Given:

4-4* A system of steel pipes is loaded and supported as shown in Fig. P4-4. If the normal stress in each pipe must not exceed 150 MPa, determine the cross-sectional areas required for each of the sections.

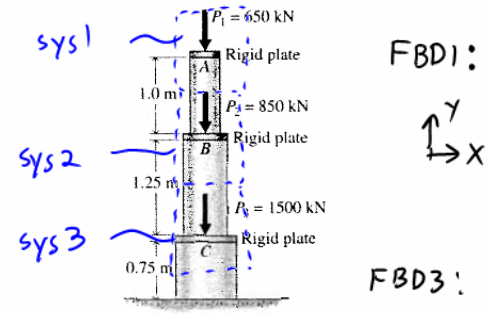


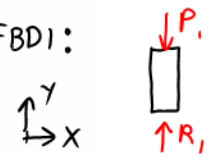
Figure P4-4

Find: A_A, A_B, A_C (cross-sectional areas)

System: (1) top of pipe A (2) bottom of A + top of B (3) bottom of B + top of C

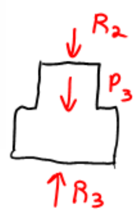
Solution:

FBD1:



FBD2:

FBD3:



Note that R_1 appears in both FBD1 + FBD2 as an equal + opposite force. Ditto for R_2 in FBD2 + FBD3.

R_1, R_2, R_3 are internal reactions.

Now use equilibrium

(sys1) $\Sigma F_y = 0$

$$R_1 - P_1 = 0 \quad \therefore \quad R_1 = P_1$$

(sys2) $\Sigma F_y = 0$

$$-R_1 - P_2 + R_2 = 0$$
$$R_2 = R_1 + P_2$$
$$R_2 = P_1 + P_2$$

(sys3) $\Sigma F_y = 0$

$$-R_2 - P_3 + R_3 = 0$$
$$R_3 = R_2 + P_3$$
$$R_3 = P_1 + P_2 + P_3$$

Now that internal reactions are found, apply stress formula to find areas

$$\sigma = \frac{F}{A}$$

(sys1) $\sigma_A = \dfrac{R_1}{A_A}$

$$A_A = \frac{R_1}{\sigma_A} = \frac{650\,kN}{150\,MPa} = 0.00433\,m^2$$
$$A_A = \boxed{4333\,mm^2}$$

(sys2) $\sigma_B = \dfrac{R_2}{A_B}$

$$A_B = \frac{R_2}{\sigma_B} = \frac{650\,kN + 850\,kN}{150\,MPa}$$
$$= 0.01\,m^2$$
$$\boxed{A_B = 10{,}000\,mm^2}$$

(sys3) $\sigma_C = \dfrac{R_3}{A_C}$

$$A_C = \frac{R_3}{\sigma_C} = \frac{650 + 850 + 1500\,kN}{150\,MPa}$$
$$= 0.02\,m^2$$
$$\boxed{A_C = 20{,}000\,mm^2}$$

Given:

4-30* A wood tension member with a 50- × 100-mm rectangular cross section will be fabricated with an inclined glued joint (45° ≤ ϕ ≤ 90°) at its midsection, as shown in Fig. P4-30. If the allowable stresses for the glue are 5 MPa in tension and 3 MPa in shear, determine
(a) The optimum angle ϕ for the joint.
(b) The maximum safe load P for the member.

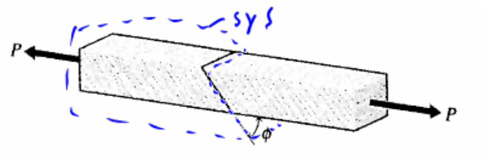


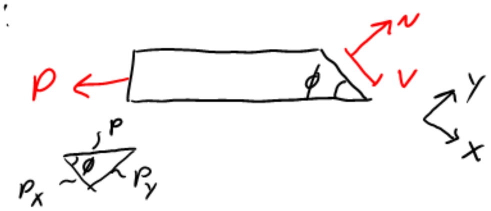
Figure P4-30

Find: optimum angle ϕ and max safe load P at that angle

System: left half at arbitrary cut ϕ

Solution: analyse at arbitrary angle

FBD:



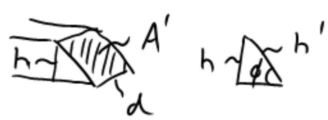
Apply equilibrium

$$\Sigma F_x = 0: \quad V - P\cos\phi = 0$$
$$V = P\cos\phi$$

$$\Sigma F_y = 0: \quad N - P\sin\phi = 0$$
$$N = P\sin\phi$$

At the optimum angle ϕ, the bar will fail in both tension + shear simultaneously!

The failure area is



$$h' = \frac{h}{\sin\phi}$$

so $A' = h'd$
$= hd/\sin\phi$

Normal stress

$$\sigma = \frac{F}{A} : \quad \sigma_{max} = \frac{N}{A'} = \frac{P\sin\phi}{hd/\sin\phi} = \frac{P\sin^2\phi}{hd} \qquad ①$$

Shear stress

$$\tau = \frac{F}{A} : \quad \tau_{max} = \frac{V}{A'} = \frac{P\cos\phi}{hd/\sin\phi} = \frac{P\sin\phi\cos\phi}{hd} \qquad ②$$

We have 2 eq'ns and 2 unknowns (ϕ, P) so solve it

Try dividing eq ① by eq ②

$$\frac{\sigma_{max}}{\tau_{max}} = \frac{\frac{P\sin^2\phi}{hd}}{\frac{P\sin\phi\cos\phi}{hd}}$$
$$= \frac{\sin\phi}{\cos\phi}$$
$$= \tan\phi$$
$$\phi = \tan^{-1}\left(\frac{\sigma_{max}}{\tau_{max}}\right)$$

$$\phi = \tan^{-1}\left(\frac{5\,MPa}{3\,MPa}\right) = \boxed{59.04°}$$

Plug ϕ back into eq ①

$$5\times10^6\,Pa = \frac{P\sin^2(59.04°)}{(50\times10^{-3}\,m)(100\times10^{-3}\,m)}$$

$$\therefore P = \boxed{34\ kN}$$

Note: other algebra methods of solving eq ① + ② also perfectly valid.