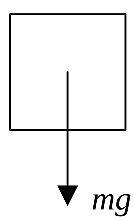
ES204 Examination 1 **Problem 1** Dec 19, 1997

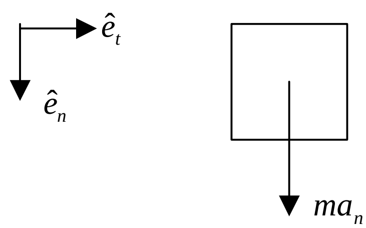
Solution strategy : We need to relate the radius of curvature to the weight (or lack thereof) of the pilot, therefore this problem involves both kinetics and kinematics. We'll probably need to use COLM(RF) and a normal/tangential CS to relate the radius of curvature to acceleration and then a polar CS to obtain $\dot{b}$ (note from the picture that $\dot{b} = \dot{q}$)

System : the pilot - note that the normal force of the chair on the pilot is 0 since he is weightless

FBD



KD



COLM(RF) : $\dfrac{d\overline{P}_{sys}}{dt} = \sum \overline{F}$

normal direction

$$mg = ma_n = m\frac{v^2}{r} \quad \Rightarrow \quad r = \frac{v^2}{g} \quad \Rightarrow \quad r = \frac{(222.2\, m/s)^2}{9.79\, m/s^2} = 5043m$$

Velocity in Polar CS : $\overline{v} = \dot{r}\,\hat{e}_r + r\dot{q}\,\hat{e}_q$

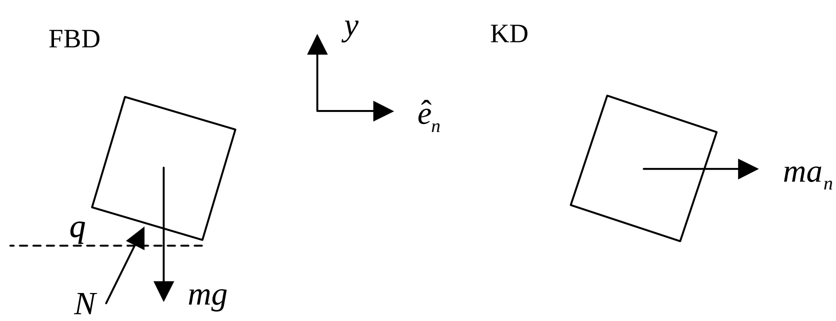
Since the radius is constant, $\dot{r} = 0$, thus

$$v = r\dot{q} \quad \Rightarrow \quad \dot{q} = \frac{v}{r} = \frac{222.2\, m/s}{5043m} = 0.044\, rad/s = 2.52\deg/s$$

ES204 Examination 1 **Problem 2** Dec 20, 1996

Solution strategy : This problem requires us to relate an angle to a velocity, thus we will probably need to use kinetics to relate the normal force, which is a function of the angle, to the acceleration and then use kinematics to relate the acceleration to the velocity.

System : the car - note that the friction force is zero



unk	eqs
N	(1)
è	(2)

COLM(RF) : $\frac{d\overline{P}_{sys}}{dt} = \sum \overline{F}$

normal direction : $$N \sin q = ma_n = m\frac{v^2}{r} \quad (1)$$

y direction : $$-mg + N\cos q = 0 \quad (2)$$

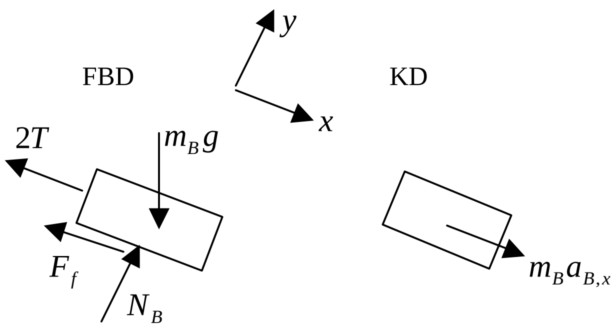
Solving in Maple (or by hand) :

$$\boxed{q = 27.37^\circ}$$

ES204 Examination 1 **Problem 3** Dec 19, 1997

Solution strategy : We need relate the forces and accelerations of both bodies. Thus we will probably need to use kinetics on both the block and the ball, use dependent motion to relate the accelerations, and use a polar CS for the ball for additional kinematic equations.

System B: the block



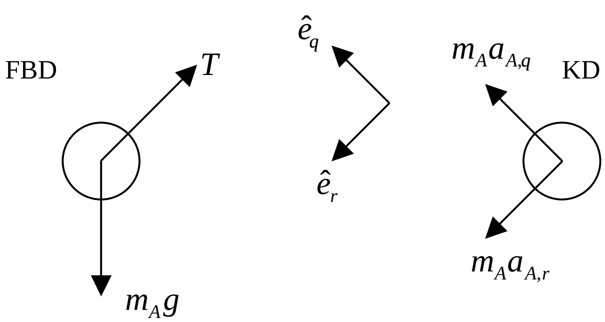
unk	eqs
$a_{B,x}$	(1)
N_B	(2)
T	(3)
F_f	(4)
$a_{A,r}$	(5)
$a_{A,è}$	(6)
$\ddot{r}$	(7)
$\dot{r}$	(8)
$\ddot{q}$	(9)

COLM(RF) : $\frac{d\overline{P}_{sys}}{dt} = \sum \overline{F}$

x direction : $$-2T - F_f + m_B g \sin 30 = m_B a_{B,x} \qquad (1)$$

y direction : $$N_B - m_B g \cos 30 = 0 \qquad (2)$$

System A : the ball



COLM(RF) : $\frac{d\overline{P}_{sys}}{dt} = \sum \overline{F}$

r direction : $$m_A g \cos q - T = m_A a_{A,r} \qquad (3)$$

q direction : $$-m_A g \sin q = m_A a_{A,q} \qquad (4)$$

We are still short an equation, let's now use kinematics :

Acceleration in Polar CS : $\bar{a} = \left(\ddot{r} - r\dot{q}\right)\hat{e}_r + \left(r\ddot{q} + 2\dot{r}\dot{q}\right)\hat{e}_q$

r component : $$a_r = \ddot{r} - r\dot{q} \qquad (5)$$

q component : $$a_q = r\ddot{q} + 2\dot{r}\dot{q} \qquad (6)$$

Dependent Motion : $L = r + 2x + C$

velocity : $$0 = \dot{r} + 2v_{B,x} \qquad (7)$$

acceleration : $$0 = \ddot{r} + 2a_{B,x} \qquad (8)$$

We still need one more equation, let's now pull out the constitutive model that relates friction force to the normal force

$$F_f = m_k N \qquad (9)$$