Course learning objectives, notes and examples

for

EM121

Statics and Mechanics of Materials I

ROSE-HULMAN
INSTITUTE OF TECHNOLOGY

Learning objectives

EM121 learning objectives

After studying the material and doing the associated activities and homework problems students of this course will be able to:

1. ☐ Define and give qualities of a **scalar**
2. ☐ Define and give qualities of a **vector**
3. ☐ Express vectors as a magnitude and direction
4. ☐ Express vectors in **Cartesian component** form
5. ☐ Use trigonometry to calculate the resultant of two or more vectors
6. ☐ Add vectors component-wise
7. ☐ Express a 3-D vector in Cartesian coordinates
8. ☐ Find the **direction cosines** of a 3-D vector
9. ☐ Find a **position vector** in 3-D space based on coordinates falling on a line
10. ☐ Find a **unit vector** from a position vector
11. ☐ Find a **force vector** from knowledge of its magnitude and a position vector with which it shares a common direction
12. ☐ Define and find the **dot product** (or **scalar product**) of two vectors.
13. ☐ Give the interpretation and one major utility of the dot product
14. ☐ Define what is meant by a **particle**
15. ☐ Determine when bodies of finite size can be treated as particles
16. ☐ Apply equilibrium equation(s) to a particle by
    - ☐ drawing a *complete and correct* **free body diagram**, and
    - ☐ writing the equilibrium equations based on the free body diagram
17. ☐ Do the above for particles in three dimensions
18. ☐ Define **stress**
    - ☐ Define and calculate **normal stress**
    - ☐ Define and calculate **shear stress**
19. ☐ Give the sign convention associated with positive and negative normal stresses
20. ☐ Apply equilibrium equation(s) to bodies subject to stress by
    - ☐ drawing a free body diagram,
    - ☐ writing the equilibrium equations based on the free body diagram, and
    - ☐ incorporating stress-force-area relationships
21. ☐ Find the normal and shear stress as a function of surface orientation for a body subject to axial stress
    - ☐ Find the plane(s) for max shear stress
    - ☐ Find the plane(s) of max normal stress
22. ☐ Explain why a body subject to only axial loading can have a shear stress
23. ☐ Explain the difference between displacements possible in rigid bodies and deformable bodies
24. ☐ Define and calculate **strain** for a member subject to axial loading
25. ☐ Graphically show how stress is related to strain; in particular define what is meant by
    - ☐ **Linearly elastic region**
    - ☐ **Elastic region**
    - ☐ **Flow region**
    - ☐ **Fracture**
    - ☐ **Yield point** and **yield stress**

26. ☐ Use **Hooke's Law** to calculate unknown stresses, strains, forces, and/or displacements for members subject to axial loads
27. ☐ Distinguish between the terms **ductile** and **brittle**, **weak** and **strong,** and **stiffer** and **less stiff**
28. ☐ Define the terms
    ☐ **thermal strain** and
    ☐ **coefficient of thermal expansion**
29. ☐ Distinguish between **thermal and mechanical strain** and know what strain to use in Hooke's Law
30. ☐ Find unknown forces, strains, stresses, and/or displacements for bodies subject to multiple axial loads, including cases where multiple materials are present
31. ☐ Determine when an axially loaded static system is **statically indeterminate**.
32. ☐ Solve for unknown forces, stresses, strains, displacements, etc. for static systems by using equilibrium and
    ☐ using **Hooke's Law** (the stress/strain relation)
    ☐ looking at the *geometry of deformation/geometric constr*aints, and
    ☐ looking at the geometry of deformation/geometric constraints when thermal effects are present
33. ☐ Identify a number of things that can cause a part to fail
34. ☐ Define and calculate a **factor of** safety (**FOS**)
35. ☐ Use a factor of safety in calculations
36. ☐ Define a **moment** in words and mathematically
37. ☐ Take the **cross product** (or **vector product**) of two vectors in 2-D and 3-D by
    ☐ Taking a formal cross product,
    ☐ using "force times a perpendicular distance", and
    ☐ breaking a force into components and calculating the contribution of each to the moment.
38. ☐ Recognize a **couple** as a pair of two oppositely-directed non-collinear forces
39. ☐ Calculate the moment due to a couple
40. ☐ Recognize that the moment due to a couple is the same about any point in space
41. ☐ Recognize when a system cannot be treated as a particle
42. ☐ "Remove" a support from a system and replace it with the appropriate reaction(s)-the force and/or moment components-by thinking about how the support restrains the motion of the system at the support location.
43. ☐ Recognize that **reaction moments** behave as couples
44. ☐ Draw complete and correct free body diagrams of systems by replacing supports with the appropriate reaction(s)
45. ☐ Draw a free body diagrams (FBDs) with all relevant forces and reaction moments on it
46. ☐ Apply the equations of equilibrium ($\Sigma \mathbf{F} = \mathbf{0}$) and $\Sigma \mathbf{M}_{point} = \mathbf{0}$) to a complete and correct FBD in order to determine unknown forces, moments, etc.
47. ☐ Identify **two-force members** in structures in order to make FBDs simpler
48. ☐ Draw several different FBDs for the same structure in order to complete equation sets for solving for unknown reactions
49. ☐ Draw FBDs and apply equilibrium to things that provide structural support (**frames**) and mechanisms that *can* move (**machines**) but aren't

50. ☐ Determine when a frame or machine in static equilibirum is **statically indeterminate**.
51. ☐ Solve for unknown forces, stresses, strains, displacements, etc. for static systems by using equilibrium and
    - ☐ using **Hooke's Law** (the stress/strain relation)
    - ☐ looking at the *geometry of deformation/geometric constr*aints, and
    - ☐ looking at the geometry of deformation/geometric constraints when thermal effects are present
52. ☐ Define what is meant by a **truss** and give the common assumptions used in **truss analysis**
53. ☐ Use the **Method of Joints** to calculate the forces in the members of a truss and indicate whether they are in **tension** or **compression**
54. ☐ Use the **Method of Sections** to calculate the forces in the members of a truss and indicate whether they are in tension or compression
55. ☐ Draw a free body diagram with all relevant forces and reaction moments on it for 3-D structures
56. ☐ Apply equations of equilibrium ($\Sigma \mathbf{F} = \mathbf{0}$) and $\Sigma \mathbf{M}_{point} = \mathbf{0}$) in vector form to 3-D FBDs in order to solve for unknown reactions, etc.
57. ☐ Recognize when **friction forces** are acting on a surface and draw them in the correct direction on an FBD
58. ☐ Recognize when friction forces can be replaced by $\mu_s N$, $\mu_k N$, or neither one
59. ☐ Apply equilibrium to systems subject to friction forces
60. ☐ Draw FBDs for systems for which **motion is impending**, either by **slipping** or **tipping**
61. ☐ Use equations of equilibrium to solve for unknown forces, moments, etc. for systems for which motion is impending
62. Explain the concepts of **center of gravity**, **center of mass,** and **centroid**, and give examples of how they are useful
63. ☐ State when center of gravity, center of mass and centroid are all the same
64. ☐ Give the criteria that a single-force acting at the center of gravity must meet in order to model a real system
65. ☐ Calculate centroids by integration
66. ☐ Find centroids of **composite shapes** from knowledge of simpler shapes that make up the composite shape
67. ☐ Define a **distributed load** and state its appropriate dimensions
68. ☐ Replace a distributed load with a single force of the appropriate magnitude at the appropriate location

Note: Terms in **bold** are key concepts or vocabulary words that you should be able to define. This is true whether or not the learning objective is explicitly to define them.

Notes and examples

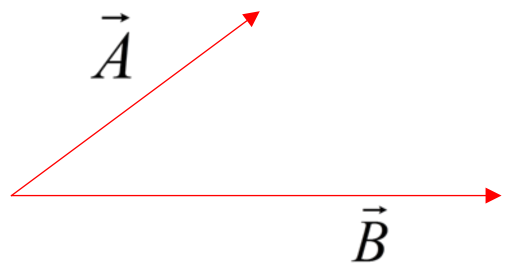
NOTES: Vector review

Scalar

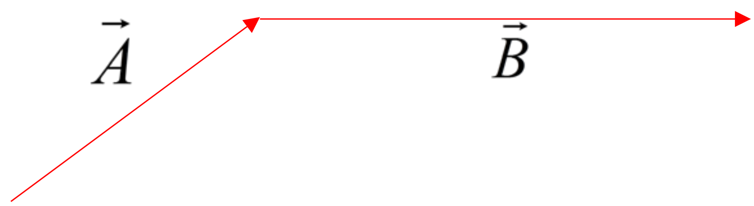
Vector

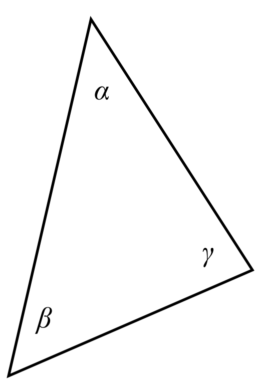
Vectors operations

Vector addition obeys _______ _________________ _______



or _________ to _______





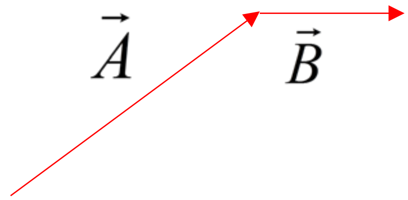
Useful tools

Law of sines

Law of cosines

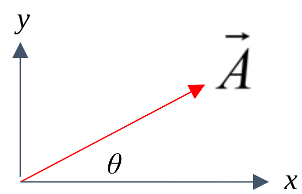
NOTES: Vector review

Adding several vectors

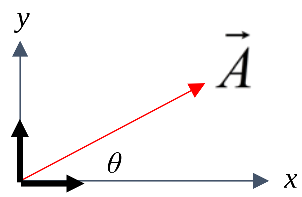


Commutative:

Associative:



Unit vectors



If |A| = 5 N and θ = 30°, write in component form

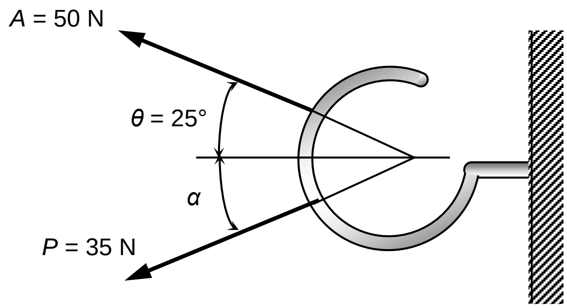
$\vec{A} =$

Vector addition works ____________________________.

Example[1]

Two forces are applied to a hook as shown. The magnitude of **P** is 35 N. Using trigonometry,

(a) find the required angle α such that the resultant **R** is horizontal, and
(b) the magnitude of **R**.
(c) Repeat (a) and (b) using vector components.

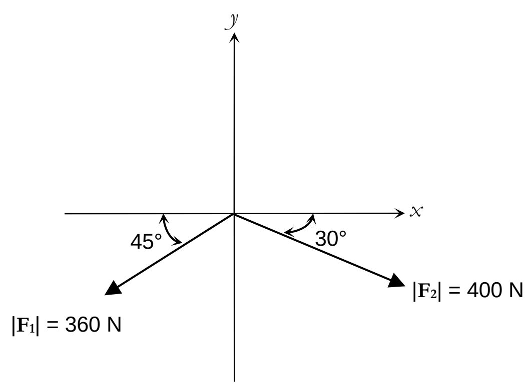


[1] From Beer and Johnston, *Vector Mechanics for Engineers, Statics*

Example

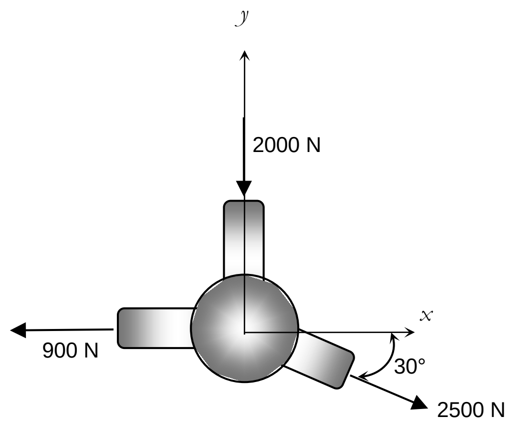
Given vectors **F_1** and **F_2** as shown, find the *resultant*. Express your answer

(a) in Cartesian vector form, and
(b) as a magnitude and an angle measured from the horizontal.



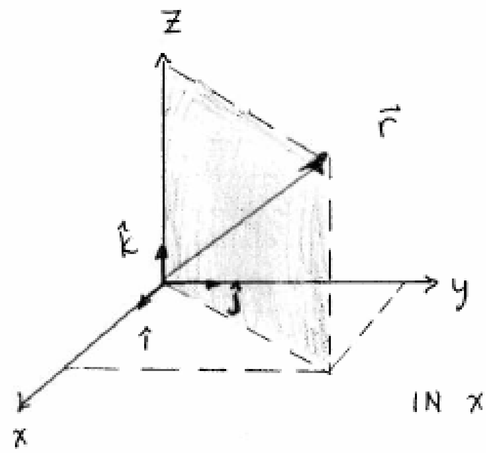
Example

Three forces act on the member as shown. Find the resultant, expressing it in Cartesian vector form.



NOTES: 3-D vectors

VECTORS.. IN... SPACE



IN COMPONENT FORM

$\vec{r} =$

MAGNITUDE?

IN x-y PLANE

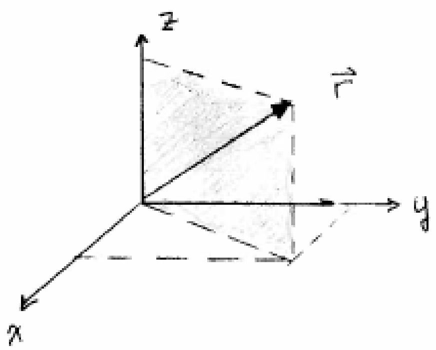
$B =$

IN SHADED PLANE

$r =$

HENCE

DIRECTION?



$\frac{r_x}{r} =$

$\frac{r_y}{r} =$

$\frac{r_z}{r} =$

$\vec{r} = \quad \hat{i} + \quad \hat{j} + \quad \hat{k}$

$= r\left[\quad \hat{i} + \quad \hat{j} + \quad \hat{k}\right]$

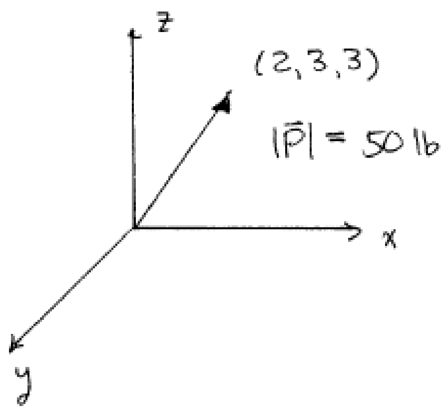
NOTES: 3-D vectors

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z \stackrel{?}{=}$$

IN DIRECTION of $\vec{F}$

$$\hat{e}_{\vec{F}} =$$

DEFINING A DIRECTION IN 3-D SPACE



1) FIND COOR. of HEAD

2) FIND COOR. of TAIL.

3) SUBTRACT ______ FROM ______. (POSITION VECTOR)

$\vec{r} =$ $\hat{i}$ + $\hat{j}$ + $\hat{k}$

4) FIND ______ ______ IN DIRECTION of $\vec{P}$

$\hat{e}_{\vec{P}} =$ ______________ =

=

$\vec{P} = P\hat{e}_p =$ $\hat{e}_p =$ $\hat{i}$ + $\hat{j}$ + $\hat{k}$

DOT PRODUCT (AKA SCALAR PRDT)

$$\vec{A} \cdot \vec{B} =$$

$\vec{A} \cdot \vec{B} =$

$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$

$= A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + \ldots$

$+ \cdots + A_y B_y \hat{j} \cdot \hat{j} + A_y B_z \hat{j} \cdot \hat{k}$

$+ \cdots + \cdots + A_z B_z \hat{k} \cdot \hat{k}$

$\hat{i} \cdot \hat{i} = ?$

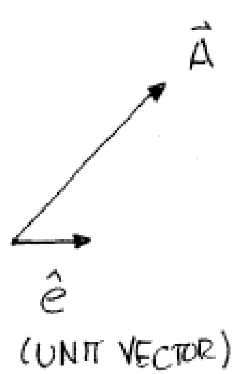
$\hat{i} \cdot \hat{j} = ?$ $\hat{j} \cdot \hat{j} = ?$

$\hat{i} \cdot \hat{k} = ?$

SO:

$$\vec{A} \cdot \vec{B} =$$

INTERPRETATION & ULTILITY:

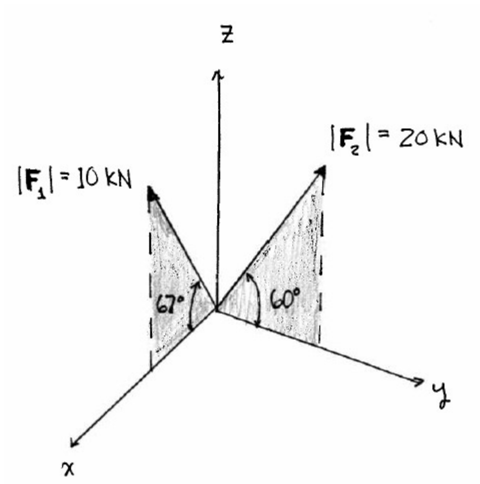


$\vec{A} \cdot \hat{e} =$

Example

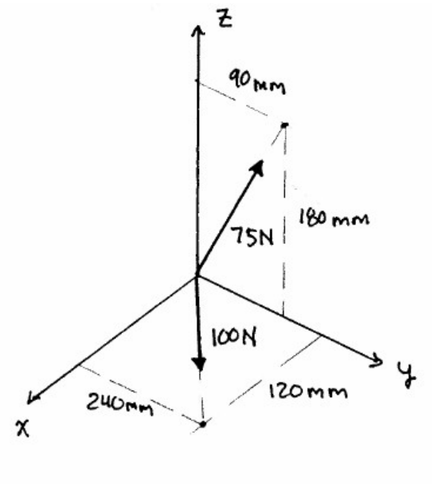
For the forces shown in the figure, find

(a) the magnitude of the resultant **R**, and
(b) the angles θ_x, θ_y, and θ_z between the line of action of the resultant and the coordinate axes.



Example

Find the magnitude and the direction of the resultant of the two forces shown.



NOTES: Particle equilibrium

EQUILIBRIUM of A PARTICLE

WHAT IS A PARTICLE?

- HAS ________ BUT NO ____________

∴ ALL __________ ACT THROUGH A ___________.

AND SO SOMETIMES WE HAVE PARTICLES

¿IS THE EARTH A PARTICLE?

DEPENDS ON __________.

FOR EQUILIBRIUM

$$\sum \vec{F} = m \frac{d\bar{v}}{dt}$$

$$\equiv$$ (DEFINES EQUILIBRIUM)

SOL'N TECHNIQUE

1. IDENTIFY __________ ⇨

   a. "____" CABLES & REPLACE W/ __________

   b. "__________" SUPPORTS & REPLACE W/ __________

   c. SHOW __________ AS DOWNWARD FORCE.

2. WRITE ____________ of ________________; IN COMPONENT FORM THESE ARE

$\sum$ =

$\sum$ =

$\sum$ =

3. SOLVE THE EQUATIONS!

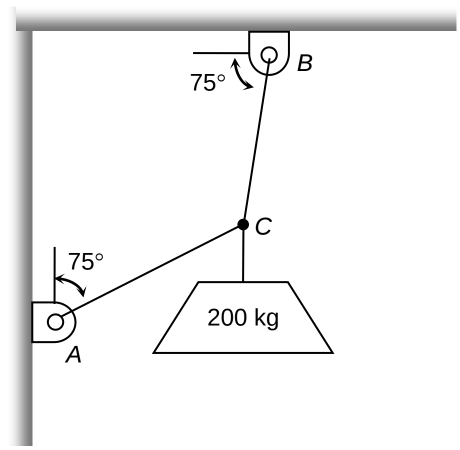
MAKE SURE

a. =

b. EQUATIONS ARE ______________ .

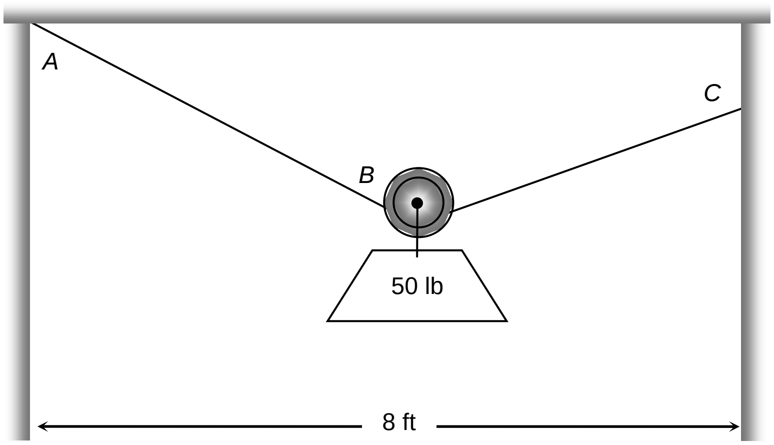
Example

A 200-kg mass is suspended from two light, inextensible cables tied together as shown. Find the tension in cable *AC* and *BC*.



Example

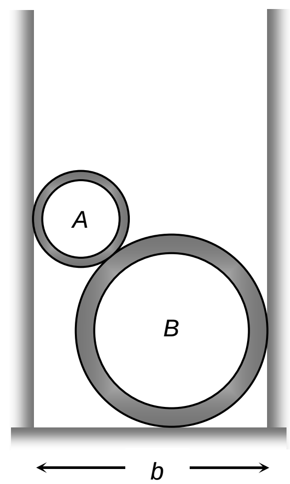
A light inextensible cable of total length 10 ft is stretched between two walls 8 ft apart. A 50-lb weight is suspended from a massless, frictionless pulley on the cable. Find the tension in the cable.



Example

Two smooth steel pipes are stacked in a box. The masses and diameters of pipe A and B are, m_A = 5 kg, m_B = 20 kg, D_A = 100 mm and D_B = 200 mm, respectively. If the distance between the walls is b = 250 mm, find

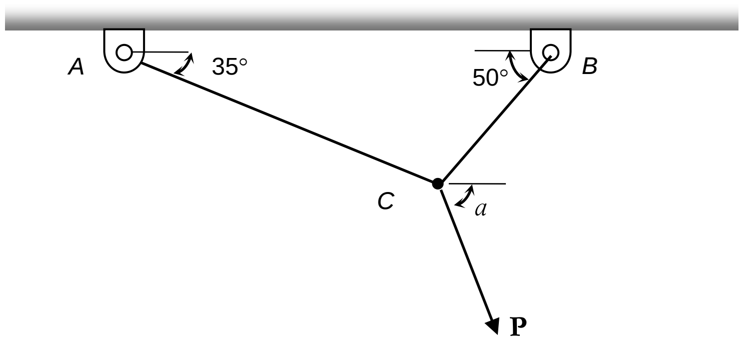
(a) the magnitude of the two forces exerted on pipe A, and
(b) the force the bottom of the box exerts on pipe B.



Example

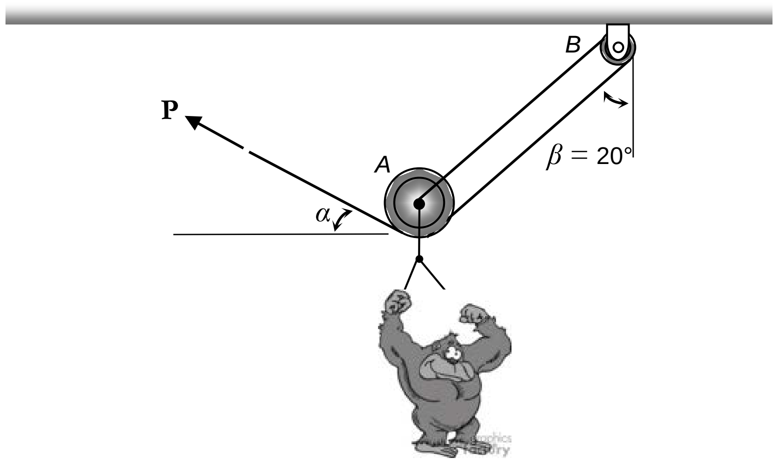
Two cables are tied together as shown. If the largest allowable tension in either cable is 800 N,

(a) what is the largest force P that can be applied at C?
(b) What is the corresponding angle α?



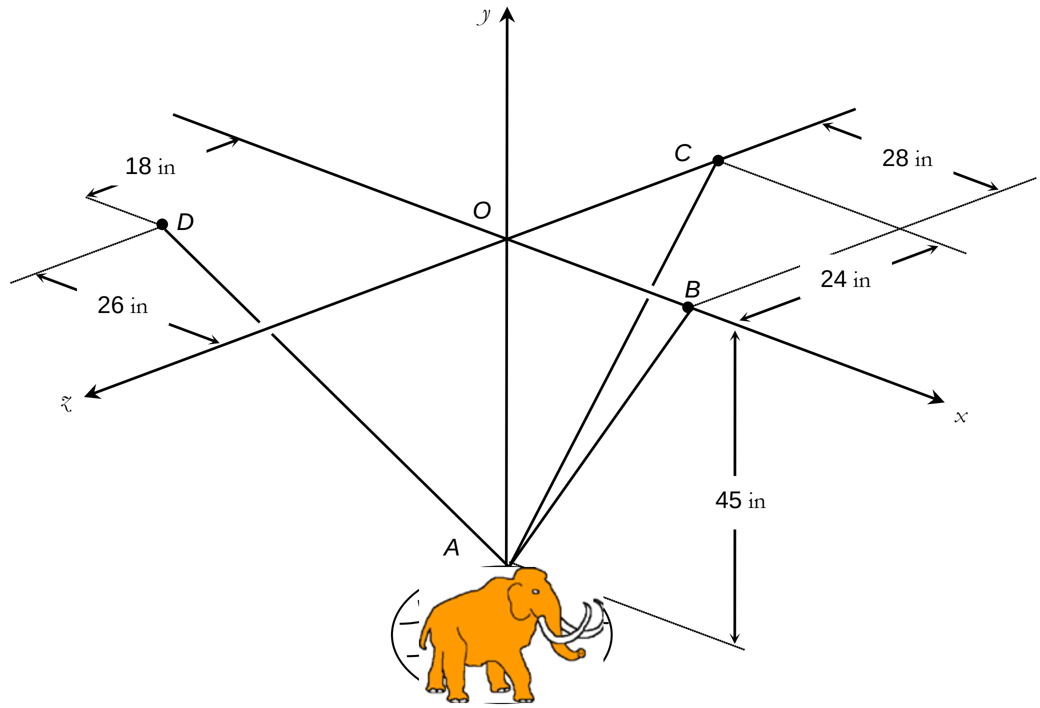
Example

A gorilla of mass 160 kg is suspended from a light, inextensible cable making use of two massless, frictionless pulleys as shown in the figure. Find the magnitude of the force **P** that must be applied to keep the gorilla stationary as well as the angle α.



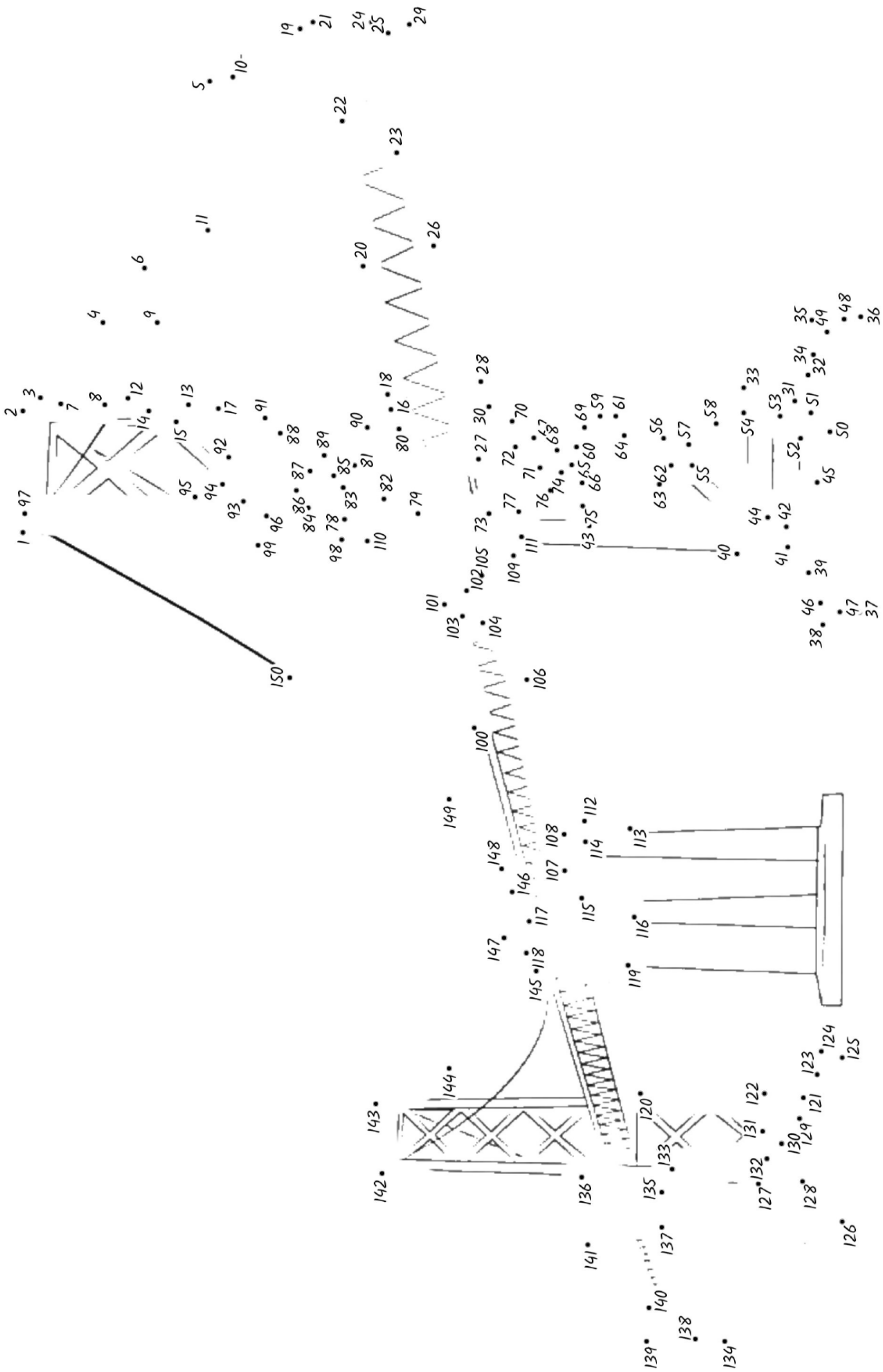
Example

A woolly mammoth has been caught up in the web of a giant alien spider. If the mammoth is suspended by three threads with the lengths/orientations shown in the figure, find the weight of the mammoth. The tension in thread AB is 1378 lb.



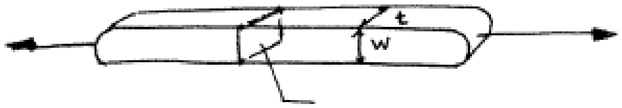
DOT TO DOT: Statics edition

Connect the dots to reveal the picture.



STRESS, STRAIN & OTHER Ss

NORMAL STRESS IN MEMBERS



___ ___ ___

___ ___ ___

A =

σ = ——— ≡

UNITS:

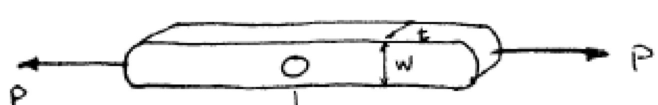
SIGN CONVENTION

( )   ( )   ( )

___   ___   ___

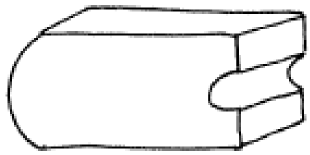
THINGS BREAK WHEN ______ IS TOO HIGH.

¿WHAT IF THERE IS A HOLE IN BAR?



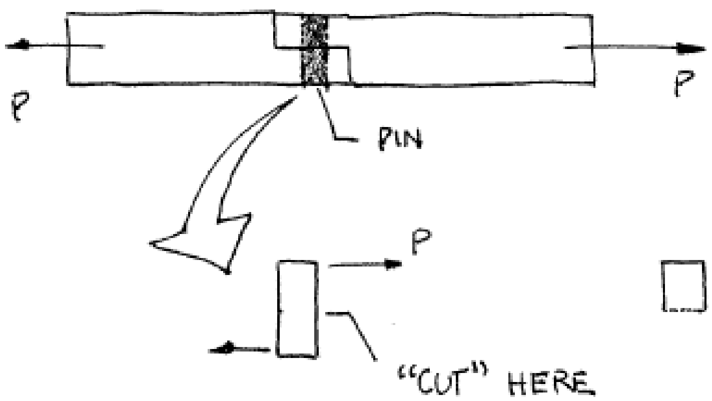
"CUT" HERE

DRAW F.B.D. FOR SECTION LEFT of "CUT".



$\sigma_{AVG} =$

SHEAR STRESS IN CONNECTIONS

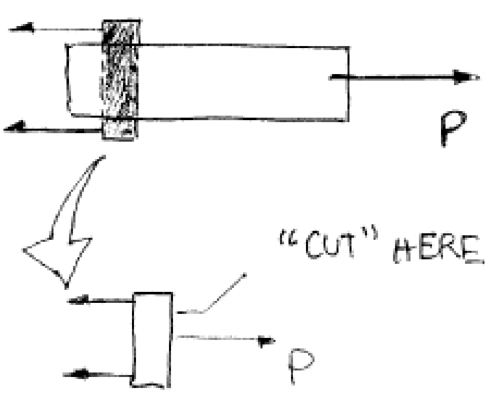


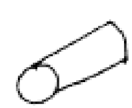
ISO VIEW

SINGLE SHEAR

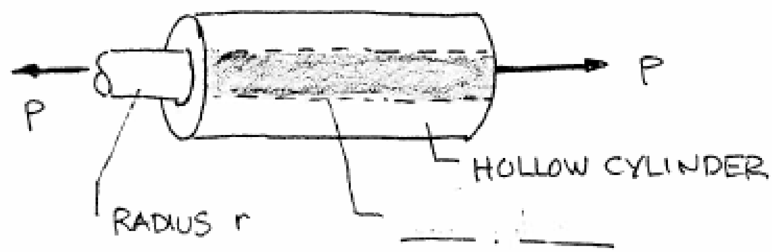
$\tau_{AV} = \frac{\quad}{\quad} \equiv$

DOUBLE SHEAR



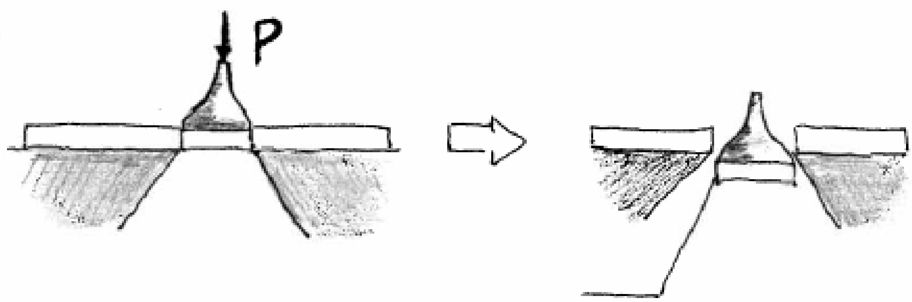


ISO-VIEW



F.B.D. of ROD:

PUNCHING SHEAR STRESS



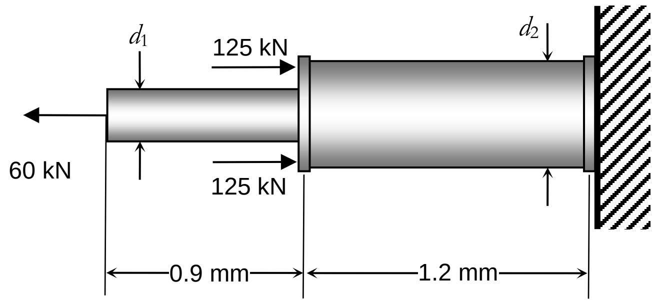
FBD PUNCHED DISK

SUMMARY

- STRESS IS __________ PER __________
  1. __________ ⇒
  2. __________ ⇒
- DRAW FBD!! NOW W/ __________ & __________.

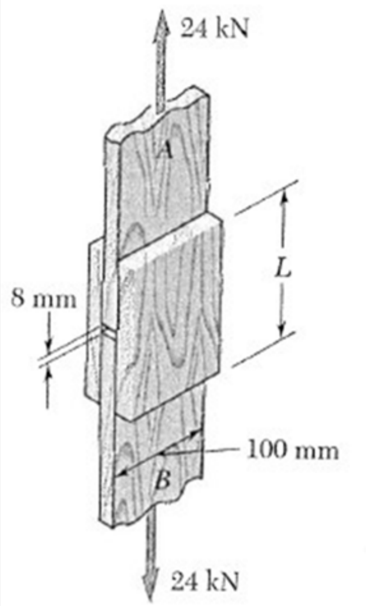
Example

The figure shows two solid cylindrical rods welded together at B. The average normal stress in either rod is not to exceed 150 MPa. For the loading shown, find the smallest allowable diameters for each rod.



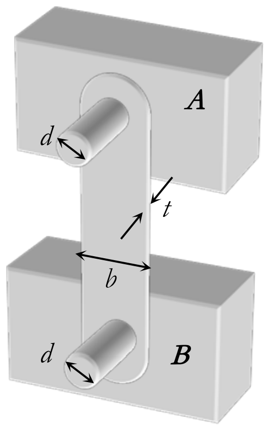
Example

Two pieces of wood are to be joined via gluing splice plates to them as shown in the figure. The clearance between the members is to be 8 mm. If the maximum allowable stress in the glue is not to exceed 800 kPa, what is the smallest allowable length, *L*?



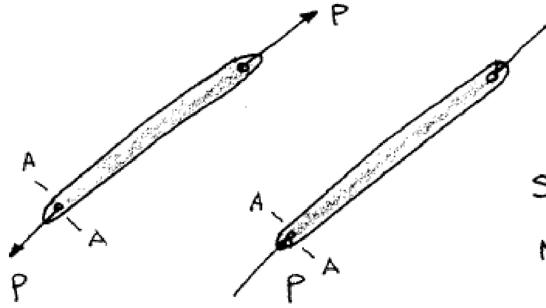
Example

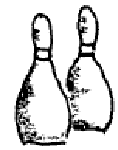
Link *AB* is used to support the end of a beam. The dimensions of the link are b = 2″ and t = ¼′′. The average **normal stress** in the link is -20 ksi and the average **shearing stress** in the two pins is 12 ksi. What is the diameter of the two pins?



NOTES: **Stress in a link**

TENSION & COMPRESSION IN A LINK W/ PINS





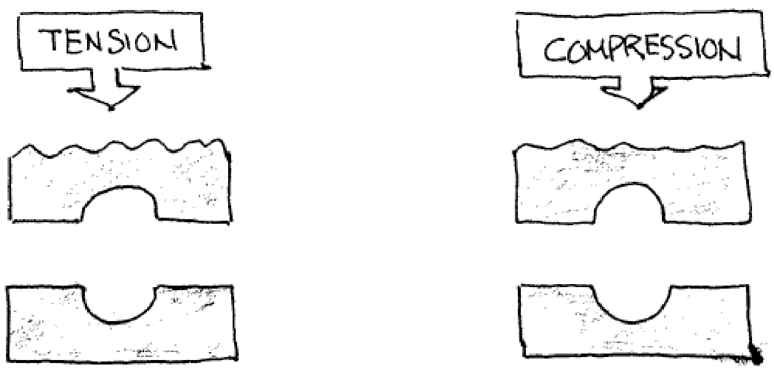
http://www.wikipedia.org

STRESS @ A-A IN TENSION ________

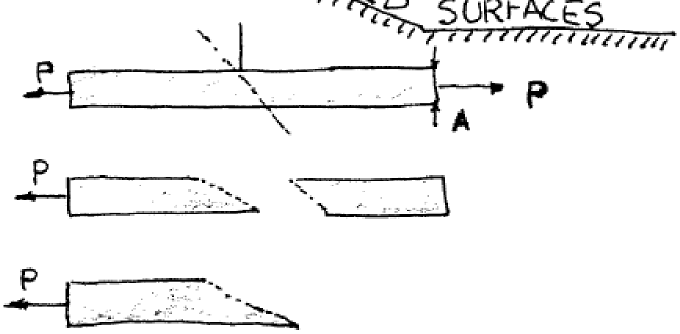
NEGATIVE of STRESS @ A-A IN COMPRESSION

IN ________ IN ________

DRAW THE FREE BODY DIAGRAMS:



FORCES ON INCLINED SURFACES



USE EQUILIBRIUM TO FIND N & V.

(HINT: TILT YOUR AXES)

NOTES: Stress in a link

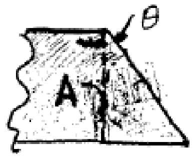
$\sum F_{x'} = 0$

$\sum F_{y'} = 0$

$N =$

$V =$

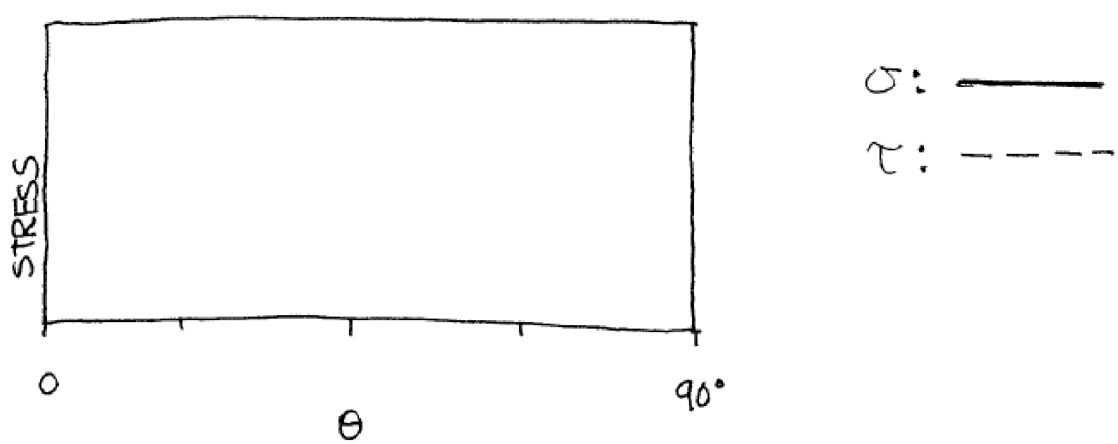
NOW CALCULATE NORMAL & SHEAR STRESSES: (HINT: THINK ABOUT WHAT AREA TO USE.)



$\sigma = \dfrac{N}{\quad} =$

$\tau = \dfrac{V}{\quad} =$

PLOT NORMAL & SHEAR STRESS AS FUNCTIONS OF θ:



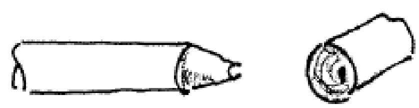
WHERE IS $\tau = \tau_{MAX}$?

NOTES:

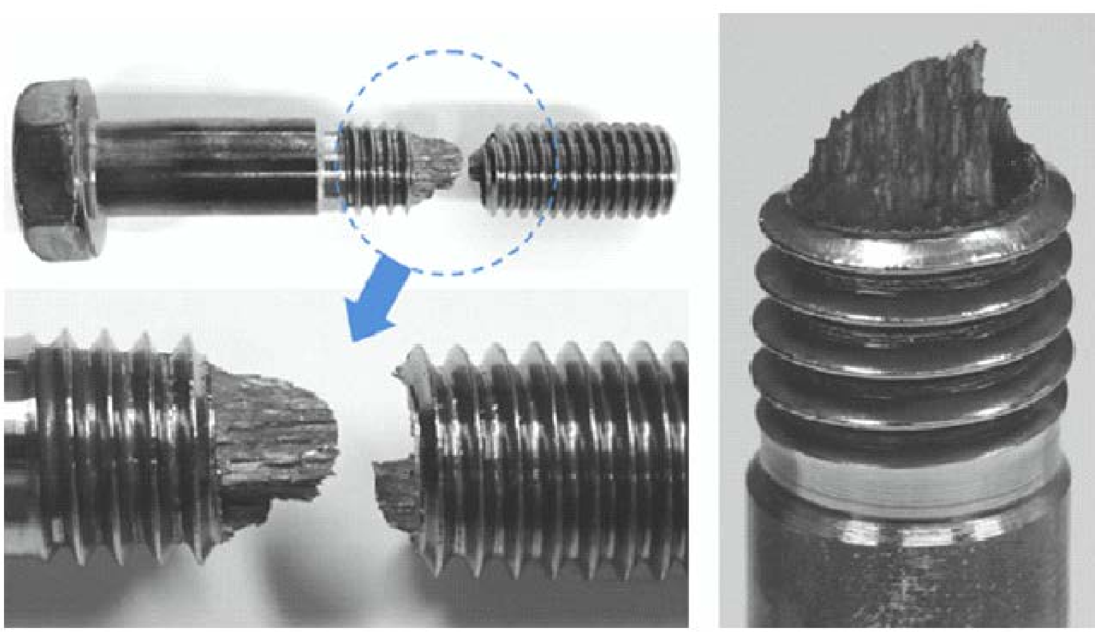
1. FOR AN ____________ LOAD, WE CAN STILL HAVE __________ ___________.

2. FAILURE MODE FOR A SPECIMEN IN TENSION IS OFTEN DESCRIBED AS ____________.

   FAILURE PLANE IS ______° FROM LINE of ACTION of FORCE.



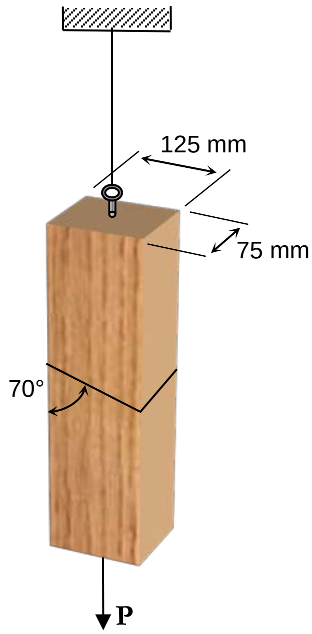
"______ & ________" FAILURE.



From Oct. 5, 2009 press release, National Institute for Materials Science (Japan)

Example

A 6-kN load **P** is applied to two wooden members with a rectangular cross section. The two members are joined by a glued scarf splice as shown in the figure. Find the normal and shearing stresses in the splice.

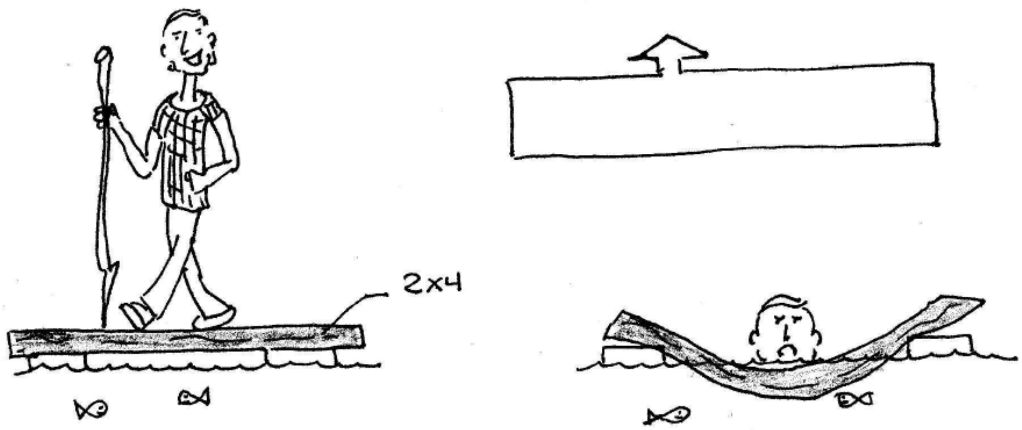


NOTES: Elasticity and thermal strain

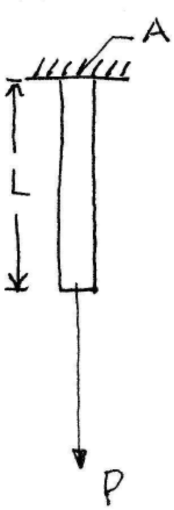
APPLY A LOAD TO A STRUCTURE AND IT CAN CAUSE IT TO ________ .

LOAD → ________ , ________ BUT NO ___ ___ ________ OR ________ .

→ ___ ___ ________ OR ________ .



DEFORMATION IN AXIAL LOADING



PULLING MAKES BAR LONGER.

DEFORMATION IS ______ ___ ________ .

= ☐

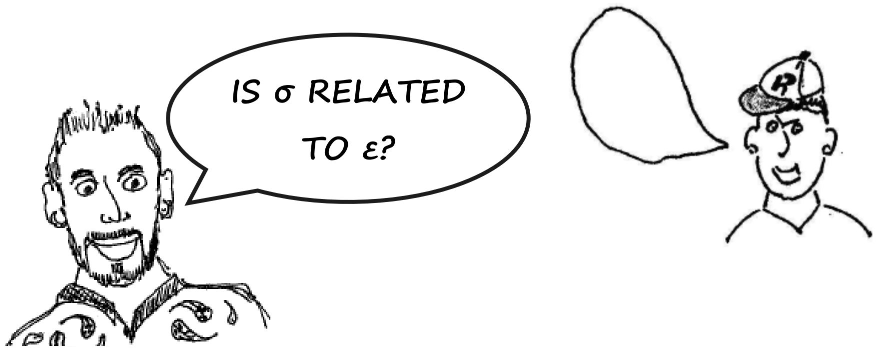
CHANGE IN LENGTH ______ ________ ________

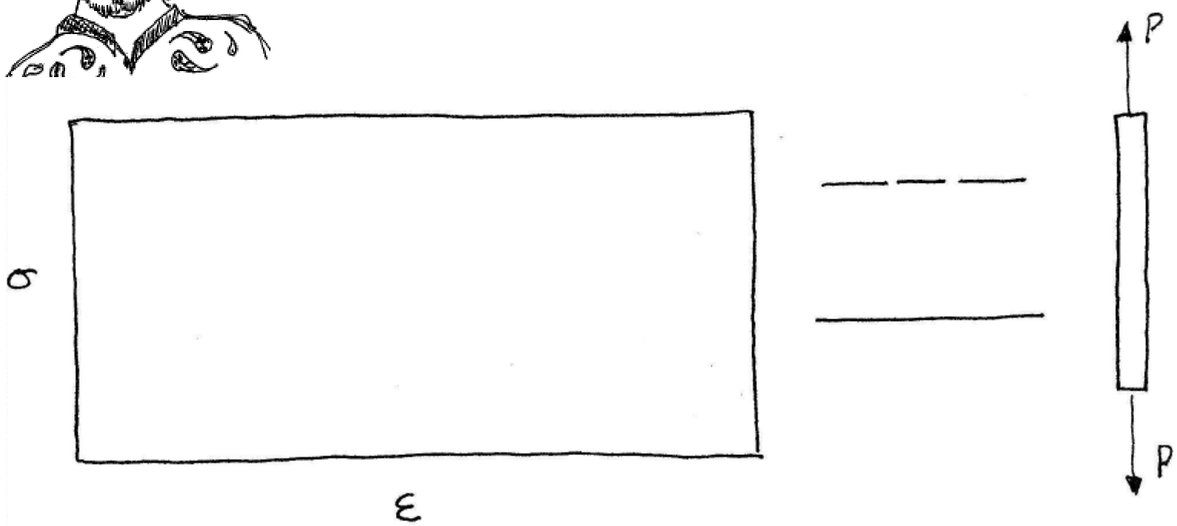
IS

≡ = ___

NOTES: **Elasticity and thermal strain**

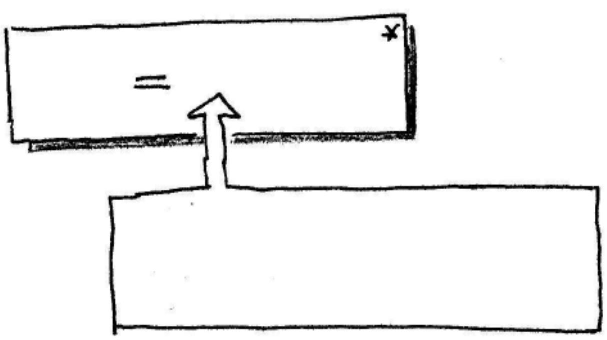
UNITS & DIMENSIONS of ε?





WHEN WE ARE IN THE LINEAR REGION, MATERIAL IS

____________ ____________ &



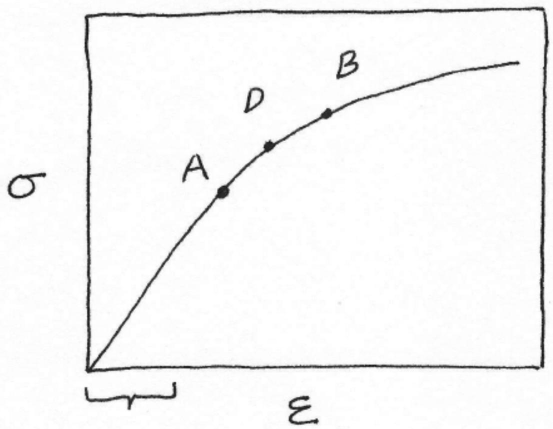
* THIS IS AN EXAMPLE of A CONSTITUTIVE RELATION, SOMETIMES CALLED "LAWS" DESPITE A LACK of UNIVERSALITY. OTHERS INCLUDE OHM'S LAW & THE IDEAL GAS EQUATION.

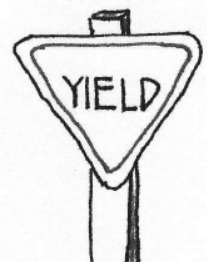
STAYING IN THE ELASTIC REGION

FOR SOME MATERIALS ⇒ ________ ________

FOR MOST MATERIALS

________ ________ ⇒ ________ ________





A:

D:

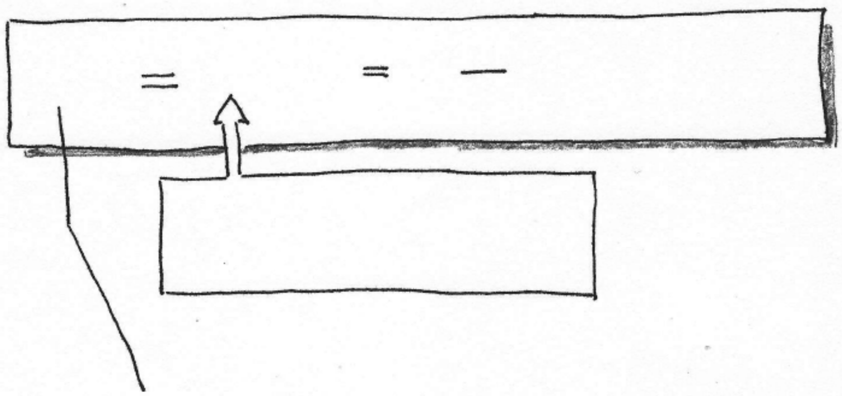
C:

NOTES: Elasticity and thermal strain

THERMAL STRAIN

ONE WAY TO CAUSE DEFORMATION IS APPLYING STRESS.

ANOTHER WAY IS TO ____________ ____________.



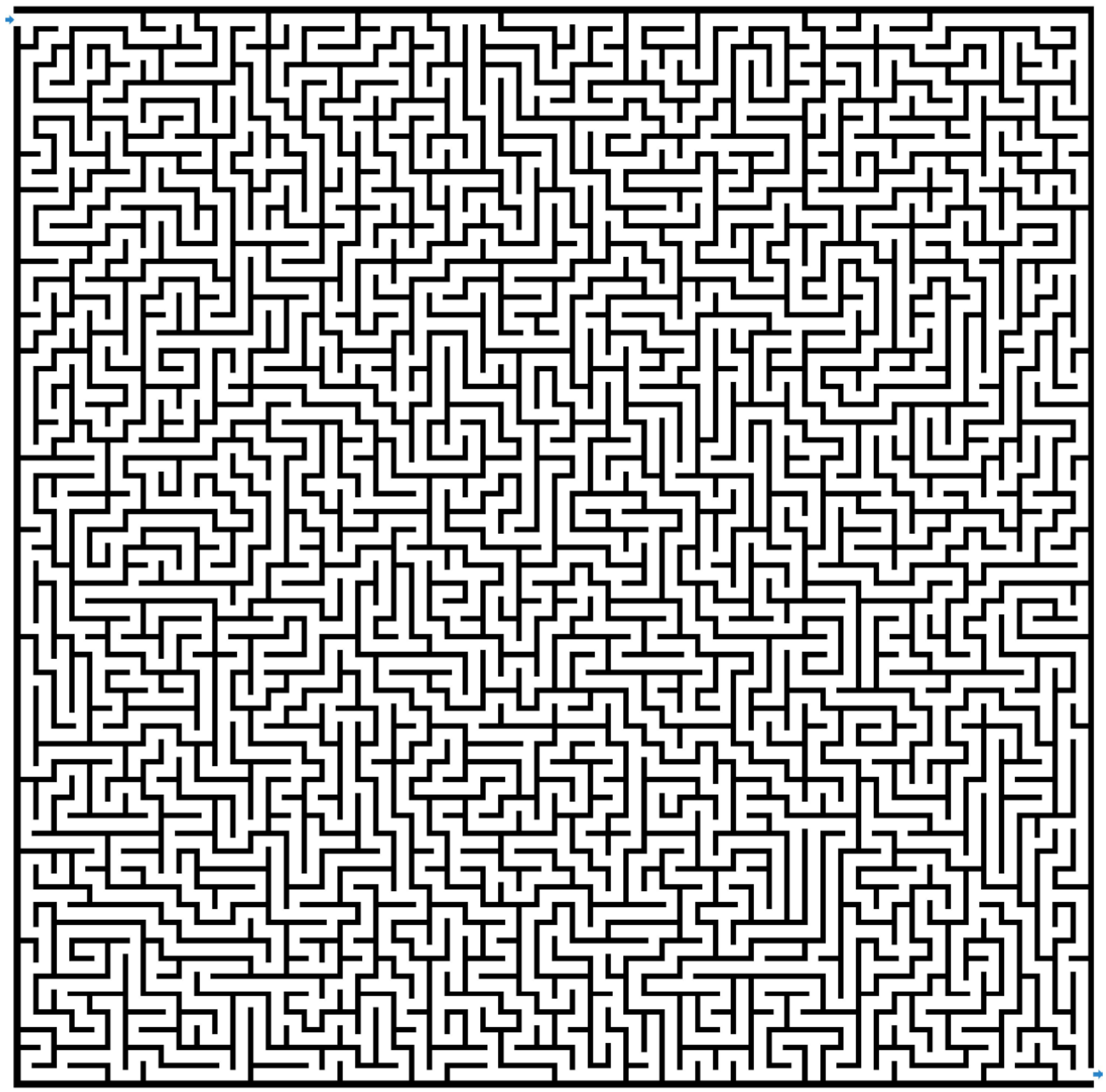
IF APPLY STRESS AND CHANGE T,

$$\varepsilon_{TOTAL} = \qquad + $$

$$= \qquad + $$

MAZE: **Statics edition**

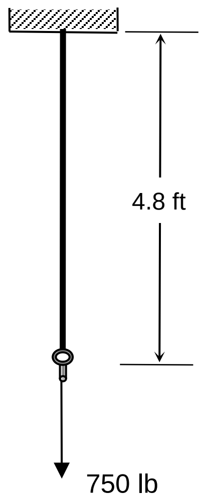
Find your way to Dr Thom’s office.



Example

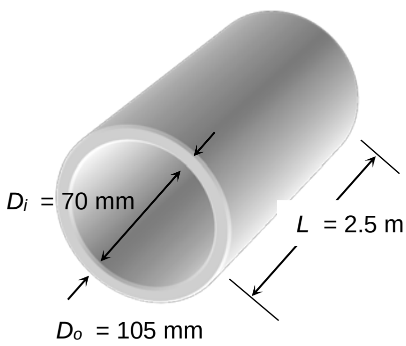
A 4.8-ft-long wire with Young's Modulus of E = 29×10^6 psi is subjected to a 750-lb tensile load. If the diameter of the wire is ¼ in, find

(a) the wire's elongation and,
(b) the resulting normal stress.



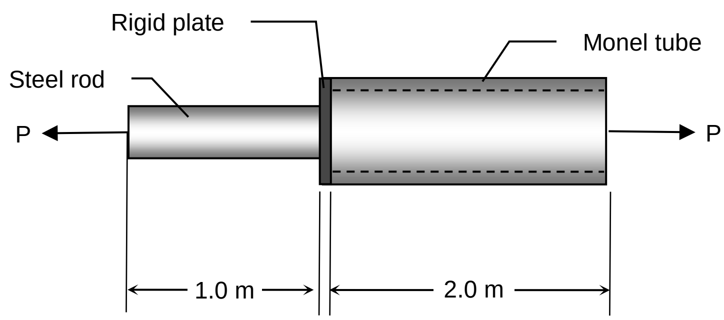
Example

A cast iron pipe has inside and outside diameters of 70 mm and 105 mm, respectively. The length of the pipe is 2.5 m and the coefficient of thermal expansion is $\alpha = 12.1\times10^{-6}$/°C. For a 70°C increase in temperature, find the new pipe dimensions.



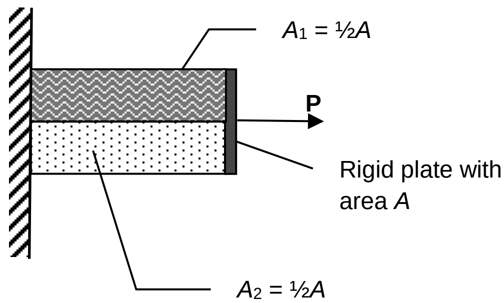
Example

A steel (E = 200 GPa) rod with diameter 30 mm and length 1.0 m is attached to a 2.0-m long Monel (E = 180 GPa) *tube* via a rigid plate. The Monel tube has internal diameter of 40 mm and a wall thickness of 10 mm. Determine the total axial load required to stretch the total assembly 3.00 mm.



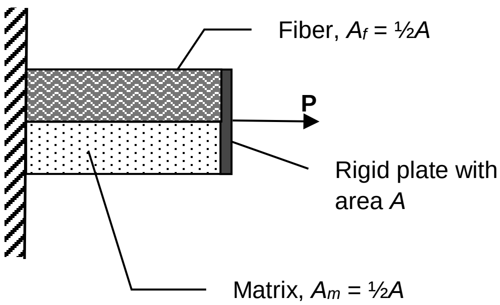
Example

Two deformable bodies are subjected to an axial load of *P* as shown in the figure. Draw a free body diagram that would help you to determine the load (axial force) in each material.



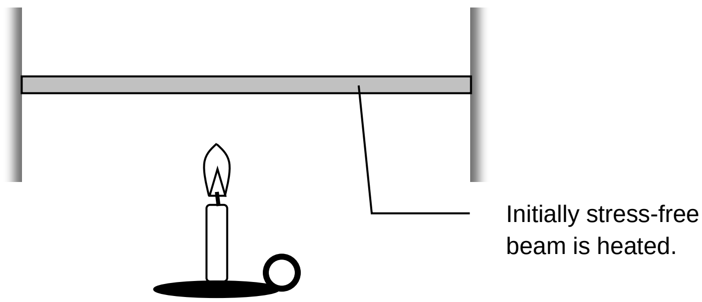
Example

A composite structure made of fiber (E_f = 231 GPa) and a matrix (E_m = 3.4 GPa) is subjected to an axial load of P as shown in the figure. Find the load carried by the fiber, the load carried by the matrix, and the total deformation of the composite.



Example

A thin rod suspended between two fixed supports is initially in a stress free state. The rod is then uniformly heated resulting in a temperature change of the rod of ΔT. Because of the heating, the rod wants to expand. However, the fixed supports prevent this from happening resulting in a compressive stress in the rod.



(a) Find an expression for the resulting stress in the rod in terms of Young's modulus E, the thermal expansion coefficient α, and the temperature change ΔT. Assume that the thermal expansion coefficient is constant.

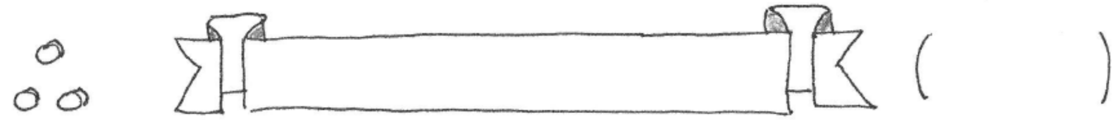
(b) If the rod is made of SiO_2 with E = 69 GPa and $\alpha = 0.55 \times 10^{-6}$ /°C, what stress will a 10°C temperature change produce? Also, find the force exerted on a rod with a square cross section with side length a = 10 μm. (1 μm = 1×10^{-6} m)

NOTES: Factor of safety

LET'S DESIGN A LINK

WHAT THINGS AFFECT WHETHER OR NOT THE LINK FAILS?

HOW ______________ WOULD YOU SAY MANY OF THESE THINGS ARE?



NOTES: Factor of safety

KEY IDEA: FOS ________ ________ ________ !

LET'S ASSUME OUR LINK WILL FAIL BY FRACTURE @ AN ULTIMATE TENSILE STRENGTH of 90 ksi

FOR A FOS = ______, WHAT STRESS SHOULD THE LINK BE DESIGNED FOR?

a. 90 ksi

b. 30 ksi

c. 270 ksi

d. SCHIFTY-FIVE

¿ WHAT FOS TO USE?

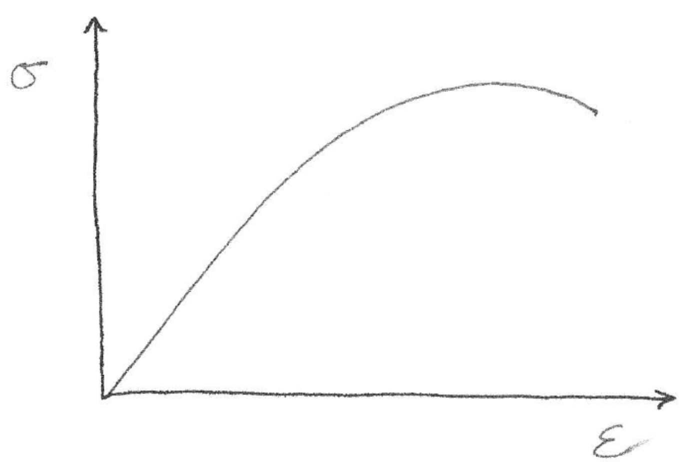
STRUCTURAL STUFF →

AIRCRAFT →

¿ WHY NOT MAKE FOS = ?

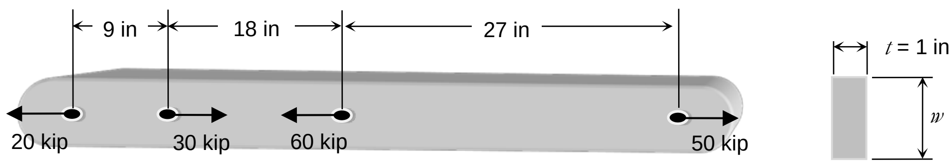
¿ WHERE DO YOU GET GUIDELINES FOR FOS?

-
-
-
-



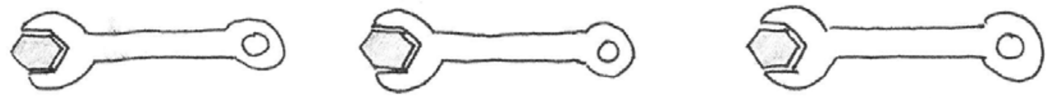
Example

A one-inch-thick 0.4% C hot-rolled steel bar is subjected to four different axial forces as shown in the figure. If the factor of safety by yielding is to be 1.75, find the minimum width w of the bar.



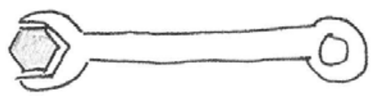
NOTES: Moments

HOW TO USE A WRENCH:



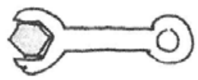
(a) (b) (c)

ONLY THE PART of THE FORCE THAT IS ____________ TO THE WRENCH IS USEFUL



____

NOW, PICK YOUR FAVORITE WRENCH:

(a) 

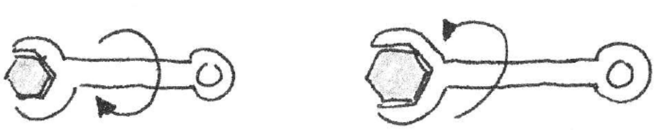
(b) 

(c) 

AND SO THE USEFUL QUANTITY IS

____

WHICH WAY TIGHTENS BOLT? WHICH WAY LOOSENS IT?



______________ & ______________ ARE IMPORTANT.

SOUNDS LIKE A ________________.

Formal Definition

≡

← ______ ______ POINT O

- | | =
- DIRECTION IS ____ TO $\vec{r}$ & $\vec{F}$ USING ______ ______ ______.

OR USE

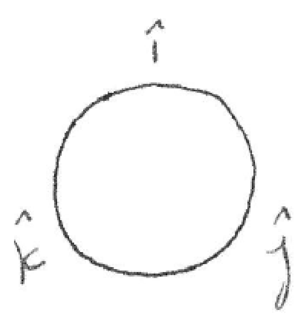
= | | MOST USEFUL IN ______.

PROPERTIES:

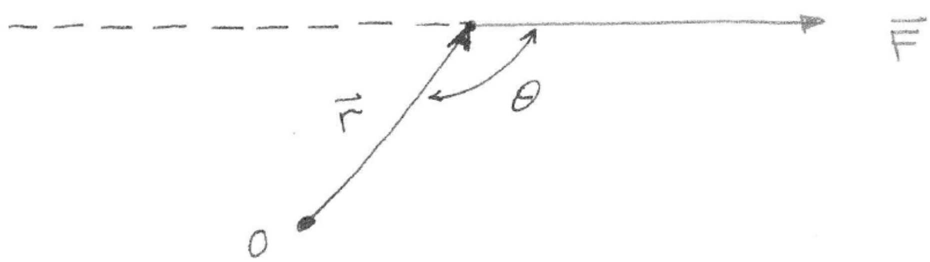
- $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$
- $\vec{A} \times \vec{B} = \vec{B} \times \vec{A}$
- IF $\vec{B} = \vec{B}_1 + \vec{B}_2$

  $\vec{A} \times \vec{B} = \vec{A} \times (\vec{B}_1 + \vec{B}_2) =$ ______

$\hat{i} \times \hat{i} =$	$\hat{i} \times \hat{j} =$	$\hat{i} \times \hat{k} =$
$\hat{j} \times \hat{i} =$	$\hat{j} \times \hat{j} =$	$\hat{j} \times \hat{k} =$
$\hat{k} \times \hat{i} =$	$\hat{k} \times \hat{j} =$	$\hat{k} \times \hat{k} =$



CONSIDER THE MOMENT ABOUT O DUE TO FORCE $\vec{F}$.



WHAT IS $|\vec{M}_o| = ?$

NOW FIND $|\vec{r}_2 \times \vec{F}|$.

$|\vec{r}| \cdot \sin\theta = |\vec{r}_2| \sin\theta_2 =$ ________.

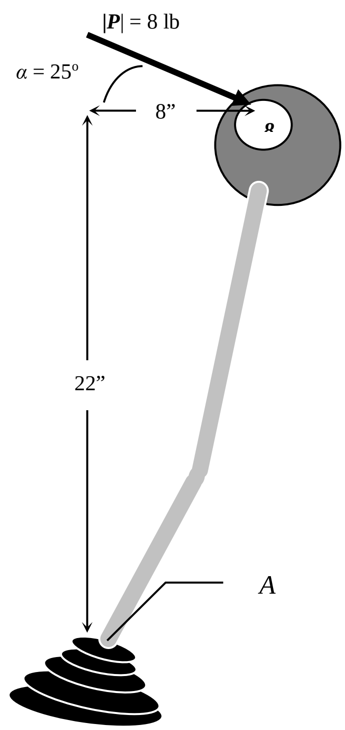
COOL THING NUMBER 1:

COOL THING NUMBER 2: INFORMAL DEFINITION of A MOMENT.

Example

A force of 8 lbs is applied to the gearshift as shown in the figure.

(a) Calculate the moment due to the applied force about pint *A* using the cross product **r×P**.
(b) Calculate the moment about point *A* by multiplying "perpendicular distance times force."
(c) Calculate the moment by breaking **P** into components.
(d) Which way was easiest, at least in this example?

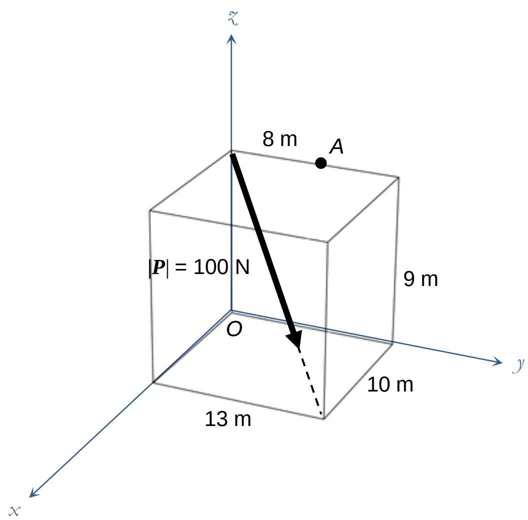


Example

For the force shown,

(a) find the moment of force **P** about the origin, and

(b) about point A.



WORD SEARCH: Statics edition

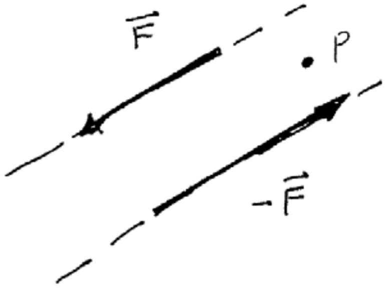
Q	L	D	X	K	S	B	N	Q	E	P	V	O	W	D
Q	D	P	P	S	R	C	O	H	A	C	S	A	U	R
O	W	C	J	O	N	J	I	R	P	T	R	C	O	J
L	I	S	T	J	P	G	T	T	R	N	T	O	I	T
L	M	C	H	L	M	I	C	A	A	I	P	K	F	M
P	E	R	Q	E	C	K	I	T	L	T	N	Y	C	O
V	W	E	V	L	A	N	R	E	E	Z	S	E	E	D
L	D	B	E	Y	P	R	F	W	L	N	U	L	N	U
K	Z	M	H	T	G	N	E	R	T	S	S	S	T	L
N	D	E	F	O	R	M	A	T	I	O	N	I	R	U
C	O	M	P	R	E	S	S	I	O	N	D	A	O	S
T	N	E	M	O	M	F	J	D	E	J	B	P	I	N
S	T	R	E	S	S	C	I	T	S	A	L	E	D	C
S	V	Q	D	A	O	L	T	P	L	A	S	T	I	C
K	M	Z	C	O	M	P	O	S	I	T	E	N	O	X

Find the following words in the puzzle above. Words can go horizontally, vertically, or diagonally.

centroid	deformation	force
member	paisley	shear
strength	vector	composite
compression	ductile	elastic
friction	load	modulus
moment	particle	plastic
statics	strain	stress
tension		

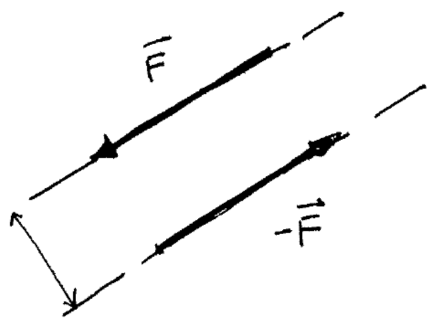
NOTES: Couples

A PAIR of OPPOSITELY-DIRECTED, NON-COLINEAR FORCES IS CALLED A ______________.



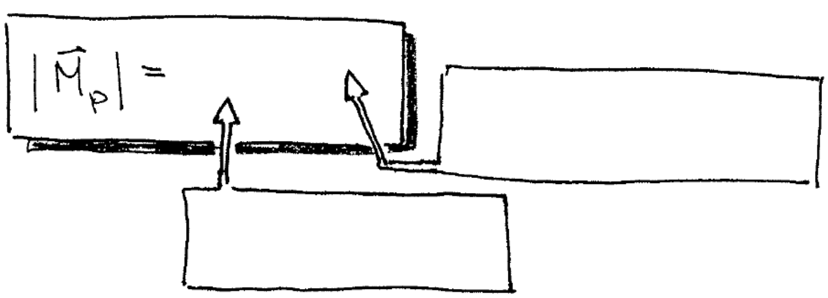
$\vec{M}_P =$

P IS ARBITRARY! THE MOMENT DUE TO A COUPLE IS THE ______________ ABOUT ________ ________ IN ______________.



$\vec{M}_P =$

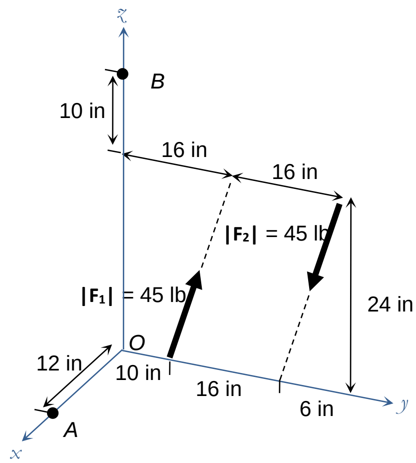
$=$



Example

Two forces, each of magnitude 45 lb, are directed as shown in the figure.

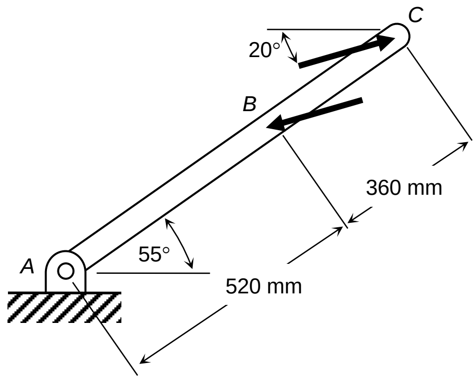
(a) Find the resultant moment due to both forces about the origin, *O*.
(b) Find the resultant moment due to both forces about the point *A*.
(c) Find the resultant moment due to both forces about the point *B*.
(d) Find the shortest distance between the lines of action between the two forces.



e) Find the moment due to the two forces about the tip of your nose.

Example

Two parallel and oppositely directed forces, each of magnitude 60 N, (and therefore a _______________!) are applied to the lever as shown in the figure. Find the moment due to the forces about point *A*.

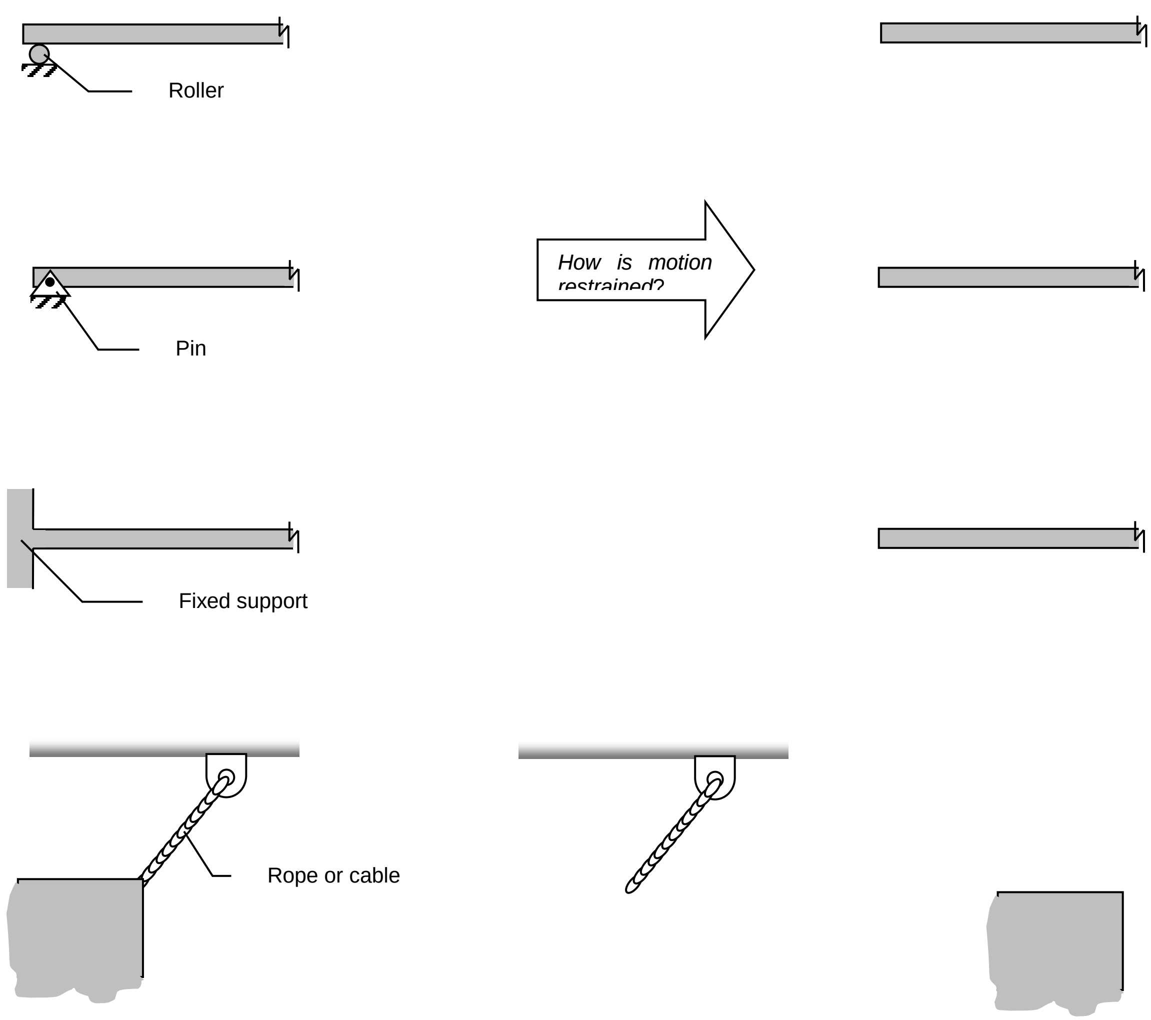


ACTIVE LEARNING EXERCISE: Types of supports and their reactions

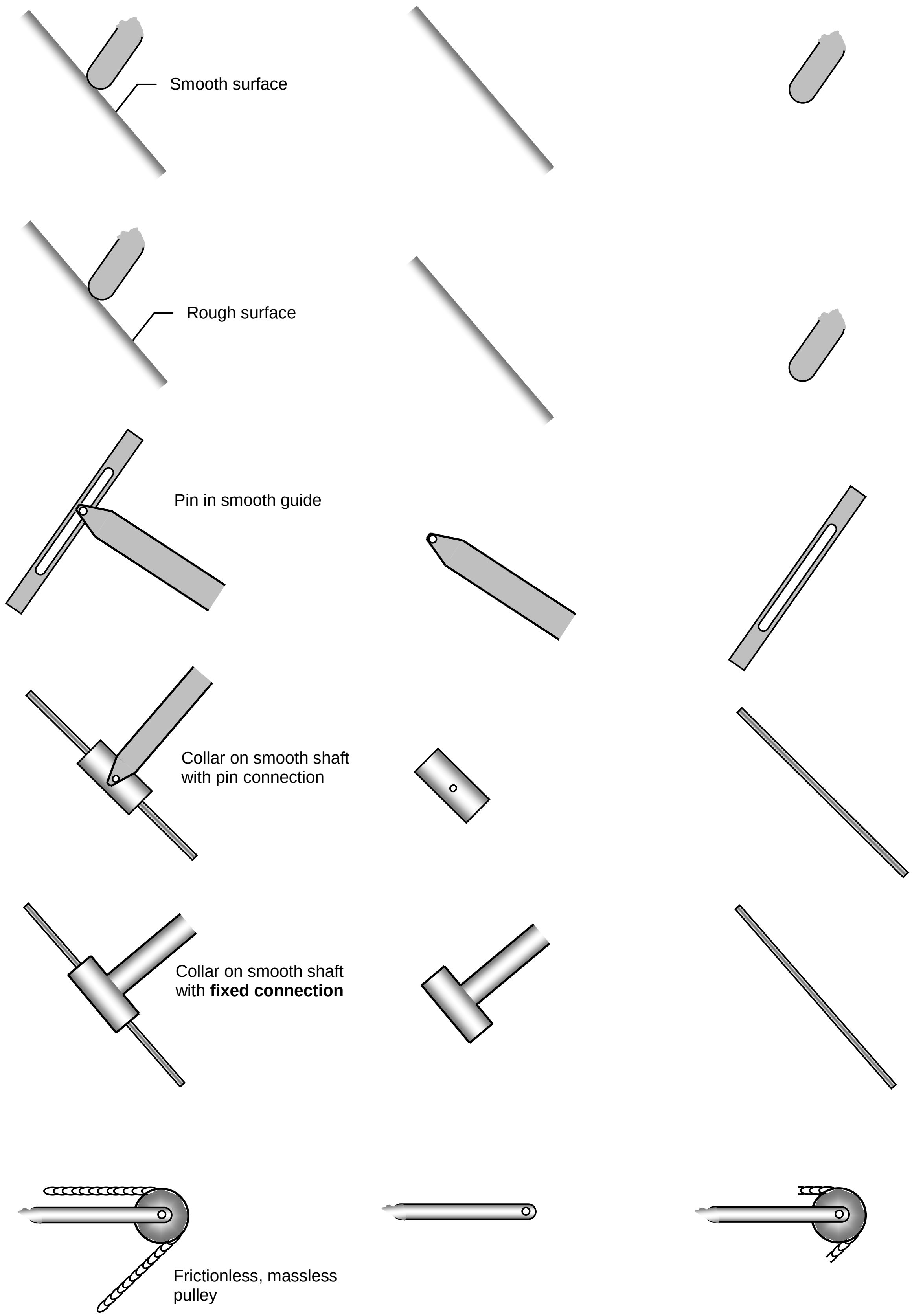
When we isolate a system for analysis, we "remove" supports and replace them with the forces and/or moments they supply to the system. Such forces/moments are called **reactions**.

When trying to figure out whether a reaction consists of forces, moments, or both, it is useful to think about the way in which the support *restrains the motion* of the system. This will also help us determine the directions these forces/moments are directed. For example, if a support keeps something from moving up and down, then a reaction force develops in the vertical direction. If a support keeps something from rotating about an axis, then a moment reaction develops about that axis.

Keeping this advice in mind, see if you can determine the reactions supplied by these three common supports.



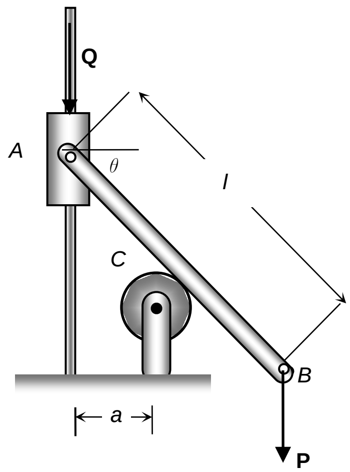
(Don't miss the exciting conclusion to this exercise on the back of this page!)



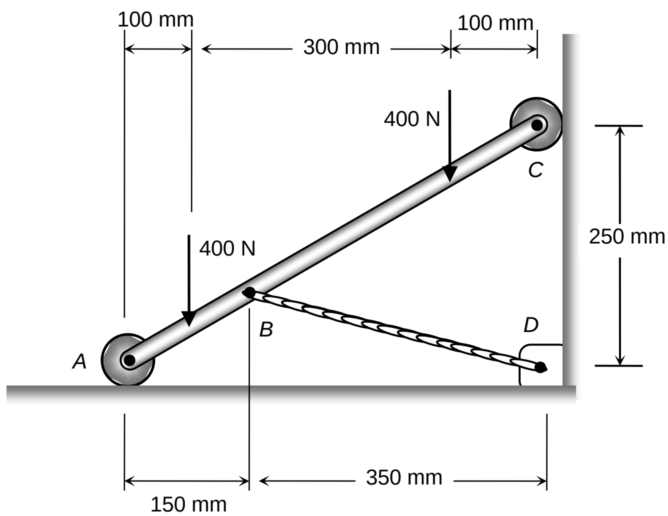
Smooth surface
Rough surface
Pin in smooth guide
Collar on smooth shaft
with pin connection
Collar on smooth shaft
with **fixed connection**
Frictionless, massless
pulley

ACTIVE LEARNING EXERCISE: **Drawing Free Body Diagrams**

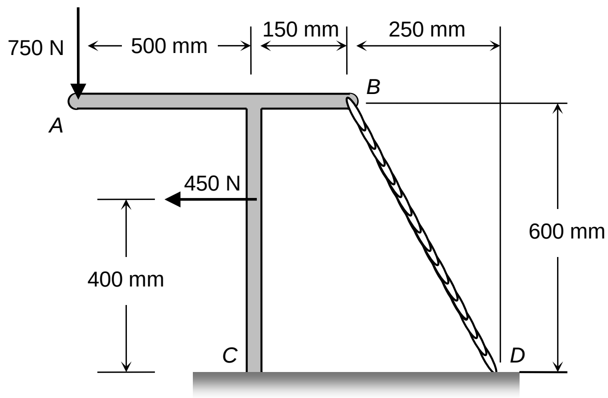
For each of the systems below, draw a free body diagram of the requested part(s).



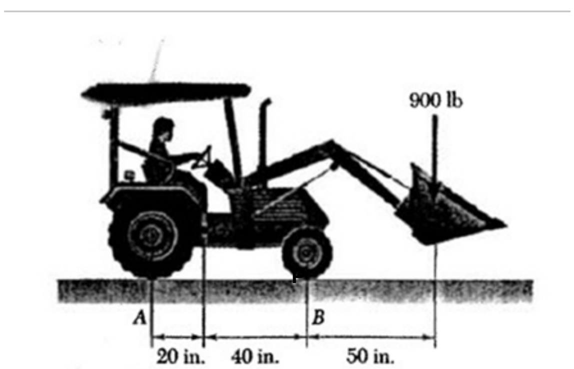
Draw FBDs of the collar and of link *AB*.



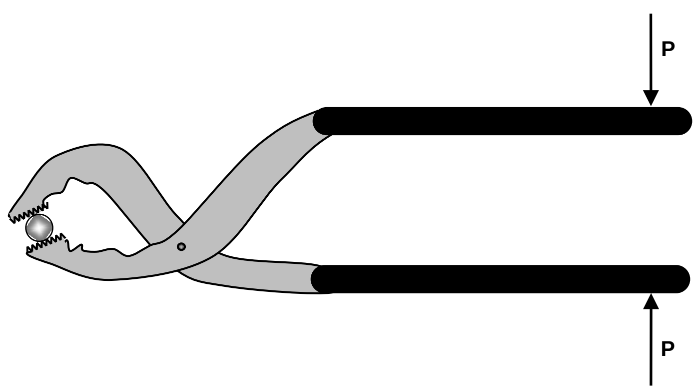
Draw FBDs of rope BD and link *AC*. (Hint: include the rollers.)



Draw a FBD of part *ABC*.



Draw a FBD of the end-loader.



Draw FBDs of each handle and of the bolt.

NOTES: Equilibrium of rigid bodies

For particles (or "particles") all the lines of action of forces ______ ________________________ ___ ________________.

For general rigid bodies this is *not* true! Therefore, _____________________ can result. Therefore, we also need

$$\Sigma \quad = 0$$

for equilibrium.

Solution plan

1) __________________ ________________ (Draw a __________!)
2) Apply equilibrium:

- $\Sigma \quad = 0$

    i. In 2-D usually easier to do in ________________ form:

    ii. Be sure to show your _________________ ____________!

- $\Sigma \quad = 0$

    What point?

    1. ________ one you want!

    2. Look for

3) Solve equations!

Dos and Don'ts

- *Do* draw the FBD.
- *Don't* assume you know the value of any reaction (force or moment) when you draw them on your FBD. Leave them as unknowns, even if it seems obvious to you what the values are. (You'll be surprised how often your intuition is wrong!)

- *Do* look at your FBD as you write the equilibrium equations. That's why you drew it!
- *Don't* write equilibrium equations first and then decide how your FBD matches your solution.

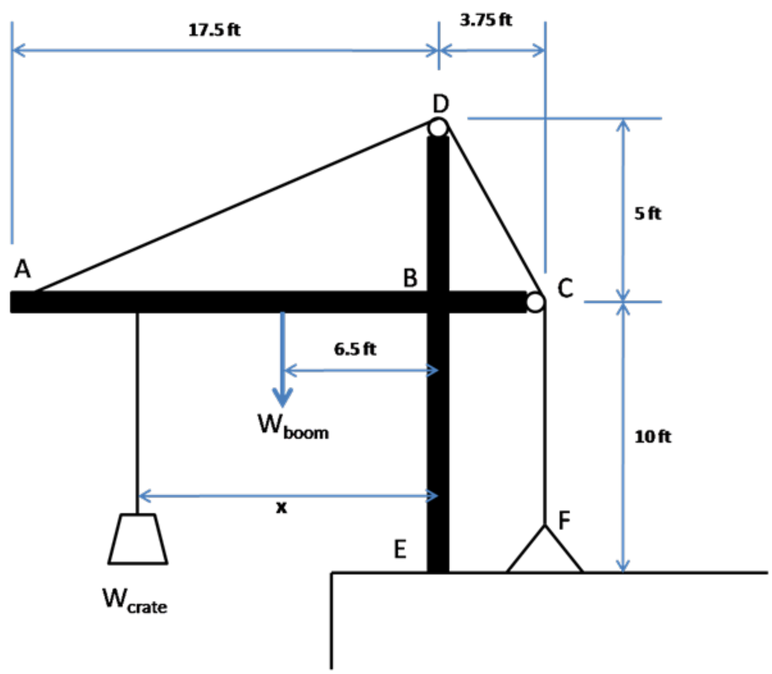
- *Do* identify your coordinate system.
- *Don't* assume it's obvious. (It's often much more convenient to use tilted axes!)

- *Do* use symbols in your solution as far as possible before plugging in numbers.
- *Don't* assume all your units work out, and so *do* write your units in each calculation.

- *Do* follow the advice above.
- *Don't* not follow the advice above.

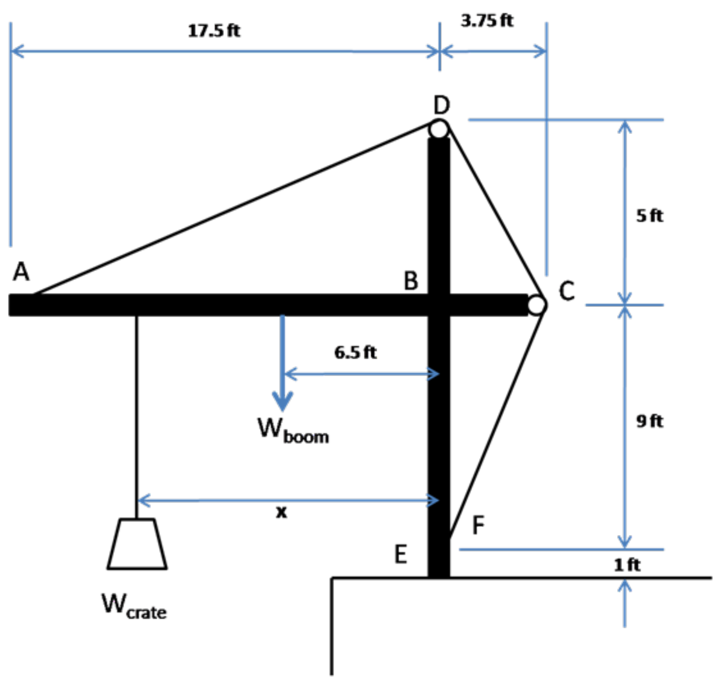
Example

The rig shown below consists of a 1200-lb boom *ABC* and a vertical member *DBE* welded together at *B*. (There are frictionless pulleys at both *C* and *D*.) The rig is being used to suspend a 3600-lb crate at a distance x = 12 ft from the vertical member. If the tension in the cable is 4 kips, determine the reaction at *E*.



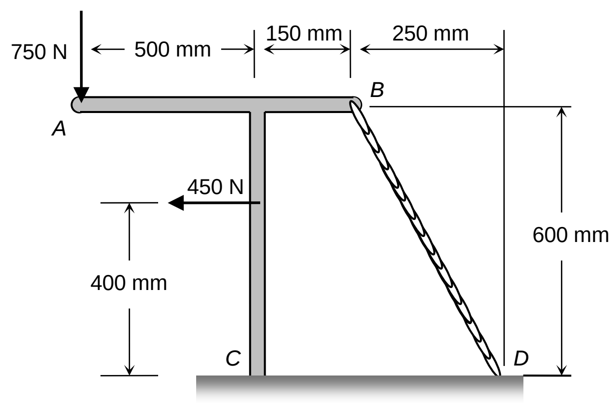
Example

If the cable attachment point in the last example is changed as shown below, find the new reaction at *E*.



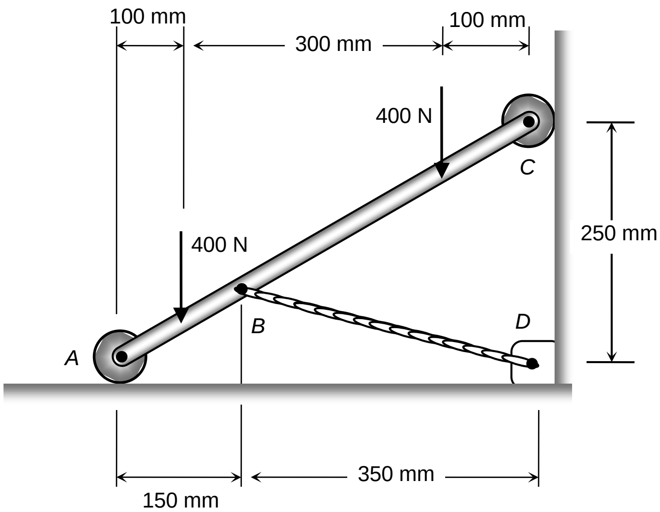
Example

Knowing that the tension in the wire *BD* is 1300 N, determine the reaction at the fixed support *C* of the structure shown. Assume that the weight of the structure is negligible.



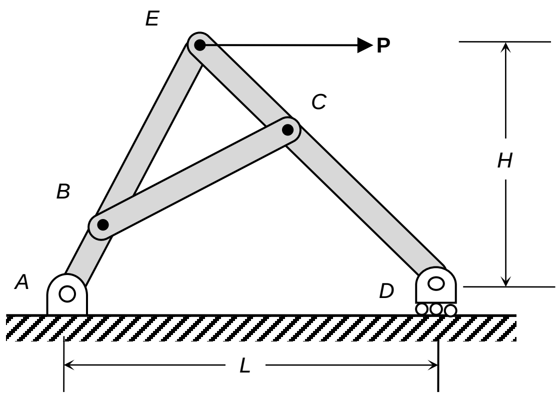
Example

Find the tension in the wire *BD*. Assume that the weight of the structure is negligible.



Example

Consider the structure below. All members can be considered massless. Set up the equations necessary to find the reactions at A and D.

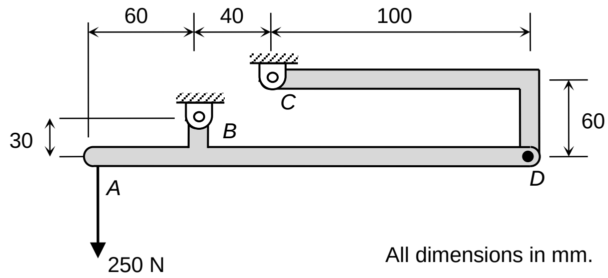


Example

Find the reactions at pins B and C in the last example. Is there anything special about those reactions?

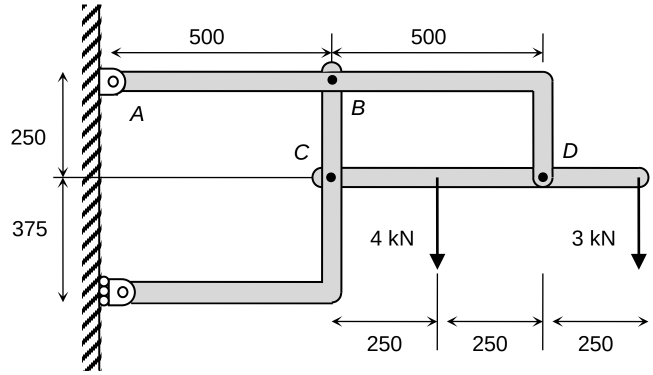
Example

Find the reactions at *B* and *C*. Assume that the weight of the structure is negligible.



Example

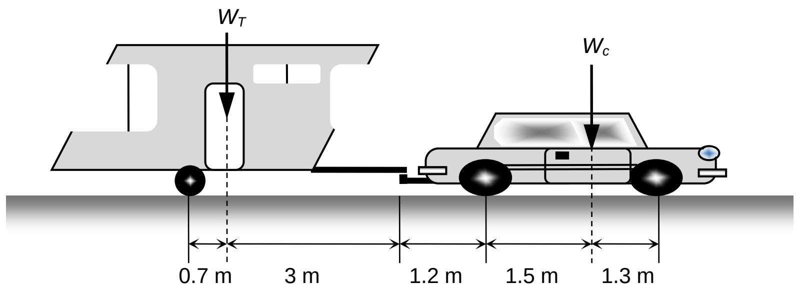
Find the components of all forces *ABD*. Assume that the weight of the structure is negligible.



All dimensions in mm.

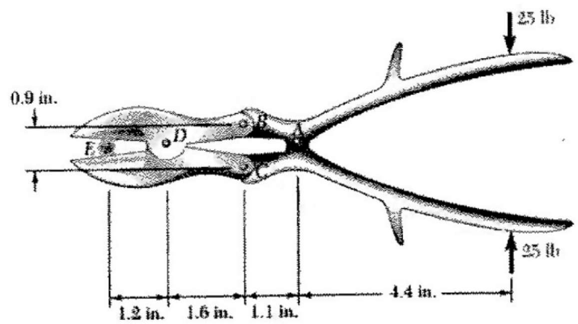
Example

The mass of the car in the figure is 1250 kg and the mass of the trailer is 1000 kg. The trailer hitch connecting the car to the trailer is a ball and socket. Find the reactions at the wheels.

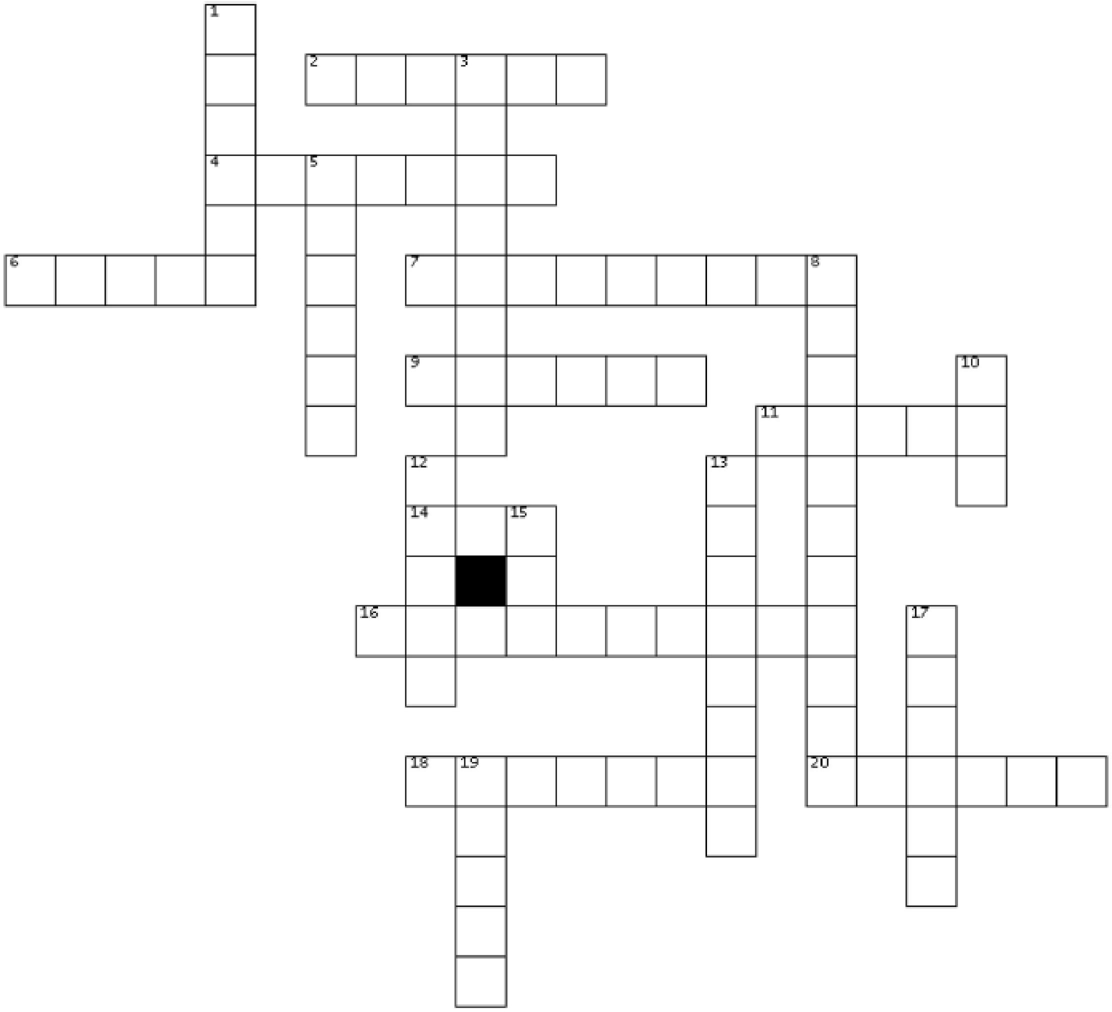


Example

The device shown in the figure is called a bone rongeur and is used in surgical procedures to cut small bones. For the 25-lb forces applied to the instrument at the locations shown, find the force applied to the bone at *E*.



CROSSWORD PUZZLE: Statics edition



ACROSS

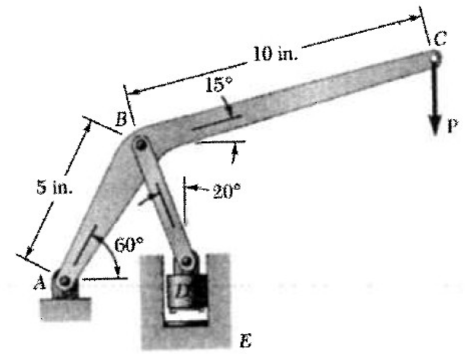
2. It actually isn’t more than the sum of its components
4. Greek letter
6. Product with a direction
7. Angle of maximum shear in an axially-load member
9. Lacks direction
11. Greek letter
14. Antonym of surname attached to a modulus
16. It's pure direction
18. Spanish Spanish
20. Small length of time or important cross product

DOWN

1. Life at Rose comes with this, as does force per unit area
3. A weightless, pinned member
5. Deformation without dimensions
8. It all adds up to nothing
10. Greek letter
12. Surname attached to a modulus
13. All lines cross at a point for this
15. Product without a direction
17. Describes a force or stress perpendicular to a plane
19. Describes a force or stress tangent to a plane

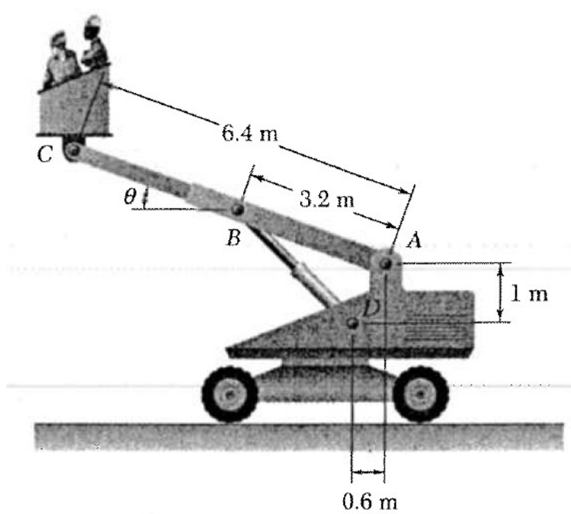
Example

The figure shows a press used to emboss a seal at *E*. If the force $P = 60$ lb, find the reaction at *A* and the vertical component of the force exerted on the seal.



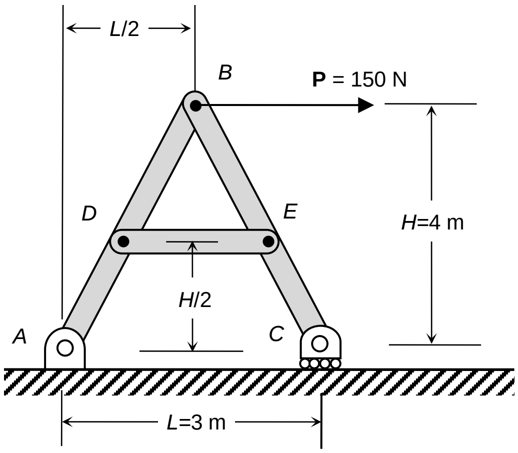
Example

The telescoping arm ABC is used to elevate workers on a platform. The combined mass of the platform and the workers is 240 kg with a combined center of gravity at *C*. If the angle $\theta = 24°$, find the force exerted by the hydraulic cylinder *BD* on the arm and the reaction at *A*.



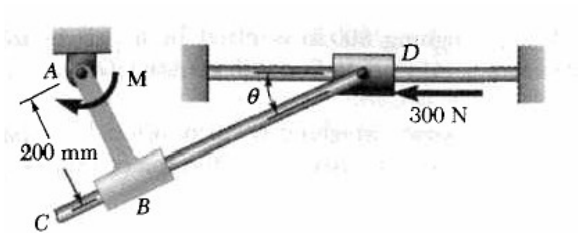
Example

The A-frame in the figure is subjected to a force of P=150 N as shown in the figure. Assuming massless members, find the reactions at A and C and the pin reactions at D and B.



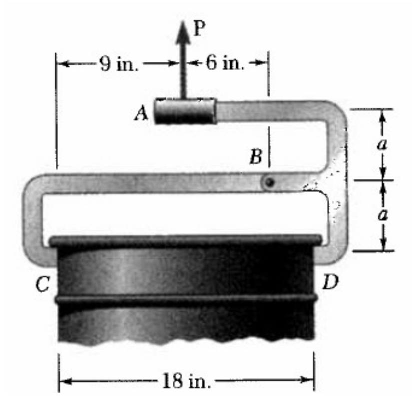
Example

The figure below shows what is known as a slider-crank mechanism, a machine that changes rotational motion into translational motion, or *vice versa*. If the angle θ=30°, find the required moment that must be supplied at *A* in order to maintain equilibrium.



Example

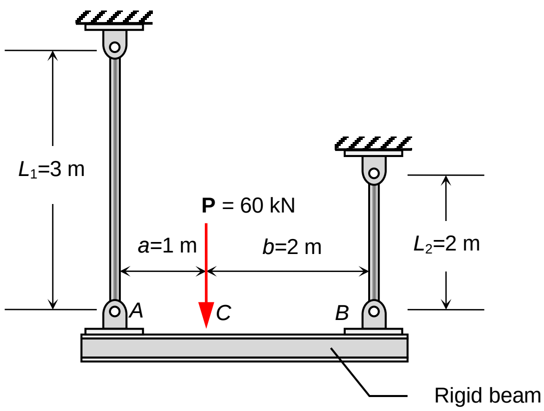
Tongs are used to lift a barrel weighing 60 lb as shown in the figure. If a=5 in, find the forces exerted on the tongs at both C and D.



Example

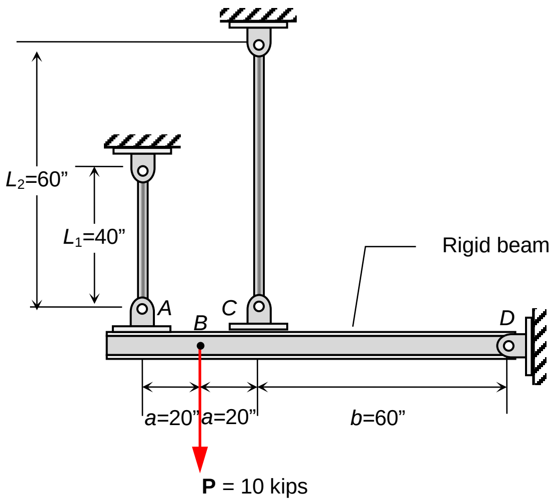
A rigid beam is supported by two vertical rods. Rod *A* has a diameter of d_A = 25 mm and rod *B* has a diameter of d_B = 10.2 mm. Both rods are made of steel (*E*=210 GPa). For the 60 kN force applied as shown,

(a) find the reactions at *A* and *B*, and
(b) the displacements of each rod.



Example

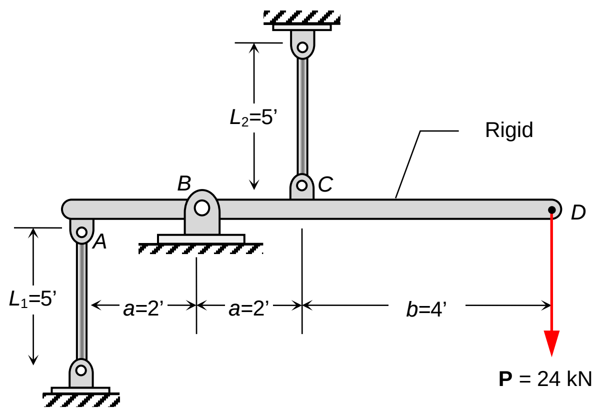
Two steel (E=30×10^3 ksi) rods both with cross sectional area A=1.0 in^2 are used to support a rigid beam connected to a wall via a smooth pin. A 10 kip point load is applied to the beam at the location shown. Neglecting the weight of the beam, find the tension in each rod.



Example

A rigid, weightless beam is supported by a smooth pin at *B*. Two aluminum (E=70 GPa) rods, both with cross sectional area A=200 mm^2, also support the rod at pins *A* and *C*. For the 24 kN load at *D*,

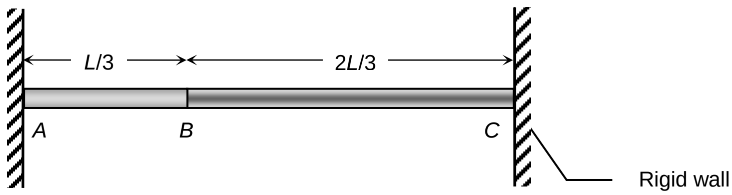
(a) find the rotation angle of the rod,
(b) the force in each rod, and
(c) the stress in each rod.



Example

Two bars, both with cross sectional areas A, are attached to rigid walls. Bar AB is made of aluminum, whereas bar BC is made of steel. At room temperature the bars are stress-free. In service the temperature of the system rises by an amount ΔT.

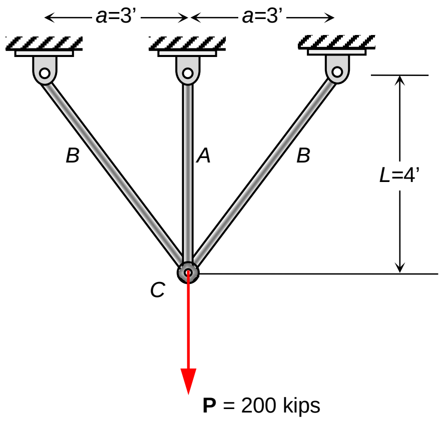
Assuming $E_{st} = 3E_{Al}$ and $\alpha_{st} = \frac{1}{2}\,\alpha_{Al}$, does point B move when heated by ΔT? If so, in which direction and how far?



Example

The structure shown in the figure consists of one cold-rolled bronze (E_b = 15×10^3 ksi, α_b = 9.4×10^{-6}/°F) bar A and two 0.2% carbon-hardened steel (E_s = 30×10^3 ksi, α_s = 6.6×10^{-6}/°F) bars B. A load P=200 kips is applied to point C while bar A experiences a temperature decrease ΔT_A = 50°F and both bars B experience a temperature increase ΔT_B = 30°F. If the cross sectional areas of bars A and B are A_b = 3.00 in^2 and A_s = 2.50 in^2, respectively,

(a) find the stress is each bar, and
(b) find the displacement of point C.



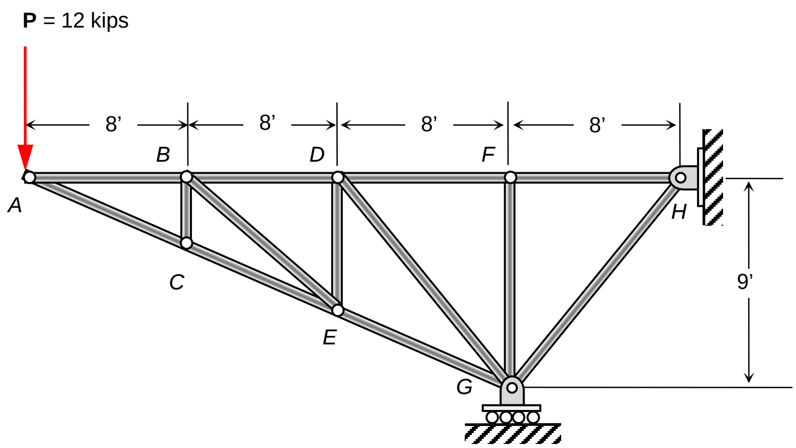
Example

Consider the structure made up of thirteen weightless members that are connected to each other via smooth pins.

(a) How many two-force members are in the structure?

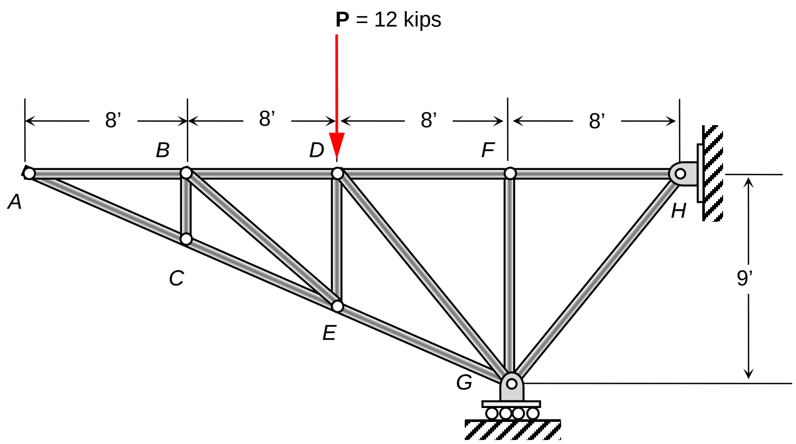
(b) Find the reactions at the pin H and the roller G.

(c) Find the internal force in each two-force member you identified in part (a). (Hint: Draw an FBD for each individual *pin* that connects two-force members. Start at a location where there are only two unknowns, such as point A.)



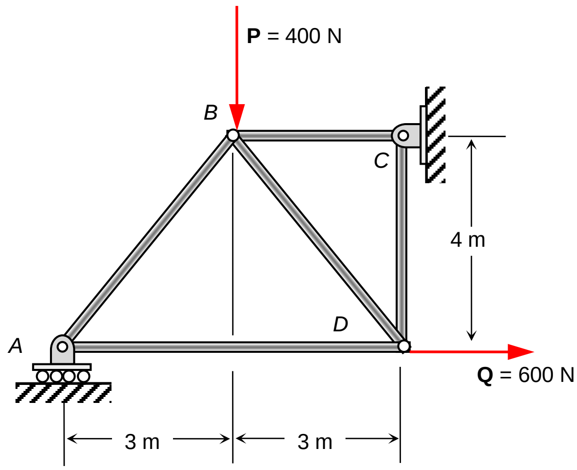
Example

Find the force in each member of the truss shown below and state whether it is in tension or compression.



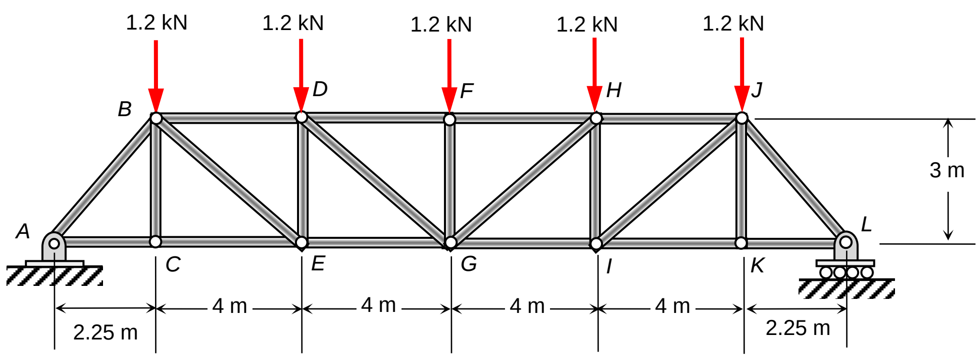
Example

Find the force in each member of the truss sown below and state whether it is in tension or compression.



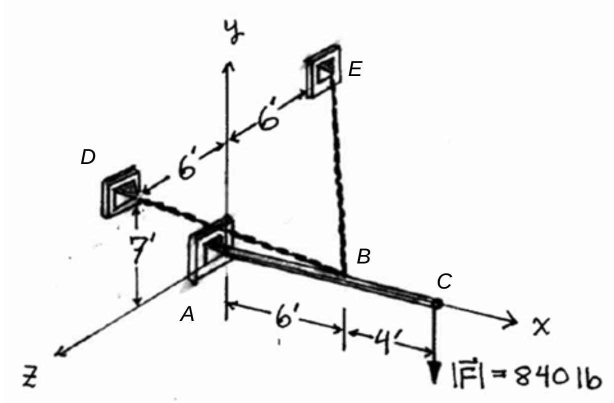
Example

The truss shown below is called a Massard roof truss. For the truss loaded as shown, find the forces in members *DF*, *DG*, and *EG* and state whether they are in tension or compression.



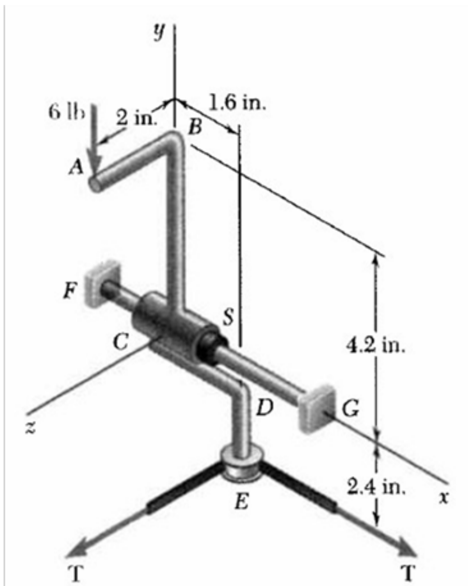
Example

The connections as A, D and E are ball and socket types. The rod AC can be modeled as weightless. Find the tension in each cable and the reaction at A.



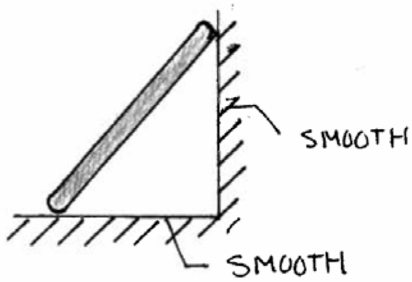
Example

For the assembly shown, find the tension T in the strap and the reactions at the **thrust bearing** C. The weight of the assembly is negligible.



NOTES: Friction

CONSIDER A LADDER ON SMOOTH SURFACES:



F.B.D.:

APPLY EQUILIBRIUM:

$$\sum F_x = 0$$

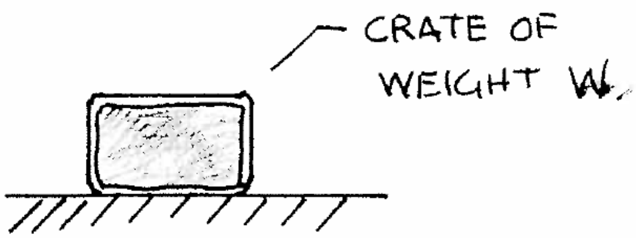
$$\sum F_y = 0$$

$$\sum M_{\_} = 0$$

DRAW THE *REAL* F.B.D.

______________ FORCES DEVELOP TO OPPOSE ______________ ________ EITHER ACTUAL OR POTENTIAL.

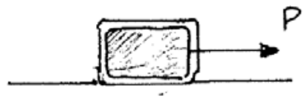
CONSIDER A CRATE ON A HORIZONTAL SURFACE



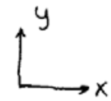
F.B.D.

NOTES: Friction

NOW APPLY A HORIZONTAL FORCE P.

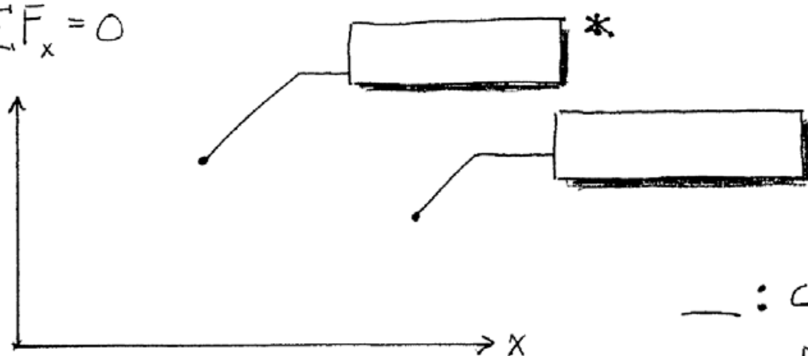


F.B.D.



FOR EQUILIBRIUM

$\sum F_x = 0$



___ : COEFF of ______ FRICTION

___ : COEFF of ______ FRICTION

* AT __________ MOTION ONLY !

FOR EQUILIBRIUM, THEN

$$\leqslant f \leqslant$$

Don't always assume:

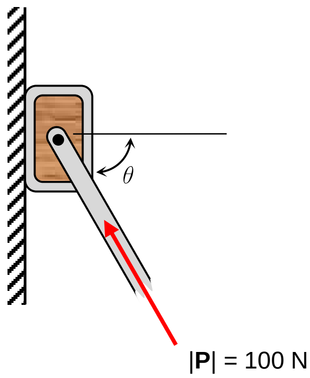
-

-

-

Example

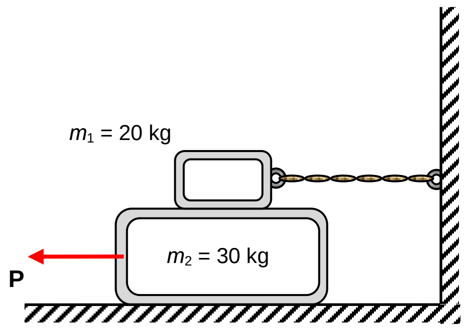
A 7.5-kg mass is subject to a force **P** as shown in the figure. The coefficients of static and kinetic friction between the mass and the wall are $\mu_s = 0.45$ and $\mu_k = 0.35$, respectively. Find the range of angles for θ for which the mass is in equilibrium.



Example

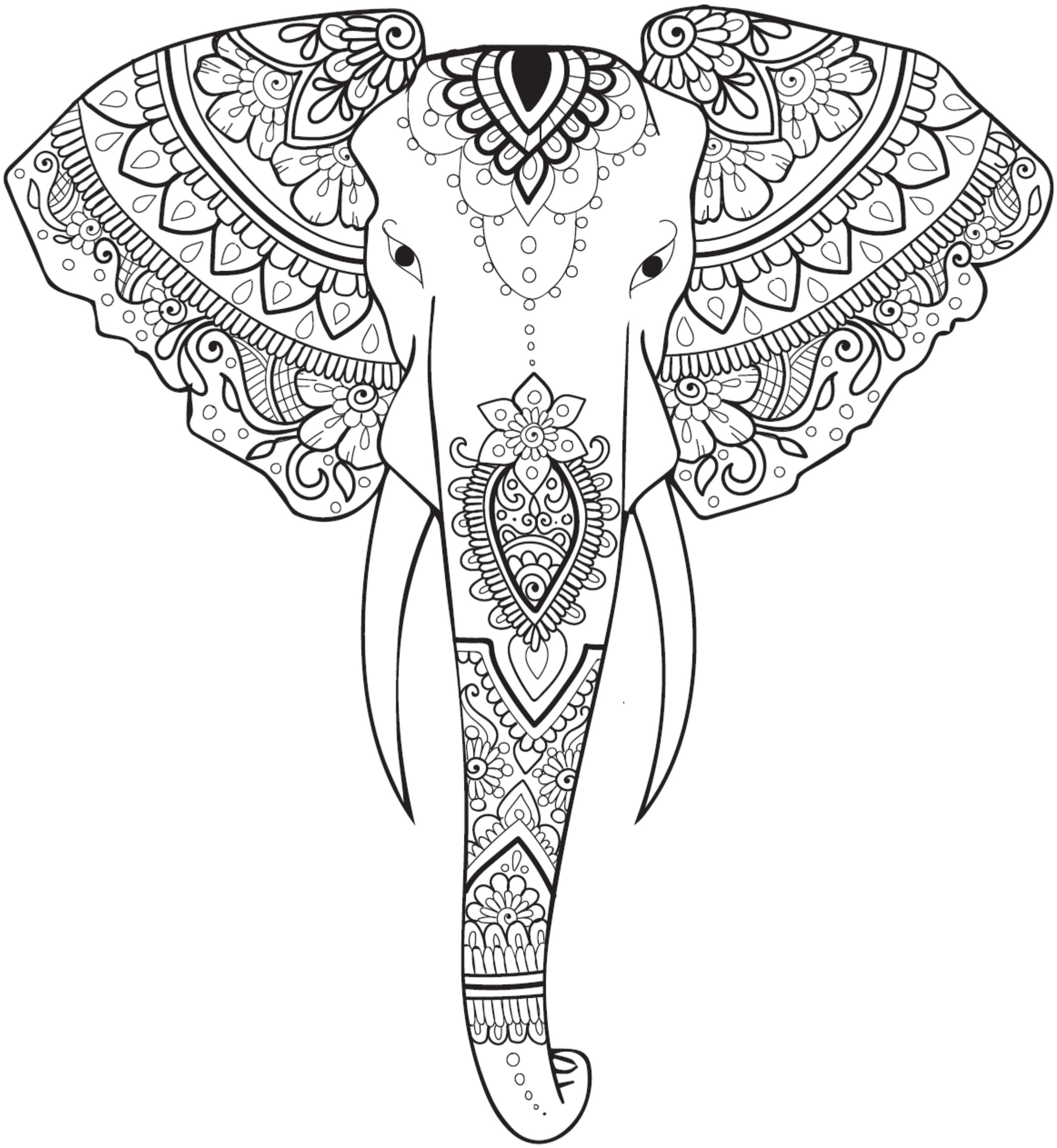
The coefficients of static and kinetic friction between all surfaces in the figure are $\mu_s = 0.40$ and $\mu_k = 0.35$, respectively.

(a) Find the smallest force P that is required to move the 30-kg block.
(b) Repeat (a) if the cable is removed.
(c) What if the friction force between the blocks for part (b)?



COLORING PAGE: Statics edition

Color the paisley elephant.

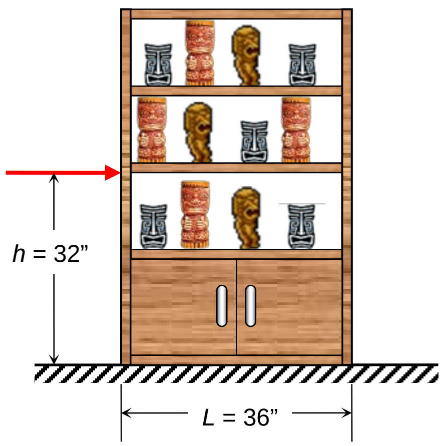


Design by Joey Bauchle. Check out her studio in Indianapolis: https://www.johennadesign.com/

Example

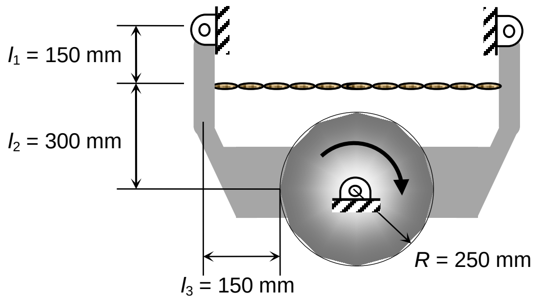
Sid Gupta's legendary Tiki mug collection is displayed in a cabinet with a total weight of W_{cab} = 120 lb. A force P is applied to the cabinet at a height of h = 32 in as shown in the figure. If the coefficient of static friction between the cabinet and the floor is μ_s = 0.30,

(a) find the minimum force P that results in the cabinet moving.
(b) Repeat (a) if shag carpet is placed under the cabinet, increasing the value of μ_s to 0.60.



Example

The coefficients of static and kinetic friction between the rotating drum and the clamps in the figure are $\mu_s = 0.40$ and $\mu_k = 0.30$. The tension in the cable holding the clamps together is $T = 3$ kN. Find the moment M that must be applied to the drum to keep it rotating clockwise at a constant speed.



DRAW A FREE BODY (-BUILDER) DIAGRAM of OUR FRIEND:

F.B.D. ⇨

NOW DRAW A F.B.D. OF ONLY HIS LEFT ARM:

F.B.D.

⬊

DISCUSS HOW YOU HANDLED THE FORCE DUE TO GRAVITY IN EACH F.B.D.

-
-
-
-

NOTES: Centroids

WE *PRETEND* LIKE WEIGHT IS ______________ AT

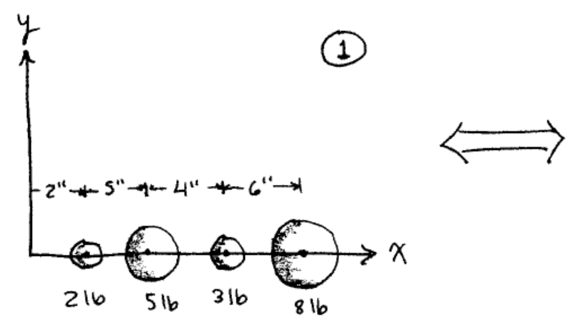
____________ ______________ . HOW CAN WE DO THIS?

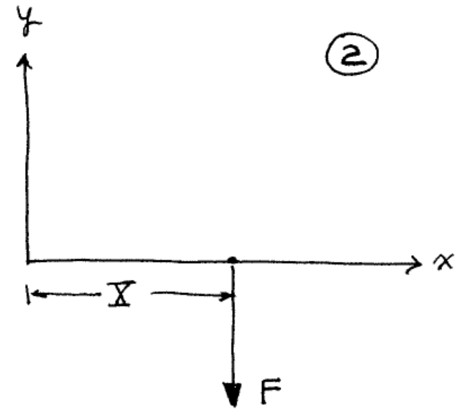
AND _____________ ______________ DO WE USE?

<u>example</u>

REPLACE THE FOUR MASSES WITH A SINGLE FORCE

AND SPECIFY ITS LOCATION.





WHAT ARE YOUR CRITERIA?

1.

2.

GENERALING:

$\bar{X} = x_c = \qquad\qquad =$

LET NUMBER of WEIGHTS BECOME INFINITE & CONTINUOUS

IN 2-D:

$x_c = \dfrac{\int \qquad}{\int \qquad} =$

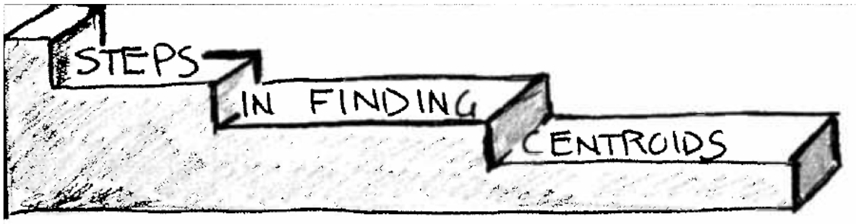
LET DENSITY BE CONSTANT.

$x_c = \dfrac{\int \qquad}{\int \qquad}$

CAN DO THE SAME FOR y-DIRECTION.

$y_c = \dfrac{\int \qquad}{\int \qquad}$

NOTES: **Centroids**



STEP 1:

STEP 2:

STEP 3:

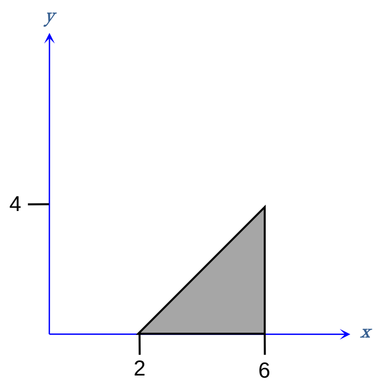
STEP 4:

STEP 5:

STEP 6:

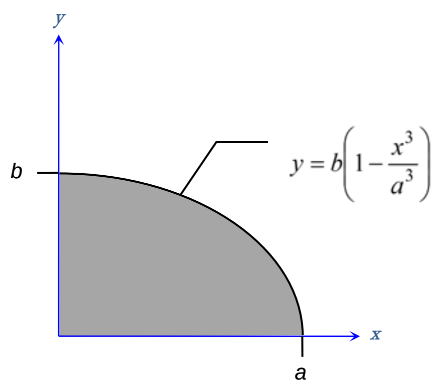
Example

Find the x-centroid for the shape below:



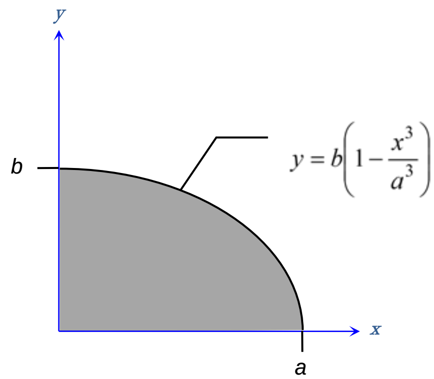
Example

Find the x-centroid for the shape below. Do you prefer a horizontal or vertical strip for your elemental area? Why?



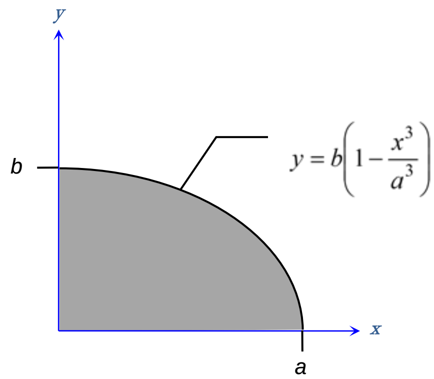
Example

Find the y-centroid for the shape below. Use the same vertical strip you did for the last example.



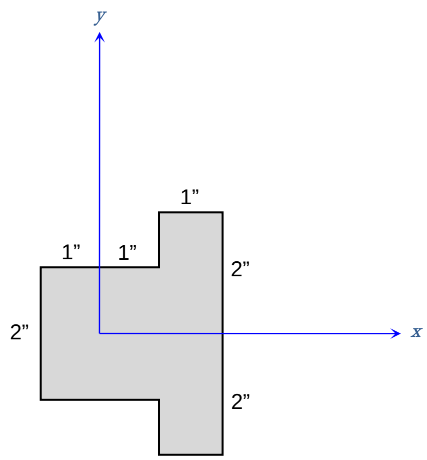
Example

Find the y-centroid for the shape below. This time use a horizontal strip for the elemental area.



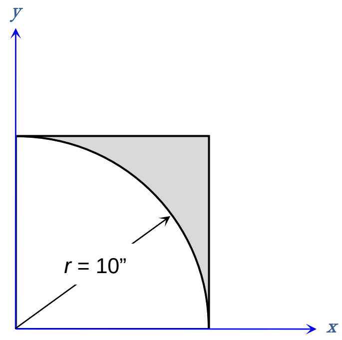
Example

Find the x- and y-centroids for the composite shape below.



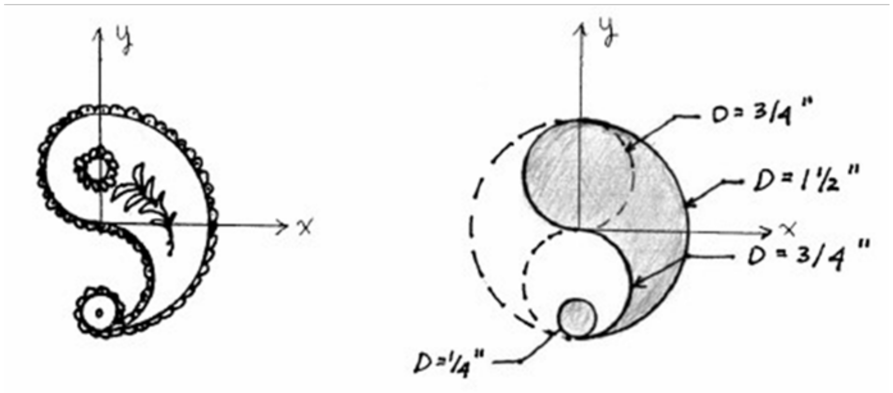
Example

Find the x- and y-centroids for the shape given by the shaded area in the figure.



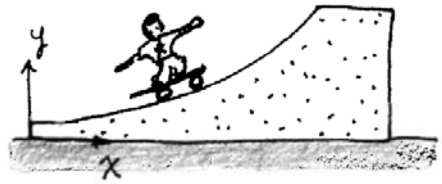
Example

Find the *x*- and *y*-centroids for a paisley by approximating it as the shape given in the right of the figure.



NOTES: Distributed loads

CONSIDER SANDBAGS -OR- A SKATE RAMP

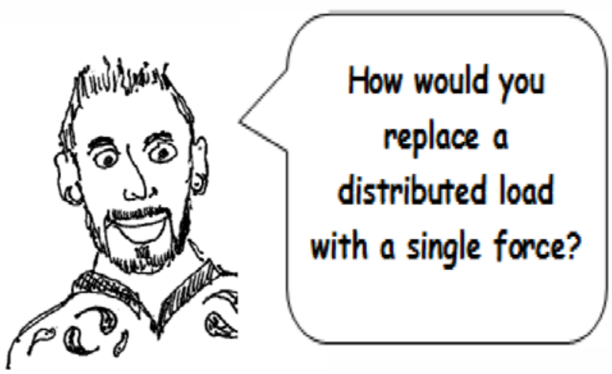


IN BOTH CASES THE FORCE EXERTED ON THE SURFACE BELOW IS __ ________________ of _______________ ( ).

THESE ARE BOTH EXAMPLES of [ ]

LET'S DRAW THESE LOADS*:

(SANDBAGS) (SKATE RAMP)



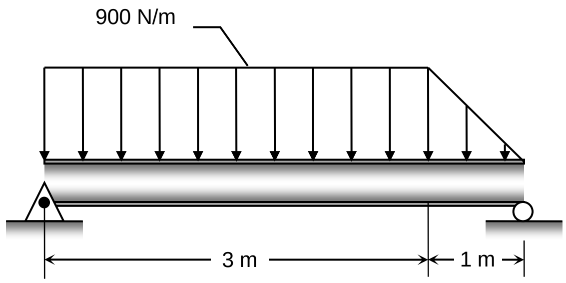
MAGNITUDE ⇒ $|\vec{F}|$ = ____________________

[ ] ⇒ = ____________________

* NOTE THAT WE CAN NOT TREAT 'THINGS' SUBJECT TO DISTRIBUTED LOADS AS PARTICLES

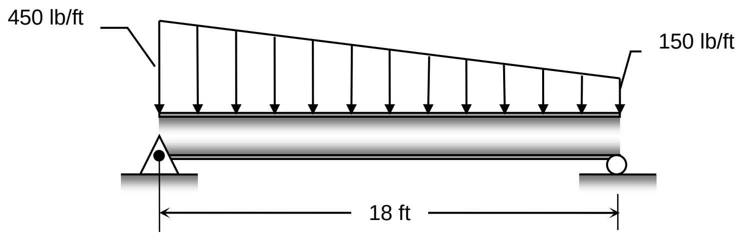
Example

For the simply supported beam below, replace the distributed load with a single force and give its location.



Example

For the simply supported beam below, replace the distributed load with a single force and give its location.



Example

The concrete structure in the figure is suggested as a design for a dam. For a one-foot thickness, find the resultant weight of the dam and give its location. The specific weight of concrete can be taken to be $\gamma_c = 150\ \text{lb/ft}^3$.