqrt{2g}\,\sqrt{h}$$

$$\int_{h=10'}^{0} \frac{1}{\sqrt{h}}\,dh = -\frac{c\sqrt{2g}\cdot 4}{\rho \pi D^2}\int_0^{t_F} dt$$

$$2h^{1/2}\Big|_{h_i}^{0} = (\qquad)\, t\Big|_0^{t_F}$$

$$-2h_i^{1/2} = (\qquad)\, t_F$$

$$t_f = \frac{-2h_i^{1/2}\,\rho \pi D^2}{-c\sqrt{2g}\,4}$$

$$= \frac{(2)(10)^{1/2}\,\cancel{ft^{1/2}}\;62.4\,\frac{\cancel{lbm}}{\cancel{ft^3}}\;\pi\;10^2\,\cancel{ft^2}}{1.23\,\frac{\cancel{lbm}}{\cancel{ft}}\cdot\sqrt{2\cdot 32.2}\;\frac{\cancel{ft^{1/2}}}{s}\cdot 4}$$

$$\approx \boxed{3140\ s = 52.3\ min}$$