SOLUTIONS

Name

Circle section:

03 [1 pm, Thom]	04 [2 pm, Thom]
05 [1 pm, Bernal]	06 [2 pm, Bernal]

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

EM121 – Statics and Mechanics of Materials I

Exam 1

Dec 15, 2023

Rules:

- Closed book/notes exam.
- Only instructor-provided help sheet allowed.
- Calculators only. No Maple, Excel, MATLAB, etc.

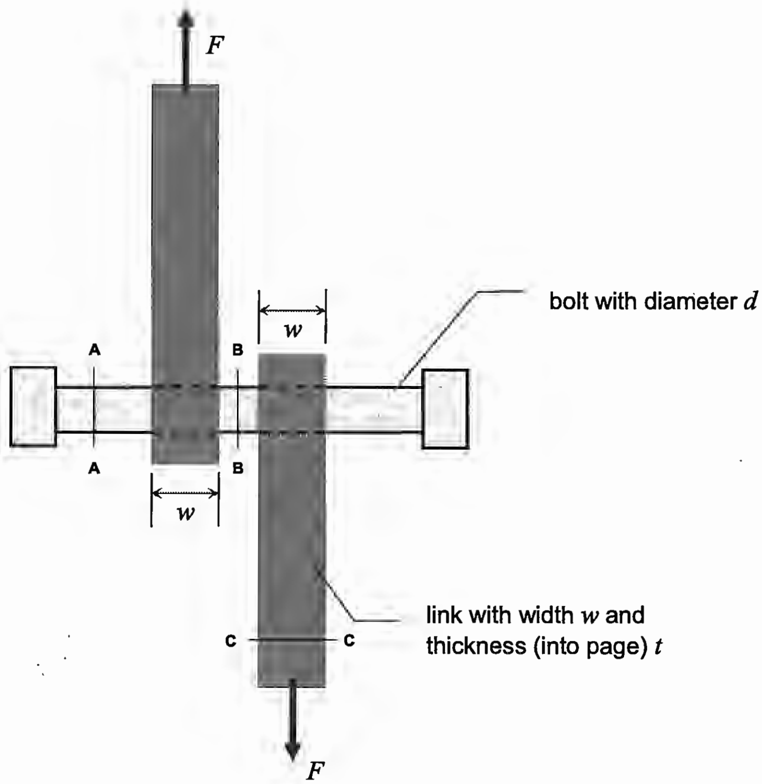
Instructions:

- Show your work to maximize the credit! But you don't have to state Given: and Find:
- For problems involving equilibrium draw a complete and correct FBD and set up the equation using vectors first.
- Helpful hint: Work in symbols for as long as you can, crunching numbers last.
- Numbers and results that appear out of nowhere are unjustified and will not receive full credit.
- If I can't follow what you're doing, I will count it wrong. Use words and phrases as needed.

Problem 1	_______/ 32
Problem 2	_______/ 40
Problem 3	_______/ 28
Total	_______/100

PROBLEM 1 [32 points]

(a) [20 pts] Two links with rectangular cross sections are connected via a bolt with a circular cross-section as shown in the figure. All weights are negligible.



i. [5pts] The shear stress at plane A-A is

- o A. 0
- o B. $\frac{F}{\pi d^2/4}$
- o C. $\frac{F}{\pi d w}$
- o D. $\frac{F}{wt}$
- o E. None of the above

ii. [5pts] The shear stress at plane B-B is

- o A. 0
- o B. $\frac{F}{\pi d^2/4}$
- o C. $\frac{F}{\pi d w}$
- o D. $\frac{F}{wt}$
- o E. None of the above

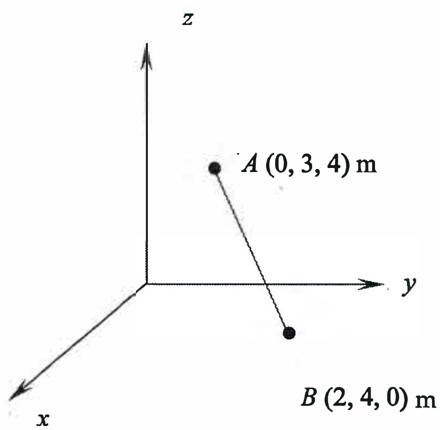
iii. [5pts] The normal stress at plane *B-B* is

- o A. 0
- o B. $\frac{F}{\pi d^2/4}$
- o C. $\frac{F}{\pi dw}$
- o D. $\frac{F}{wt}$
- o E. None of the above

iv. [5pts] The normal stress at plane *C-C* is

- o A. 0
- o B. $\frac{F}{\pi d^2/4}$
- o C. $\frac{F}{\pi dw}$
- o D. $\frac{F}{wt}$
- o E. None of the above

(b) [12 pts] A 50 N force acts in the direction going from point *A* to point *B* in the figure below. **Express the force in Cartesian vector form.**



$$\hat{e}_{AB} = \frac{\vec{r}_{AB}}{|\vec{r}_{AB}|} = \frac{(2-0)\hat{i} + (4-3)\hat{j} + (0-4)\hat{k}}{\sqrt{2^2+1^2+(-4)^2}}$$

$$= 0.436\,\hat{i} + 0.218\,\hat{j} - 0.873\,\hat{k}$$

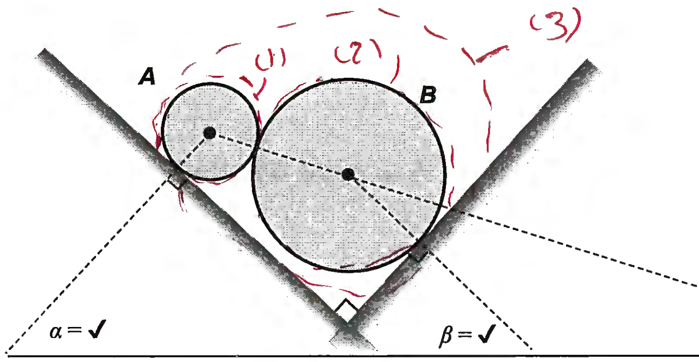
$$\vec{F} = |\vec{F}|\,\hat{e}_{AB} = (50\text{N})(0.436\hat{i} + 0.218\hat{j} - 0.873\hat{k})$$

$$= 21.8\,\hat{i} + 10.9\,\hat{j} - 43.65\,\hat{k} \text{ N}$$

PROBLEM 3 [40 points]

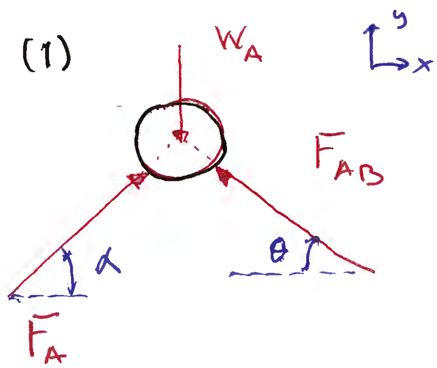
Two smooth (no friction) spheres of known weights W_A and W_B rest on smooth planes that make right angles with each other as shown in the figure. The angles α and β are both known quantities.

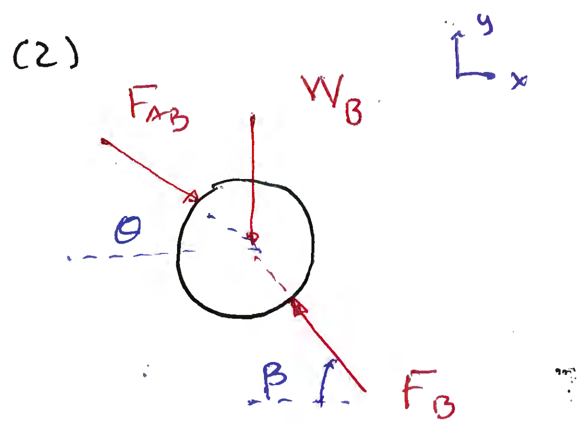
We would like to know the values of the forces the two planes exert on the spheres, the force *between* the spheres, and the angle θ that the line connecting the centers of the spheres makes with the horizontal.

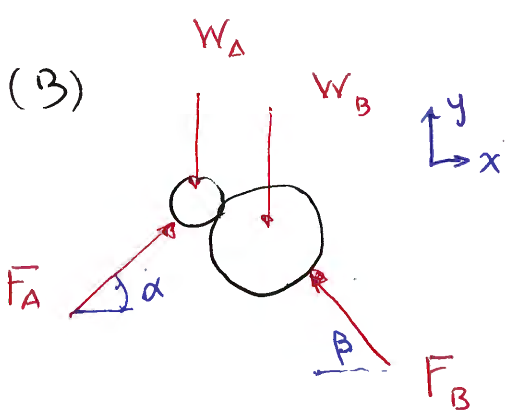


Known: W_A, W_B, α, β

(a) [20 pts] Draw **two** free body diagrams that can help you find the requested quantities.







ANY TWO OF THESE ARE OK TO USE IN THE ANALYSIS

You only need two of these three for the analysis.

(b) [20 pts] Apply equilibrium to the FBDs you drew in (a) in order to find the equations needed to solve for the unknowns. **You do not need to solve the equations** but be sure to number the equations and label which variables are unknowns.

FBD 1

$\Sigma F_x = 0 = F_A \cos\alpha - F_{AB} \cos\theta$ [1]

$\Sigma F_y = 0 = F_A \sin\alpha + F_{AB} \sin\theta - W_A$ [2]

FBD 2

$\Sigma F_x = 0 = F_{AB} \cos\theta - F_{AB} \cos\beta = 0$ [3]

$\Sigma F_y = 0 = -F_{AB} \sin\theta + F_{AB} \sin\beta - W_B$ [4]

FBD 3

$\Sigma F_x = 0 = F_A \cos\alpha - F_B \cos\beta$ [5]

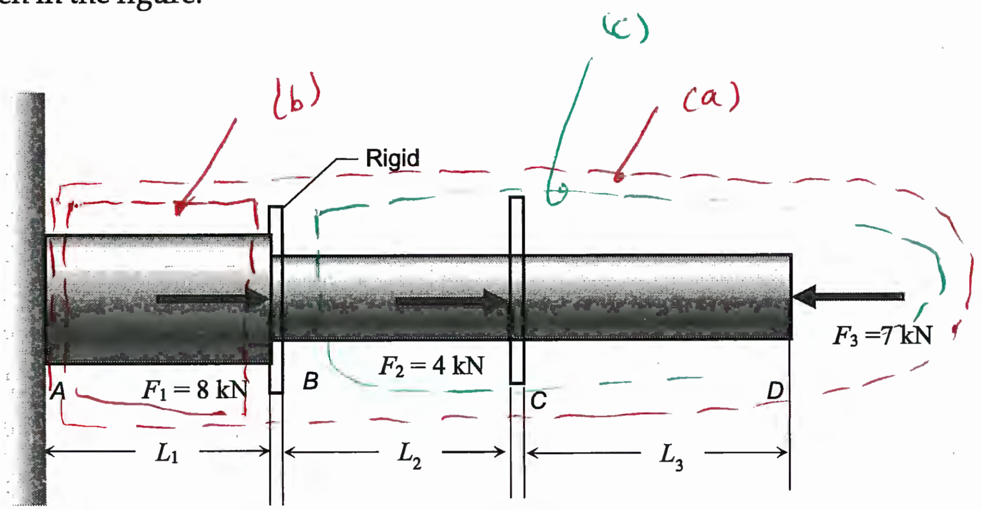
$\Sigma F_y = 0 = F_A \sin\alpha + F_B \sin\beta - W_A - W_B$ [6]

UNKNOWNS	EQUATIONS
F_A	[1], [2], [3] [4]
F_B	OR
F_{AB}	[1], [2], [5], [6]
θ	OR
	[3], [4], [5], [6]

You need a total of four equations for the four unknowns. These come from writing two equilibrium equations (one x and one y direction) for any two of the FBDs.

PROBLEM 3 [28 points]

Known loads are applied to a composite massless aluminum rod as shown in the figure. The left end is fixed to the wall and the loads are applied to rigid connectors. Material properties and dimensions are given in the figure.



$E = 69$ GPa

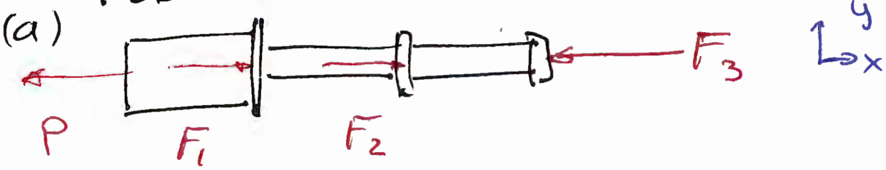
$L_1 = 200$ mm
$L_2 = 210$ mm
$L_3 = 220$ mm

Cross sectional areas

$A_{AB} = 1.6$ cm^2
$A_{BC} = 1.5$ cm^2
$A_{CD} = 1.5$ cm^2

(a) [4 pts] Find the force the wall exerts on the rod at A.
(b) [9 pts] Find the normal stress in section AB and state whether it is in tension or compression.
(c) [9 pts] Find the normal stress in section BC and state whether it is in tension or compression.

(a) FBD

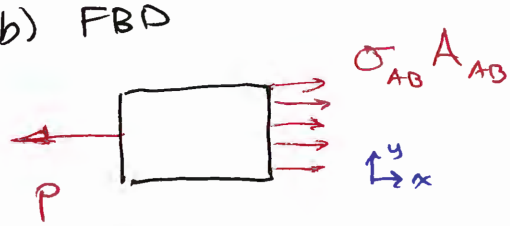


$\sum F_x = 0 \qquad -P + F_1 + F_2 - F_3 = 0$

$P = F_1 + F_2 - F_3 = (8 + 4 - 7)$ kN

$= 5$ kN ← ANS

(b) FBD



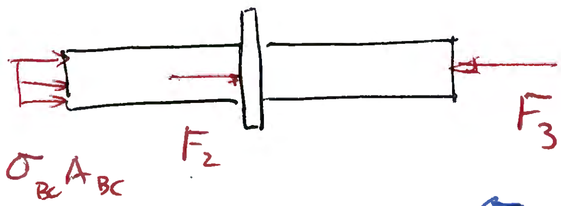
$\sum F_x = 0 = -P + \sigma_{AB} A_{AB}$

$\sigma_{AB} = \frac{P}{A_{AB}} = \frac{5\text{ kN}}{1.6\text{ cm}^2} \cdot \frac{100^2\text{ cm}^2}{\text{m}^2}$

$= 31{,}250$ kN/m^2 $= 31{,}250$ kPa

(TENSION)

(c) FBD



$\Sigma F_x = 0$

$$\sigma_{BC} A_{BC} + F_2 - F_3 = 0$$

$$\sigma_{BC} = \frac{F_3 - F_2}{A_{BC}} = \frac{(7-4)\text{ kN}}{1.5\text{ cm}^2}\left(\frac{100^2\text{ cm}^2}{\text{m}^2}\right)$$

$$= 20{,}000 \frac{\text{kN}}{\text{m}^2} = 20{,}000 \text{ kPa}$$

(COMPRESSION)

(d) [2 pts] **True** | **False** If the force at point *C* were changed from 4 kN to 10 kN, the deflection at *B* would change.

(e) [2 pts] **True** | **False** If the force at point *C* were changed from 4 kN to 10 kN, the normal stress in section *CD* would change.

(f) [2 pts] **True** | **False** If the rod temperature were increased by 5°C the deflection at *B* would change.