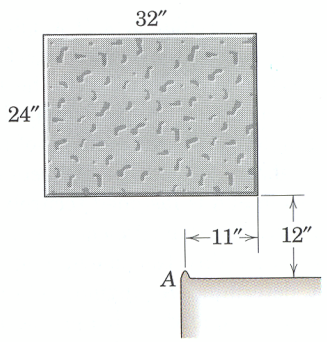
Review Problems - Final Exam

4. The uniform concrete block, which weighs 171 lb and falls from rest in the horizontal position shown, strikes the fixed corner A and pivots around it with no rebound. Calculate the angular velocity of the block immediately after it hits the corner and the percentage loss *n* of energy due to the impact. *(taken from Dynamics by Meriam and Kraige, Fourth Edition)*



ans: $\overline{w} = 1.593\ rad/s\ CCW, \quad n = 91.7\%$

Strategy: COE(FT) *process 1 to 2*
COAM, COLM (FT) *process 2-3*
Calculate KE at states 2 and 3

process 1-2

Kinetics:

COE(FT)

$$\Delta E_{sys} = W \quad \Rightarrow \quad (E_{K2} + E_{G2}) - (E_{K1} + E_{G1}) = 0$$

$$E_{K1} = 0 \quad E_{K2} = \frac{1}{2} m v_G^2$$

$$E_{G1} = 0 \quad E_{G2} = mgh$$

Solving

$$\boxed{v_G = \sqrt{2gh} = 8.02\ ft/s}$$

process 2-3

unk	eqs
v'_{Gx}	1
v'_{Gy}	2
$F_x\Delta t$	3
$F_y\Delta t$	4
ω'	5
I_G	6

Kinetics:

COLM(FT)

x-dir: $$-mv'_{Gx} = F_x \Delta t \qquad (1)$$

y-dir: $$-mv'_{Gy} + mv_{Gy} = F_y \Delta t \qquad (2)$$

What assumptions have been made about the weight as an impulsive force?

COAM(FT) at G

$$I_G w' = F_x \Delta t\left(\frac{12}{12}\right) + F_y \Delta t\left(\frac{11}{12}\right) \tag{3}$$

Kinematics:

$$\bar{v}'_G = \bar{v}'_O + \bar{w}' \times \bar{r}_{G/O}$$

Expanding and equating components knowing that the velocity at the point of contact is zero

$$\begin{aligned} \hat{i}: \quad & v'_{Gx} = -w' r_{G/O_y} \\ \hat{j}: \quad & v'_{Gy} = w' r_{G/O_x} \end{aligned} \tag{4,5}$$

Constraints and Geometry:

$$\bar{r}_{G/O} = -\frac{5}{12}\hat{i} + \frac{12}{12}\hat{j} \quad \Rightarrow \quad r_{G/O_x} = -\frac{5}{12}, \quad r_{G/O_y} = \frac{12}{12}$$

Other:

$$I_G = \frac{1}{12} m\left(h^2 + w^2\right) \tag{6}$$

Solving:

$$\boxed{w' = 1.59\, rad/s, \quad v'_{Gx} = 1.59\ ft/s, \quad v'_{Gy} = 0.66\ ft/s}$$

To find the percentage loss in energy:

Before: $$E_{Bef} = \frac{1}{2} m v_G^2 = 170.8\, lb \cdot ft$$

After: $$E_{Aft} = \frac{1}{2} m v_G'^2 + \frac{1}{2} IG w'^2 = 14\, lb \cdot ft$$

Percent Loss: $$\boxed{\%Loss = \frac{E_{Bef} - E_{Aft}}{E_{Bef}} 100 = 91.7\%}$$