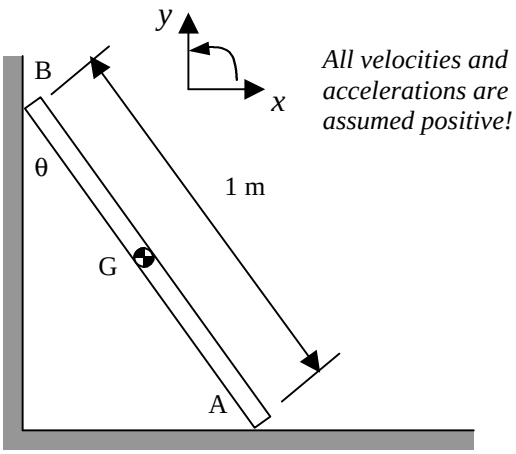
Quiz - Le 22

Name: _____



All velocities and accelerations are assumed positive!

1. Point A has a constant velocity of 10 m/s to the right. Determine the velocity and acceleration of point B when θ = 30°. Hint: Don't you dare say the acceleration of point B is zero!

Velocity Solution:

$$\bar{v}_B = \bar{v}_A + \bar{v}_{B/A} = \bar{v}_A + \bar{w}_{AB} \times \bar{r}_{B/A}$$

Expanding and equating components:

$$\begin{aligned} v_{B_x}\hat{i} + v_{B_y}\hat{j} &= v_{A_x}\hat{i} + v_{A_y}\hat{j} + w_{AB}\hat{k} \times \left(r_{B/A_x}\hat{i} + r_{B/A_y}\hat{j} \right) \\ &= v_{A_x}\hat{i} + v_{A_y}\hat{j} + w_{AB} r_{B/A_x}\hat{j} - w_{AB} r_{B/A_y}\hat{i} \end{aligned}$$

$$\begin{aligned} \hat{i}: &\quad v_{B_x} = v_{A_x} - w_{AB} r_{B/A_y} \\ \hat{j}: &\quad v_{B_y} = v_{A_y} + w_{AB} r_{B/A_x} \end{aligned} \qquad (1,2)$$

From the constrained motion and geometry, $v_{B_x} = v_{A_y} = 0$ and $\bar{r}_{B/A} = -0.5\hat{i} + 0.866\hat{j}$. Knowing that $v_{A_x} = 10$ allows us to solve (1,2) and get

$$\boxed{\bar{w}_{AB} = 11.54\hat{k}\, rad/s \quad \bar{v}_B = 0\hat{i} - 5.77\hat{j}\, m/s}$$

Acceleration Solution:

$$\bar{a}_B = \bar{a}_A + \bar{a}_{B/A} = \bar{a}_A + \bar{a}_{AB} \times \bar{r}_{B/A} - w_{AB}^2 \bar{r}_{B/A}$$

Expanding and equating components:

$$\begin{aligned} a_{B_x}\hat{i} + a_{B_y}\hat{j} &= a_{A_x}\hat{i} + a_{A_y}\hat{j} + a_{AB}\hat{k} \times \left(r_{B/A_x}\hat{i} + r_{B/A_y}\hat{j} \right) - w_{AB}^2 \left(r_{B/A_x}\hat{i} + r_{B/A_y}\hat{j} \right) \\ &= a_{A_x}\hat{i} + a_{A_y}\hat{j} + a_{AB} r_{B/A_x}\hat{j} - a_{AB} r_{B/A_y}\hat{i} - w_{AB}^2 r_{B/A_x}\hat{i} - w_{AB}^2 r_{B/A_y}\hat{j} \end{aligned}$$

$$\begin{aligned} \hat{i}: &\quad a_{B_x} = a_{A_x} - a_{AB} r_{B/A_y} - w_{AB}^2 r_{B/A_x} \\ \hat{j}: &\quad a_{B_y} = a_{A_y} + a_{AB} r_{B/A_x} - w_{AB}^2 r_{B/A_y} \end{aligned} \qquad (3,4)$$

From the constrained motion and geometry, $a_{B_x} = a_{A_y} = 0$ and $\bar{r}_{B/A} = -0.5\hat{i} + 0.866\hat{j}$. Knowing that $a_{A_x} = 0$ allows us to solve (3,4) and get

$$\boxed{\bar{a}_{AB} = 77\hat{k}\, rad/s^2 \quad \bar{a}_B = 0\hat{i} - 154\hat{j}\, m/s^2}$$